Section 2.2 – Standard Deviation

Consider this:

- There are 30 students in a math and science class and the mean of a test in Math and in Science is 60%. If Dawson scored 75% in Math and 80% in Science, in which class did he do better compared to the other students?

- The answer isn't easy to determine because you don't know the spread of the test scores with respect to the mean in the two classes.

- That spread is what is known as:

STANDARD DEVIATION

- The standard deviation is based on the deviations from the mean.
  - First we square the difference between each value and the mean (squaring eliminates negatives)
  - Then total the squared deviations and divide them by the number of values
  - Then take the square root of the value and you get the desired Standard Deviation

Standard Deviation Definition

$\sigma = \text{Greek symbol sigma for standard deviation of a population}$

$$\sigma = \sqrt{\frac{\text{sum of the squares of the differences from the mean}}{\text{number of values}}}$$

$$\sigma = \sqrt{\frac{(x_1-\mu)^2+(x_2-\mu)^2+\cdots+(x_n-\mu)^2}{n}} \quad (\textit{Basic Formula})$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2} \quad (\textit{Summation Notation})$$

- As you can imagine, this is a tedious process on a regular scientific calculator
- We will try a few to see if we can manage and comprehend the concepts
- But will also explore the STATS function of the calculator and how it simplifies the input of data

Example: Calculate the Standard Deviation from the following set of values:

a) 5, 6, 7, 8, 9 b) 3, 5, 7, 9, 11

Solution: Using the formula given above

a) First find the mean:

$$\mu = \frac{5+6+7+8+9}{5} = 7$$

$$\sigma = \sqrt{\frac{(5-7)^2+(6-7)^2+(7-7)^2+(8-7)^2+(9-7)^2}{5}}$$

$$\sigma = \sqrt{\frac{4+1+0+1+4}{5}} = \sqrt{\frac{10}{5}} = \sqrt{2}$$

b) First find the mean:

$$\mu = \frac{3+5+7+9+11}{5} = 7$$

$$\sigma = \sqrt{\frac{(3-7)^2+(5-7)^2+(7-7)^2+(9-7)^2+(11-7)^2}{5}}$$

$$\sigma = \sqrt{\frac{16+4+0+4+16}{5}} = \sqrt{\frac{40}{5}} = \sqrt{8} = 2\sqrt{2}$$

- Notice that (3, 5, 7, 9, 11) are **two units apart** compared to (5, 6, 7, 8, 9) which are **one unit apart**
- So it only makes intuitive sense that the **Standard Deviation is twice as large** for the second set of data

A **Small Standard Deviation** means:

The measures are **clustered close to the mean**

A **High Standard Deviation** means:

The measures are **widely scattered from the mean**

Example 2: Calculate the Standard Deviation for the following set of data:

a)

	Daily Commute time in Minutes	Number of Employees
5	0 to less than 10	4
15	10 to less than 20	9
25	20 to less than 30	6
35	30 to less than 40	4
45	40 to less than 50	2
	Total:	25

b)

	Number of Orders	Number of Days
11	10 – 12	4
14	13 – 15	12
17	16 – 18	20
20	19 – 21	14
	Total:	50

Solution 2:

a) First find the mean:

$$\mu = \frac{4(5) + 9(15) + 6(25) + 4(35) + 2(45)}{25} = \frac{535}{25} = 21.4$$

$$\sigma = \sqrt{\frac{4(5-21.4)^2 + 9(15-21.4)^2 + 6(25-21.4)^2 + 4(35-21.4)^2 + 2(45-21.4)^2}{25}}$$

$\sigma = 11.6207$

b) First find the mean:

$$\mu = \frac{4(11) + 12(14) + 20(17) + 14(20)}{50} = \frac{832}{50} = 16.64$$

$$\sigma = \sqrt{\frac{[4(11-16.64)^2 + 12(14-16.64)^2 + 20(17-16.64)^2 + 14(20-16.64)^2]}{50}}$$

$\sigma = 2.7259$

Section 2.2 – Practice Questions

1. The Value of Standard Deviation is:

a) Never Negative
b) Never Positive
c) Never Zero

Why?

2. Find the Standard Deviation of: 2, 3, 5, 6, 9

3. Find the Standard Deviation

Score	Frequency
1	1
2	3
3	5
4	4
5	2

4. Find the Standard Deviation

Score	Frequency
$0 \leq x < 10$	1
$10 \leq x < 20$	4
$20 \leq x < 30$	3
$30 \leq x < 40$	2

5. What do each of the following signify?
   a) A small Standard Deviation

   b) A large Standard Deviation

   c) A Standard Deviation of Zero