Section 2.3 – Practice Problems

1. What is the Domain of the following functions?

a) $f(x) = \dfrac{1}{x-2}$

Denominator cannot equal 0

$\boxed{x \neq 2}$

b) $f(x) = \dfrac{x-3}{x^2-9}$ $\rightarrow \dfrac{x-3}{(x+3)(x-3)}$

$\boxed{x \neq \pm 3}$

c) $f(x) = \sqrt{x+2}$

no negatives under the radical

$\boxed{x \geq -2}$

d) $f(x) = \sqrt{3-x}$

$3 - x \geq 0$

$\boxed{3 \geq x}$

e) $f(x) = \dfrac{1}{\sqrt{x}}$

no negatives and no zero denominator

$\boxed{x > 0}$

f) $f(x) = \sqrt{x^2-1}$

no negatives under radical

$x^2 - 1 \geq 0$

$x^2 \geq 1$

✓ ✗ ✓

-1 1

$\boxed{x \geq 1}$ or $\boxed{x \leq -1}$

g) $f(x) = \sqrt{1-x^2}$

no negatives

$1 - x^2 \geq 0$

$1 \geq x^2$

$\pm 1 \geq x$

✗ ✓ ✗

-1 1

$\boxed{-1 \leq x \leq 1}$

h) $f(x) = \sqrt{x(x-2)}$ 2 cases

if $x \geq 2$

always positive

$\boxed{x \geq 2}$

$x \leq 0$

neg(neg) positive

$\boxed{x \leq 0}$

2. Let $f(x) = 2x^2 - 3x + 1$, $g(x) = x + 1$, $h(x) = 5$, $j(x) = \dfrac{x-1}{x+1}$

Evaluate the following

a) $(f \circ g)(2)$

$g(2) = 2 + 1$
$= 3$

$f(3) = 2(3)^2 - 3(3) + 1$
$= 18 - 9 + 1 \rightarrow \boxed{10}$

b) $(h \circ j)(-3)$

$j(-3) = \dfrac{-3-1}{-3+1} = \dfrac{-4}{-2} = 2$

$h(2) = 5$

$\boxed{5}$

c) $(j \circ h)(2)$

$h(2) = 5$

$j(5) = \frac{5-1}{5+1} = \frac{4}{6} = \boxed{\frac{2}{3}}$

d) $(j \circ g)(0)$

$g(0) = 0+1$
$= 1$

$j(1) = \frac{1-1}{1+1} = \frac{0}{2}$

$j(1) = \boxed{0}$

e) $(h \circ j)(-1)$

$j(-1) = \frac{-1-1}{-1+1}$

$= \frac{-2}{0}$

$= \text{undefined}$

therefore;

