Section 2.3 – Practice Problems

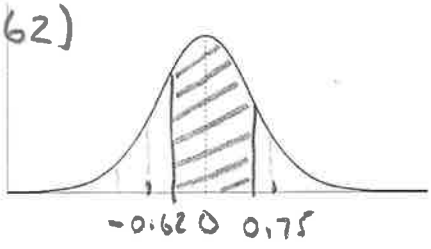
Find the Area under the Standard Normal Curve

1. Between $z = -0.62$ and $z = 0.75$

$P(-0.62 < z < 0.75) = P(z < 0.75) - P(z < -0.62)$

$= 0.7734 - 0.2676$

$= 0.5058$



2. Between $z = -2.35$ and $z = 1.42$

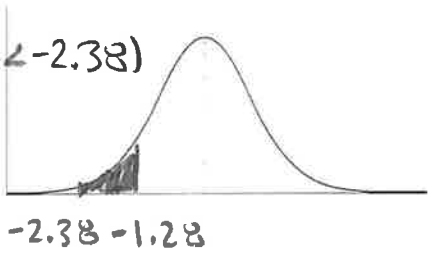
$P(-2.35 < z < 1.42) = P(z < 1.42) - P(z < -2.35)$

$= 0.9128$

3. Between $z = -1.42$ and $z = -2.38$

$P(-1.42 < z < -2.38) = P(z < -1.42) - P(z < -2.38)$

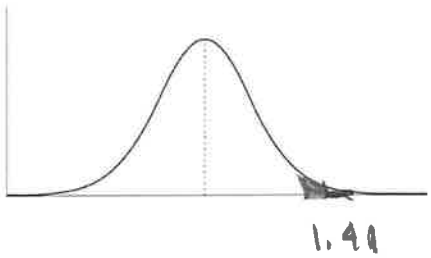
$= 0.06915$



4. To the right of $z = 1.46$

$P(z > 1.46) = 1 - P(z < 1.46)$

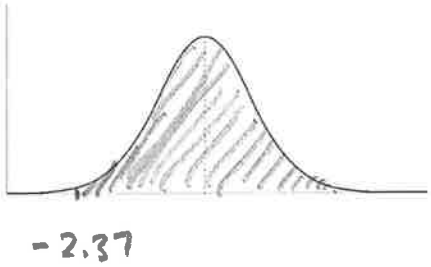
$= 0.0721$



5. To the right of $z = -2.37$

$P(z > -2.37) = 1 - P(z < -2.37)$

$= 0.9911$



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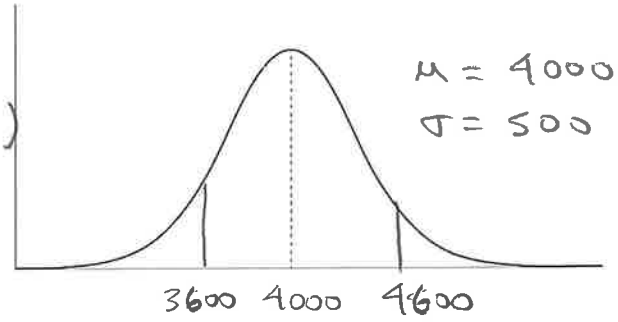
6. The attendance for a week at the local theatre is normally distributed with a mean of 4000 and a standard deviation of 500. What percent of attendance figures fall between 3600 and 4600?

$$P(3600 < X < 4600)$$
$$= P(X < 4600) - P(X < 3600)$$
$$= P\left(Z < \frac{4600-4000}{500}\right) - P\left(Z < \frac{3600-4000}{500}\right)$$

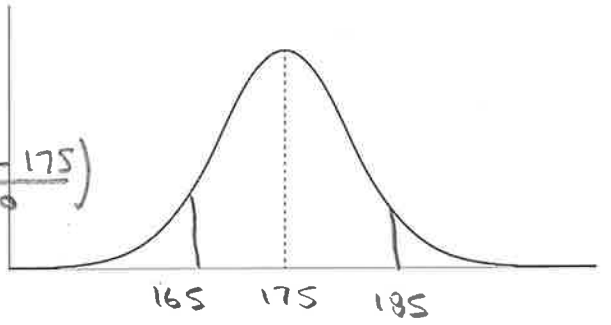


$$= P(Z < 1.2) - P(Z < -0.8) = 0.8849 - 0.2119$$
$$= 0.673$$

7. The average height of humans in Victoria is 175 cm with a standard deviation of 10 cm.

a) How many people would have a height between 160 cm and 180 cm?

