

Section 2.4 – Confidence Intervals

We are going to find out how to estimate the population mean from a sample.

Our estimate is not 100% certain. An error is involved. We can estimate this error and state an estimate of the population mean within a certain level of confidence.

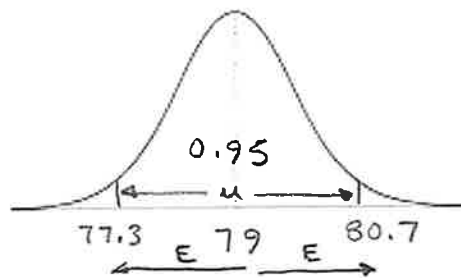
Let's say that we are interested in the average resting heart rate for women in beats per minute. We measure the heart rate for one hundred women.

The sample average is $\bar{x} = 79$

And the sample standard deviation is $s = 8.6$

If we were to take another sample, we would get a different mean, so this sample mean is an estimate

We can use this sample mean to estimate the population mean within a certain level of confidence



$79 - E$ } where E is the
 $79 + E$ } error within a
 } 95% confidence
 } level

$$\bar{x} - E < \mu < \bar{x} + E$$

We would say that we are 95% confident that the population mean is between 77.3 and 80.7 beats per minute. This is what we call a confidence interval.

We could also describe this confidence interval as the population mean is 79 bpm, within 1.7 bpm, 19 times out of 20.

To calculate this confidence interval, we use the Confidence Level Theorem

Confidence Level Theorem

$$\bar{x} - Z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < \bar{x} + Z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

where,

Note: a Sample Size of $n > 30$ is required for this formula

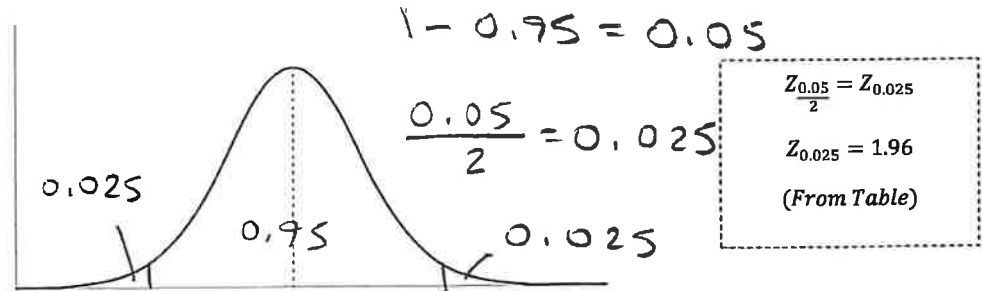
μ = population mean

\bar{x} = sample mean

$Z_{\frac{\alpha}{2}}$ = Standard Deviation Z-score, separated into two tails

n = sample size

s = Standard Deviation of the sample



$$\bar{x} = 79$$

$$n = 100$$

$$s = 8.6$$

$$\bar{x} - Z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} < \mu < \bar{x} + Z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

$$79 - (1.96) \frac{8.6}{\sqrt{100}} < \mu < 79 + (1.96) \frac{8.6}{\sqrt{100}}$$

$$79 - 1.6856 < \mu < 79 + 1.6856$$

$$77.3 < \mu < 80.7$$

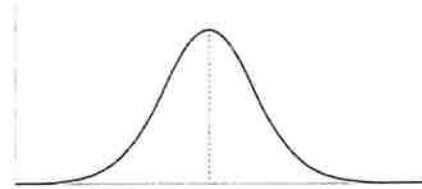
- With these results we can say that with 95% confidence that the population average is between 77.3 bpm and 80.7 bpm.

Or...

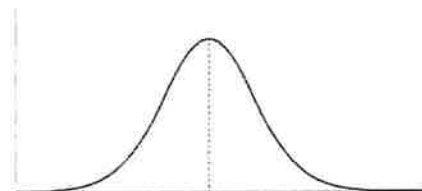
- The true mean of the population is 79 bpm, within 1.7 bpm, 19 times out of 20.

Section 2.4 – Practice Problems

1. A study of 50 English teachers found the average time spent marking a term paper was 15.2 minutes with a standard deviation of 2.8 minutes. Find a 95% confidence interval of the mean time for all term papers

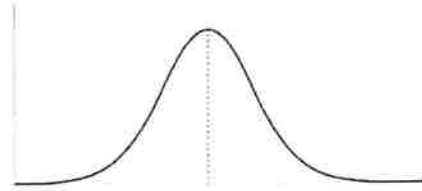


2. Forest companies bid on a large tract of land in the Prince George forest district. A random sample of 150 trees yields a mean diameter of 48 cm with a standard deviation of 5.6 cm. Find a 90% confidence interval for the mean diameter of all trees



Foundations of Math 11

3. A real estate firm in Winnipeg takes a random sample of 60 homes. This sample yield a mean of 1800 square feet of living space with a standard deviation of 280 square feet. Construct a 99% confidence level for the mean square footage of living space for all Winnipeg homes.



4. A high school teacher wishes to estimate the number of hours a student spends studying each week. The standard deviation from a previous study was 2.5 hours. How large a sample must be selected if the teacher wants to be 98% confident that the true mean differs from the sample mean by 0.75 hours?