$(h \circ j)(-1)$ is $\boxed{\text{undefined}}$

f) $(f \circ j)(3)$

$j(3) = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$

$f(\frac{1}{2}) = 2(\frac{1}{2})^2 - 3(\frac{1}{2}) + 1$

$= 2(\frac{1}{4}) - \frac{3}{2} + 1$

$= \frac{1}{2} - \frac{3}{2} + 1 \rightarrow -1+1 = \boxed{0}$

g) $(h \circ g \circ g)(2)$

$g(2) = 2+1$
$= 3$

$g(3) = 3+1$
$= 4$

$h(4) = 5$

$\boxed{5}$

h) $(f \circ f \circ f)(-1)$

$f(-1) = 2(-1)^2 - 3(-1) + 1$
$= 2+3+1 = 6$

$f(6) = 2(6)^2 - 3(6) + 1 \rightarrow 72 - 18 + 1 = 55$

$f(55) = 2(55)^2 - 3(55) + 1 = \boxed{5886}$

i) $(j \circ h \circ g)(-3)$

$g(-3) = -3+1$
$= -2$

$h(-2) = 5$

$j(5) = \frac{5-1}{5+1} = \frac{4}{6} \rightarrow \boxed{\frac{2}{3}}$

j) $(g \circ j \circ f)(4)$

$f(4) = 2(4)^2 - 3(4) + 1$
$= 32 - 12 + 1 \rightarrow 21$

$j(21) = \frac{21-1}{21+1} = \frac{20}{22} = \frac{10}{11}$

$g(\frac{10}{11}) = \frac{10}{11} + 1$

$\frac{10}{11} + \frac{11}{11} = \boxed{\frac{21}{11}}$

k) $(f \circ h \circ j)(2)$

$j(2) = \frac{2-1}{2+1} = \frac{1}{3}$

$h(\frac{1}{3}) = 5$

$f(5) = 2(5)^2 - 3(5) + 1$
$= 50 - 15 + 1 \Rightarrow \boxed{36}$

l) $(j \circ j \circ g \circ f)(-2)$

$f(-2) = 2(-2)^2 - 3(-2) + 1$
$= 8+6+1 = 15$

$g(15) = 15+1$
$= 16$

$j(16) = \frac{16-1}{16+1} = \frac{15}{17}$

$j(\frac{15}{17}) = \frac{\frac{15}{17} - 1}{\frac{15}{17} + 1} \rightarrow \frac{\frac{15-17}{17}}{\frac{15+17}{17}} \rightarrow \frac{-2}{32} = \boxed{-\frac{1}{16}}$

3. Use f and g by the following table of values to evaluate the following:

x	-2	0	3	7
$f(x)$	0	1	4	6

x	-1	1	4	6
$g(x)$	3	2	-2	-4

a) $f(0)$ $\boxed{1}$

b) $g(1)$ $\boxed{2}$

c) $(f \circ g)(-1)$ $f(3) = \boxed{4}$

d) $(f \circ g)(4)$ $f(-2) = \boxed{0}$

e) $(g \circ f)(0)$ $g(1) = \boxed{2}$

f) $(g \circ f)(7)$ $g(6) = \boxed{-4}$

g) $(f \circ g)(1)$ $f(2) =$ undefined

h) $(g \circ f)(-2)$ $g(0) =$ undefined

4. For each pair of functions, find $(f \circ g)(x)$ and $(g \circ f)(x)$. State the Domain of the result

a) $f(x) = \sqrt{x+2}$ and $g(x) = 2x - 3$

$(f \circ g)(x) \rightarrow \sqrt{2x-3+2} = \boxed{\sqrt{2x-1}}$

$\boxed{D: x \geq \frac{1}{2}}$

$(g \circ f)(x) \rightarrow \boxed{2\sqrt{x+2} - 3}$

$\boxed{D: x \geq -2}$

b) $f(x) = \dfrac{2}{x}$ and $g(x) = \dfrac{x}{x-1}$

$(f \circ g)(x) = \dfrac{2}{\frac{x}{x-1}} \rightarrow \boxed{\dfrac{2(x-1)}{x}}$

$\boxed{D: x \neq 0, \; x \neq 1}$

$(g \circ f)(x) = \dfrac{\frac{2}{x}}{\frac{2}{x} - 1} \rightarrow \dfrac{\frac{2}{x}}{\frac{2-x}{x}} \rightarrow \dfrac{2x}{x(2-x)}$