$$P(160 < X < 180)$$
$$= P(X < 180) - P(X < 160)$$
$$= P\left(Z < \frac{180-175}{10}\right) - P\left(Z < \frac{160-175}{10}\right)$$
$$= P(Z < 0.5) - P(Z < -1.5)$$
$$= 0.6915 - 0.0668$$



$$= 0.6247$$

b) If you were to random choose a person, what is the probability that they have a height between 160 cm and 180 cm?

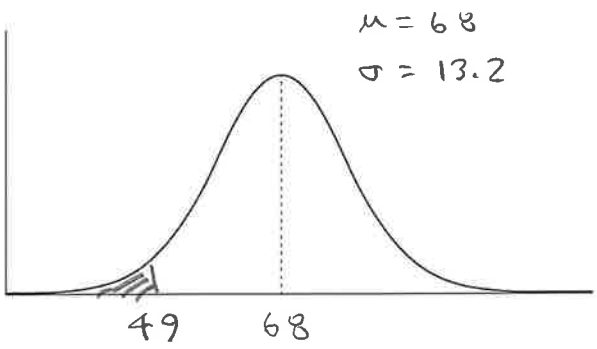
$$P(160 < X < 180) = 0.6247 \quad = 62.47\%$$

c) If the population of Victoria is 86 000, how many would you expect to have a height between 160 cm and 180 cm?

$$E(X) = n\,P(X)$$
$$E(160 < X < 180) = n\,P(160 < X < 180)$$
$$= 86000\,(0.6247)$$
$$= 53724$$

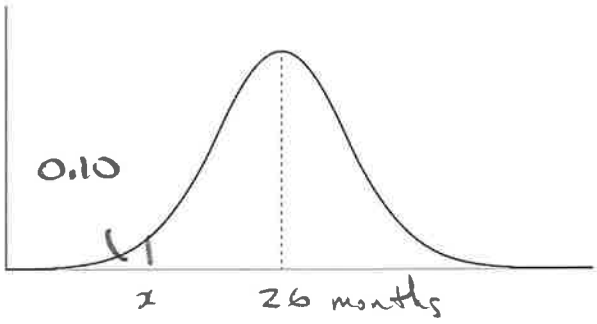
8. A provincial math exam has a mean of 68 and a standard deviation of 13.2. If 30000 students take the exam, and a score of 49 or less fails, how many students fail the exam?

$$P(x < 49) = P\left(z < \frac{49 - 68}{13.2}\right)$$
$$= P(z < -1.4391)$$
$$= 0.0749$$
$$E(x < 49) = n\ P(x < 49)$$
$$= 30000(0.0749)$$
$$= 2247$$



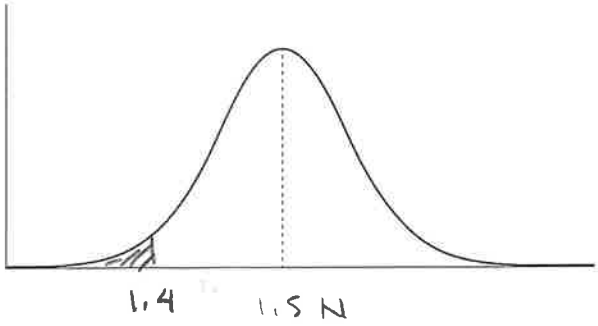
9. A manufacture of iPhone indicated a mean of 26 months before there is need of repairs with a standard deviation of 6 months. What length of time for the warranty should the manufacture set such that less that 10% of all iPhone will need repairs during this warranty period?

$$\frac{x - \mu}{\sigma} = z_{0.10}$$
$$\frac{x - 26}{6} = -1.29$$
$$x = 18.26 \text{ months}$$