$\boxed{\dfrac{2}{2-x}}$

$\boxed{x \neq 2, \; x \neq 0}$

c) $f(x) = x^2 - 2x$ and $g(x) = x + 3$

$(f \circ g)(x) = (x+3)^2 - 2(x+3)$

$= x^2 + 6x + 9 - 2x - 6$

$= \boxed{x^2 + 4x + 3}$ $\boxed{D: \mathbb{R}}$

$(g \circ f)(x) = \boxed{x^2 - 2x + 3}$ $\boxed{D: \mathbb{R}}$

d) $f(x) = x^2 + x - 3$ and $g(x) = x + 2$

$(f \circ g)(x) = (x+2)^2 + x + 2 - 3$

$= x^2 + 4x + 4 + x + 2 - 3$

$= \boxed{x^2 + 5x + 3}$ $\boxed{D: \mathbb{R}}$

$(g \circ f)(x) = x^2 + x - 3 + 2$

$= \boxed{x^2 + x - 1}$ $\boxed{D: \mathbb{R}}$

e) $f(x) = \sqrt{x - 2}$ and $g(x) = 3x + 2$

$(f \circ g)(x) \rightarrow \sqrt{3x + 2 - 2}$

$= \sqrt{3x}$ $\boxed{D: x \geq 0}$

$(g \circ f)(x) = \boxed{3\sqrt{x-2} + 2}$

$\boxed{D: x \geq 2}$

f) $f(x) = |x| - 3$ and $g(x) = -2x + 3$

$(f \circ g)(x) = \boxed{|-2x+3| - 3}$

$\boxed{D: \mathbb{R}}$

$(g \circ f)(x) = -2[|x| - 3] + 3$

$= -2|x| + 6 + 3$

$= \boxed{-2|x| + 9}$

$\boxed{D: \mathbb{R}}$

g) $f(x) = \frac{3}{x}$ and $g(x) = \frac{1}{x-4}$

$(f \circ g)(x) = \dfrac{3}{\frac{1}{x-4}} \rightarrow 3(x-4)$

$= \boxed{3x-12}$ $\boxed{D: x \neq 4}$

$(g \circ f)(x) = \dfrac{1}{\frac{3}{x}-4} \rightarrow \dfrac{1}{\frac{3-4x}{x}}$

$\boxed{\dfrac{x}{3-4x}}$ $\boxed{D: x \neq 0, \; x \neq 3/4}$

h) $f(x) = |x-2| - 3$ and $g(x) = \frac{1}{x}$

$(f \circ g)(x) = \left|\frac{1}{x} - 2\right| - 3$

$= \boxed{\left|\dfrac{1-2x}{x}\right| - 3}$ $\boxed{D: x \neq 0}$

$(g \circ f)(x) = \boxed{\dfrac{1}{|x-2|-3}}$

$|x-2|-3=0$ when $|x-2| = \pm 3$

$x = 5$ or $x = -1$

$\boxed{D: x \neq 5, \; x \neq -1}$

5. Find two functions $f(x)$ and $g(x)$ such that $h(x) = (f \circ g)(x)$. Answers may vary.

a) $h(x) = (2x-3)^2$ ← outer function

↑ inner function

let $g(x) = 2x-3$

let $f(x) = x^2$

b) $h(x) = \sqrt[3]{3x^2-2}$ ← outer

↑ inner

Let $g(x) = 3x^2-2$

Let $f(x) = \sqrt[3]{x}$

c) $h(x) = \frac{1}{3x-4}$

inner

Let $g(x) = 3x-4$

Let $f(x) = \frac{1}{x}$

d) $h(x) = \frac{2}{x^2+4}$

inner

Let $g(x) = x^2+4$

Let $f(x) = \frac{2}{x}$

e) $h(x) = \sqrt{\underbrace{x^2+1}_{\text{inner}}} + 3$

Let $g(x) = x^2+1$

Let $f(x) = \sqrt{x} + 3$

f) $h(x) = \sqrt[3]{3x+4} - 1$

inner

Let $g(x) = 3x+4$

Let $f(x) = \sqrt[3]{x} - 1$

g) $h(x) = 3\underbrace{(2x-3)}_{\text{inner}}^4 - \underbrace{(2x-3)}_{\text{inner}}^7$

Let $g(x) = 2x-3$

Let $f(x) = 3x^4 - x^7$

h) $h(x) = 3\underbrace{(2x+4)}_{\text{inner}}^3 + 2\underbrace{(2x+4)}_{\text{inner}}^6$