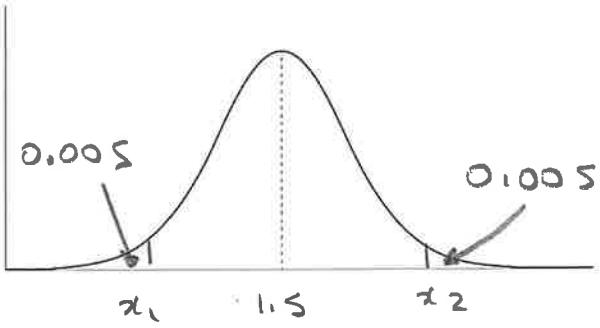


10. Nylon strands are manufactured to a mean tensile strength of 1.5 N, with a standard deviation of 0.04 N. If the tensile strength is normally distributed,
a) what percent of the strands would have a strength less than 1.4 N?

$$P(x < 1.4) = P\left(z < \frac{1.4 - 1.5}{0.04}\right)$$
$$= P(z < -2.5)$$
$$= 0.0062$$
$$0.62\%$$



b) what range of strengths, symmetrical about the mean, would you expect 99.0% of the strands to have?



$$\frac{x_1 - \mu}{\sigma} = z_{0.005}$$

$$\frac{x_1 - 1.5}{0.04} = -2.57$$

$$x_1 = 1.40$$

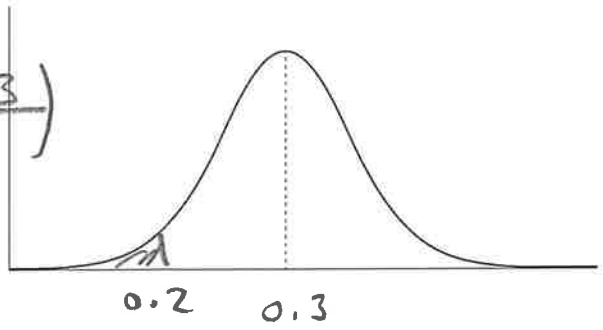
$$\frac{x_2 - \mu}{\sigma} = z_{0.995}$$

$$x_2 = 1.60$$

$$1.40 < x < 1.60$$

$$\frac{x_2 - 1.5}{0.04} = 2.57$$

11. The accuracy of an automatic pitching machine for batting practice is based on the off-line distance that a pitch is from a target line that is 30 m away. The off-line distance is normally distributed with a mean of 0.3 m and a standard deviation of 0.05 m. What percent of the pitches fall within 0.2 m of the target line?



$$P(x < 0.2) = P\left(z < \frac{0.2 - 0.3}{0.05}\right)$$

$$= P(z < -2)$$

$$= 0.02275$$

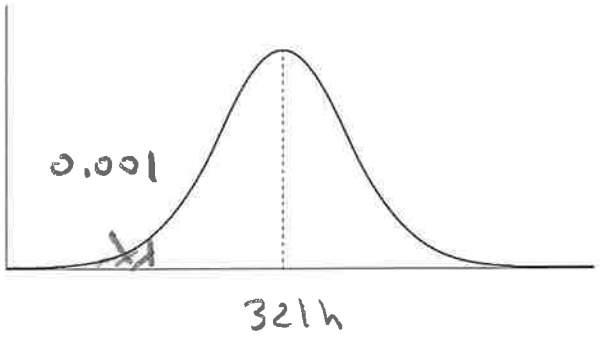
$$= 2.28\%$$

12. Major manufacturing companies operate on the principle of preventive maintenance to avoid a complete shutdown of the assembly line if a component fails. The lifetime of one component is normally distributed with a mean of 321 h an d a standard deviation of 23 h. How frequently should the component be replaced so that the probability of its failing during operation is less than 0.001?