Let $g(x) = 2x+4$

Let $f(x) = 3x^3 + 2x^6$

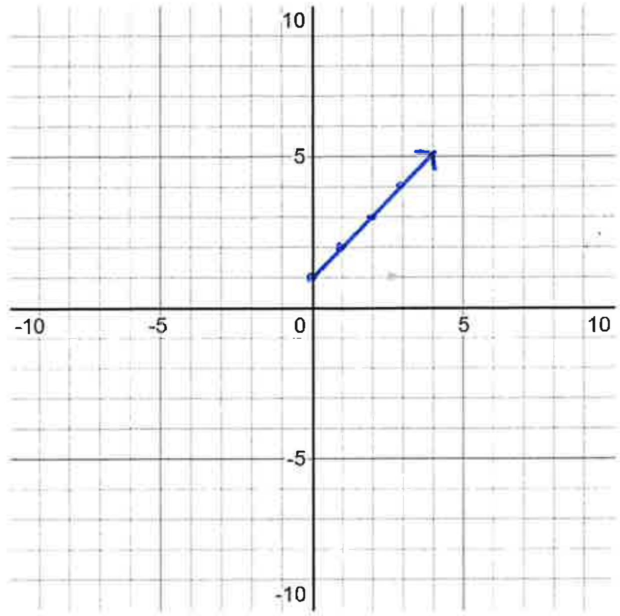
6. Sketch the graph of the $(f \circ g)(x)$ for the following. State the Domain.

a) $f(x) = x^2 + 1$ and $g(x) = \sqrt{x}$

$$(f \circ g)(x) = \sqrt{x}^2 + 1$$

$$= x + 1$$

but $g(x)$ has Domain: $x \geq 0$



Domain: $x \geq 0$

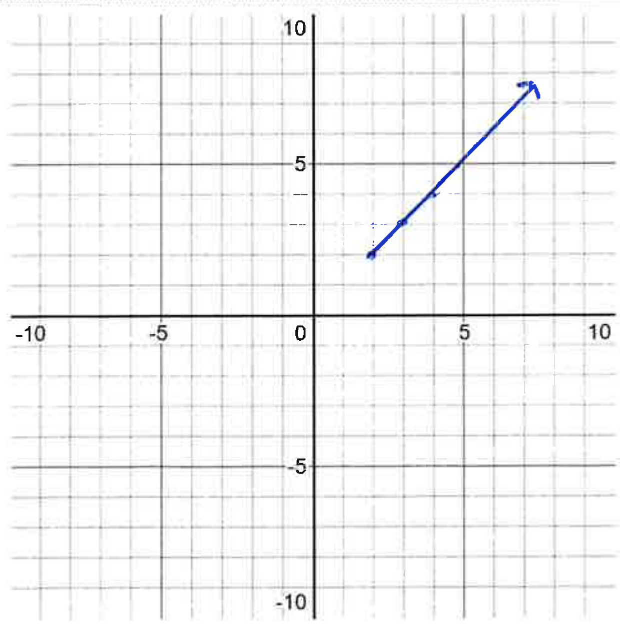
b) $f(x) = x^2 + 2$ and $g(x) = \sqrt{x - 2}$

$$(f \circ g)(x) = \sqrt{x-2}^2 + 2$$

$$= x - 2 + 2$$

$$= x$$

but $g(x)$ has Domain: $x \geq 2$



Domain: $x \geq 2$

Sketch the graph of the $(f \circ g)(x)$ for the following. State the Domain.

c) $f(x) = 1 - x^2$ and $g(x) = \sqrt{1-x}$

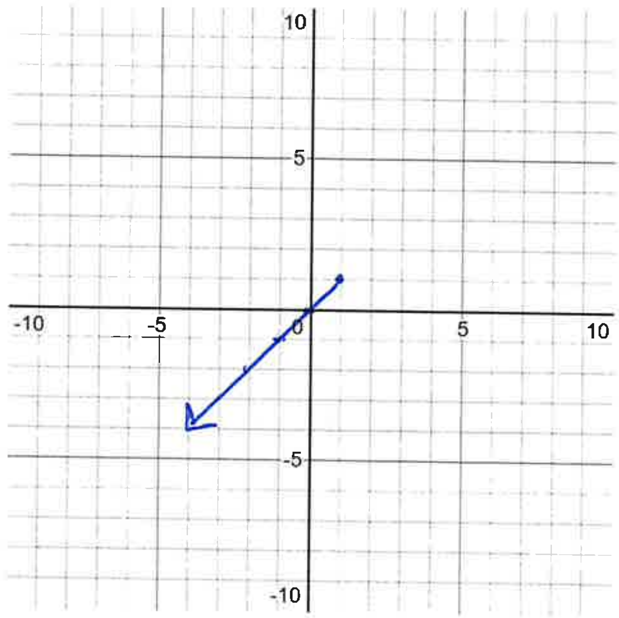
$(f \circ g)(x) = 1 - \sqrt{1-x}^{\,2}$

$= 1 - (1 - x)$

$= 1 - 1 + x$

$= x$

$g(x)$ has $D: x \leq 1$



Domain: $x \leq 1$

d) $f(x) = x^2 + 1$ and $g(x) = \sqrt{4 - x^2}$

$(f \circ g)(x) = \sqrt{4-x^2}^{\,2} + 1$

$= 4 - x^2 + 1$

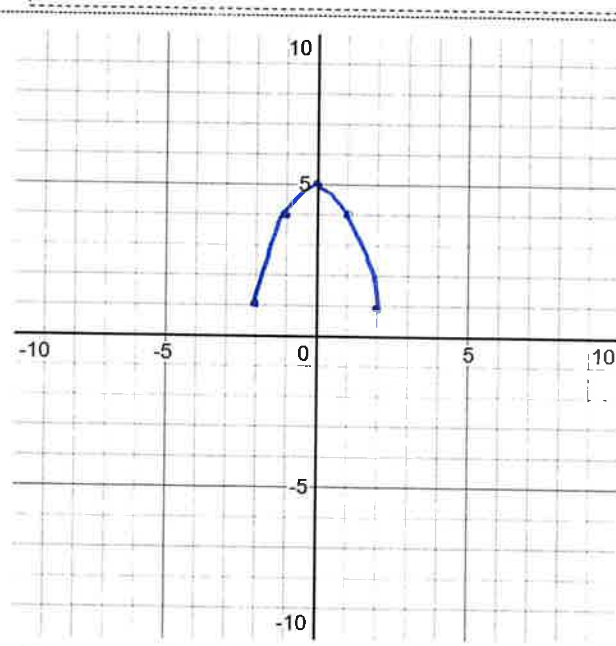
$= -x^2 + 5$

$= -(x^2 - 5)$

$g(x)$ has Domain: $4 - x^2 \geq 0$

$4 \geq x^2$

$-2 \leq x \leq 2$



Domain: $-2 \leq x \leq 2$

7. The first of the two graphs shows two functions f and g. The second shows two functions h and k. Use the graphs to compute the following:

a) $(g \circ f)(-4) =$ $g(0) = -3$

b) $(f \circ g)(3) =$ $f(0) = 2$

c) $(f \circ f)(-2) =$ $f(3) = 1$

d) $(g \circ g)(3) =$ $g(0) = -3$

e) $(g \circ f)(-5) =$ $g(-3) = -5$

f) $(g \circ f)(-3) =$ $g(\frac{3}{2}) = -3/2$

g) $(h \circ k)(0) =$ $h(5) = 4$

h) $(h \circ k)(-1) =$ $h(3) = 3.5$

i) $(h \circ k)(2) =$ $h(0) = 2$

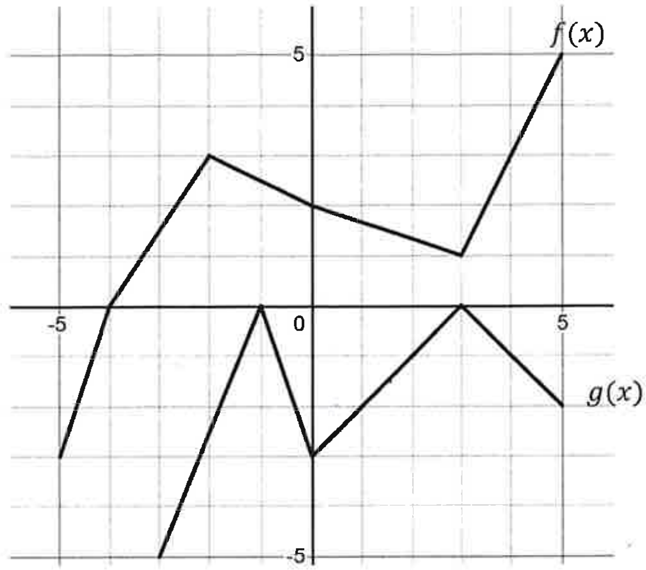
j) $(h \circ k)(-3) =$ $h(-5) = -\frac{1}{2}$