$$\frac{x-\mu}{\sigma} = z_{0.001}$$

$$\frac{x-321}{23} = -3.1$$

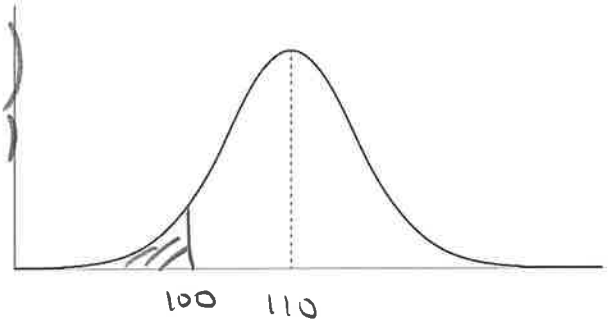
$$x = 249.7\text{ h}$$



13. The Trans-Canada highway stretches from St. Hon's to Victoria. On one section of the highway it has been found that motorists drive at speeds that are normally distributed with a mean of 110 km/h and a standard deviation of 16 km/h.

a) What percent of motorists are driving less than or at the posted speed limit of 100 km/h on this section?

$$P(x<100) = P\left(z < \frac{100-110}{16}\right)$$
$$= P(z < -0.625)$$
$$= 0.266$$
$$= 26.6\%$$

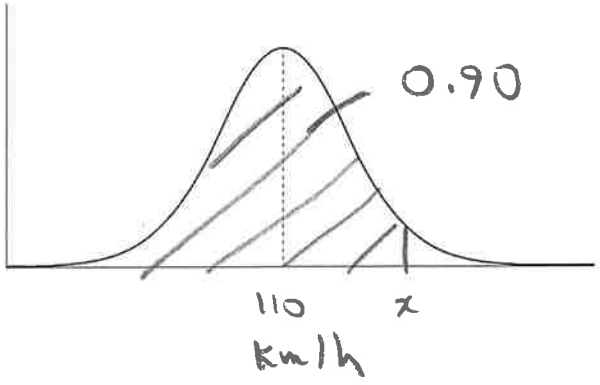


b) Below what speed do 90% of the motorists drive?

$$\frac{x-\mu}{\sigma} = z_{0.90}$$

$$\frac{x-110}{16} = 1.28$$

$$x = 130.5\text{ km/h}$$



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14. Students' marks on a test were normally distributed with a mean of 70 and a standard deviation of 8

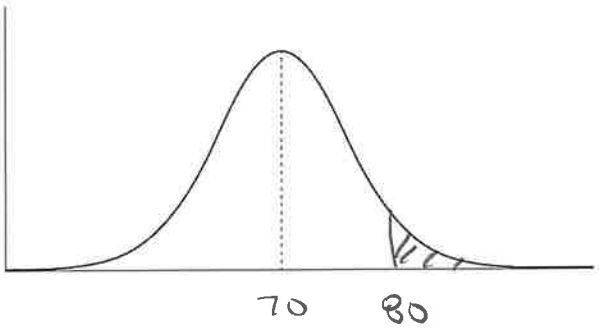
a) What percent of the students obtained a mark above 80?

$$P(X>80) = 1 - P(X<80)$$

$$= 1 - P\left(z < \frac{80-70}{8}\right)$$

$$= 1 - P(z < 1.25)$$

$$= 1 - 0.8944 = 0.1056 = 10.56\%$$



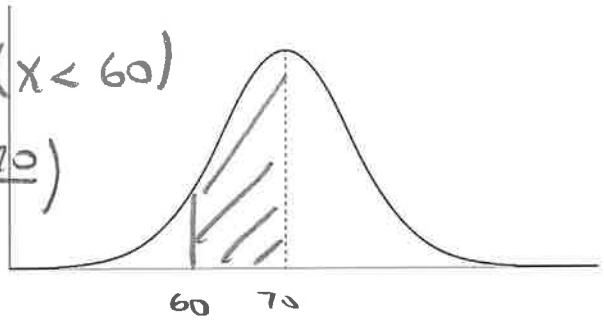
b) What percent of the students obtained a C grade (60 to 70)?

$$P(60 < X < 70) = P(X<70) - P(X<60)$$

$$= P\left(z < \frac{70-70}{8}\right) - P\left(z < \frac{60-70}{8}\right)$$

$$= P(z<0) - P(z<-1.25)$$

$$= 0.5000 - 0.1056 = 0.3944 = 39.44\%$$



c) Determine the mark under which 75% of the students' marks occur. This is referred to the 75th percentile

$$\frac{x-\mu}{\sigma} = z_{0.75}$$

$$\frac{x-70}{8} = 0.68$$

$$x = 75.4$$

75.4%