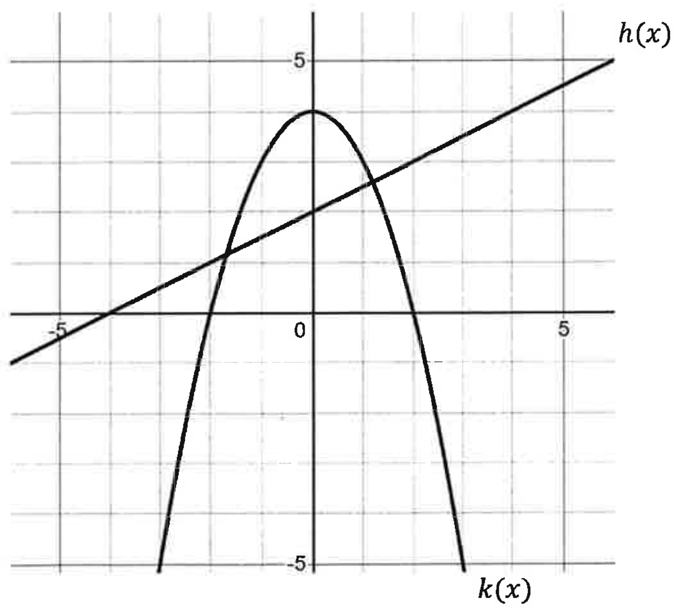
k) $(k \circ h)(0) =$ $k(2) = 0$

l) $(k \circ h)(2) =$ $k(3) = -5$

m) $(k \circ h)(-4) =$ $k(0) = 4$

n) $(k \circ h)(-2) =$ $k(-1) = 3$





8. If $f = \{(3,4),(4,5),(5,6),(6,7)\}$ and $g = \{(5,3),(6,4),(7,-2),\ (8,0)\}$, determine:

a) $(f \circ g)(x)$

$f(g(5)) = f(3) = 4 \quad (5,4)$

$f(g(6)) = f(4) = 5 \quad (6,5)$

$f(g(7)) = f(-2) \rightarrow \text{undefined}$

$f(g(8)) = f(0) \rightarrow \text{undefined}$

$(f \circ g)(x) = \{(5,4),(6,5)\}$

b) $(g \circ f)(x)$

$g(f(3)) = g(4) = \text{undefined}$

$g(f(4)) = g(5) = 3 \quad (4,3)$

$g(f(5)) = g(6) = 4 \quad (5,4)$

$g(f(6)) = g(7) = -2 \quad (6,-2)$

$(g \circ f)(x) = \{(4,3),(5,4),(6,-2)\}$

9. If $f(x) = 3x - 2$ *and* $g(x) = 3x + b$, find b such that $(f \circ g)(x) = (g \circ f)(x)$ for all real numbers x.

$f(x) = 3x - 2$

$g(x) = 3x + b$

$(f \circ g)(x) = 3(3x+b) - 2$

$= 9x + 3b - 2$

$(g \circ f)(x) = 3(3x-2) + b$

$= 9x - 6 + b$

if $(f \circ g)(x) = (g \circ f)(x)$

$9x + 3b - 2 = 9x - 6 + b$

$3b - 2 = -6 + b$

$-3b \quad +6 \qquad +6 \quad -3b$

$4 = -2b$

$\boxed{b = -2}$

10. Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ $h \neq 0$ for the given function f

a) $f(x) = 2x + 3$

$f(x+h) = 2(x+h) + 3 \rightarrow 2x + 2h + 3$

so $\dfrac{f(x+h) - f(x)}{h}$

$\dfrac{2x+2h+3-[2x+3]}{h}$

$\dfrac{2x+2h+3-2x-3}{h} \rightarrow \dfrac{2h}{h} = \boxed{2}$

b) $f(x) = x^2 + x$

$f(x+h) = (x+h)^2 + x + h$

$= x^2 + 2xh + h^2 + x + h$

$\dfrac{f(x+h) - f(x)}{h} \rightarrow \dfrac{x^2 + 2xh + h^2 + x + h - [x^2 + x]}{h}$

$\dfrac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$

$\dfrac{2xh + h^2 + h}{h} \rightarrow \boxed{2x + h + 1}$

c) $f(x) = -3x^2 + 2x$

$f(x+h) = -3(x+h)^2 + 2(x+h)$

$= -3(x^2 + 2xh + h^2) + 2x + 2h$

$= -3x^2 - 6xh - 3h^2 + 2x + 2h$

$\dfrac{-3x^2 - 6xh - 3h^2 + 2x + 2h - (-3x^2 + 2x)}{h}$

$\dfrac{-3x^2 - 6xh - 3h^2 + 2x + 2h + 3x^2 - 2x}{h}$

$\dfrac{-6xh - 3h^2 + 2h}{h} = \boxed{-6x - 3h + 2}$

d) $f(x) = \dfrac{1}{x}$

$f(x+h) = \dfrac{1}{x+h}$

$\dfrac{\frac{1}{x+h} - \frac{1}{x}}{h} \rightarrow \dfrac{\frac{x}{x(x+h)} - \frac{(x+h)}{x(x+h)}}{h}$

$\dfrac{\frac{x - x - h}{x(x+h)}}{h} = \dfrac{\frac{-h}{x(x+h)}}{h}$

$\dfrac{-h}{hx(x+h)} = \boxed{\dfrac{-1}{x(x+h)}}$

e) $f(x) = \dfrac{4}{2x-1}$

$$f(x+h) = \frac{4}{2(x+h)-1} = \frac{4}{2x+2h-1}$$

$$\frac{\frac{4}{(2x+2h-1)} - \frac{4}{(2x-1)}}{h}$$

$$\frac{\frac{4(2x-1)-4(2x+2h-1)}{(2x+2h-1)(2x-1)}}{h}$$

$$\frac{8x-4-8x-8h+4}{h(2x+2h-1)(2x-1)}$$

$$\frac{-8h}{h(2x+2h-1)(2x-1)}$$

$$\boxed{\frac{-8}{(2x+2h-1)(2x-1)}}$$

f) $f(x) = \dfrac{1}{\sqrt{x}}$

$$f(x+h) = \frac{1}{\sqrt{x+h}}$$

$$\frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$\frac{\frac{\sqrt{x}-\sqrt{x+h}}{(\sqrt{x+h})(\sqrt{x})}}{h}$$

Radicalize the numerator

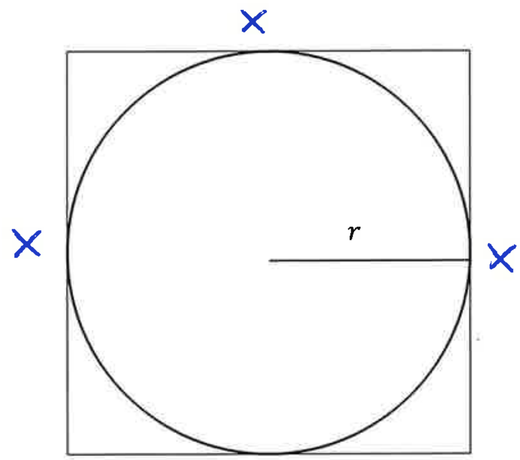
$$\frac{(\sqrt{x}-\sqrt{x+h})}{h\sqrt{x+h}\sqrt{x}} \cdot \frac{(\sqrt{x}+\sqrt{x+h})}{(\sqrt{x}+\sqrt{x+h})}$$

$$\frac{x + \cancel{\sqrt{x}\sqrt{x+h}} - \cancel{\sqrt{x}\sqrt{x+h}} - (x+h)}{h\sqrt{x+h}\sqrt{x}\sqrt{x} + h\sqrt{x+h}\sqrt{x}\sqrt{x+h}}$$

$$\frac{-h}{hx\sqrt{h+x} + h\sqrt{x}(x+h)}$$

$$\boxed{\frac{-1}{x\sqrt{x+h} + \sqrt{x}(x+h)}}$$

11. A circle inscribed in a square.



a) Write the radius of the circle as a function of the length x of the sides of the square.

$r = \frac{1}{2}d$ but $d = x$

so $\boxed{r = \frac{1}{2}x}$

b) Write the area A of the circle as a function of the radius.

$A = \pi r^2$

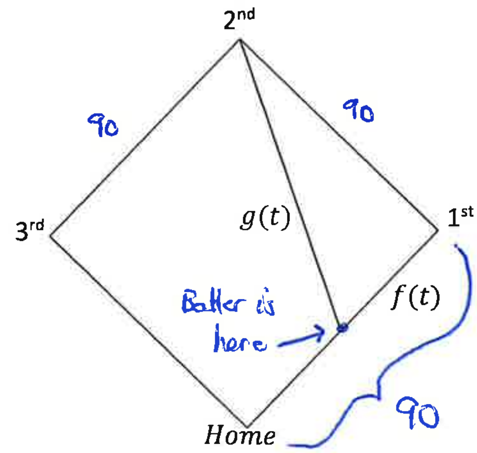
c) Find $(A \circ r)(x)$.

$r(x) = \frac{1}{2}x$

$A(r(x)) = \pi\left(\frac{1}{2}x\right)^2$

$= \boxed{\frac{\pi x^2}{4}}$

12. A baseball diamond is a square $90ft$ on each side. A batter is running to first base at a rate of $27\ ft/sec$



a) Find the function $f(t)$ for the distance x of the batter from first base in terms of time t

$f(t) = 90 - 27t$

* Since he's running at 27 ft/sec the t gives a time that will determine distance travelled

b) Find a function $g(f)$ for the distance the batter is from second base in terms of the distance f

use Pythagorean Theorem

$g(t)^2 = 90^2 + f(t)^2$

$(g \circ f)(t) = g(f)$

so

becomes $g(f)^2 = 90^2 + f(t)^2$

$\boxed{(g \circ f)(t) = \sqrt{90^2 + (90 - 27t)^2}}$

c) Find $(g \circ f)(t)$ and explain the meaning of the function.

$(g \circ f)(t) = \sqrt{\underbrace{90^2}_{\text{distance to second from } 1^{st}} + \underbrace{(90 - 27t)^2}_{\text{changing distance from home to } 1^{st}}}$

So overall we get the distance from home to second as the runner runs around the bases.

See Website for Detailed Answer Key

Extra Work Space