Section 2.2 – Simplifying Radicals

This booklet belongs to:____________________________Block: ______

- Remember: **Radicals** can be written as **fractional exponents**! (Flower Power)

Examples: $\sqrt{2} = 2^{\frac{1}{2}}$ $\sqrt[4]{x} = x^{\frac{1}{4}}$ $\sqrt[n]{a} = a^{\frac{1}{n}},\ n\ is\ positive$

- There are 3 Important Relationships of Radicals to remember and understand

1. $\sqrt[n]{a^n} = a, \quad a \geq 0, because\ \sqrt[n]{a^n} = (a^n)^{\frac{1}{n}} = a$ (This is also true for $a < 0$ when n is odd)

2. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}, \quad a, b \geq 0, because\ \sqrt[n]{ab} = (ab)^{\frac{1}{n}} = (a)^{\frac{1}{n}} \cdot (b)^{\frac{1}{n}} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

3. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, \quad a \geq 0, b > 0, because\ \sqrt[n]{\frac{a}{b}} = \left(\frac{a}{b}\right)^{\frac{1}{n}} = \frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Simplifying Expressions with Radicals

- A radical is **simplified** when **no perfect square factor** is still **under the radical sign**

Example: Simplify $\sqrt{20}$

Solution: There is a perfect square that is a factor of 20. It's 4!

$$\sqrt{20} = \sqrt{4 \cdot 5} \quad \rightarrow \quad \sqrt{4} \cdot \sqrt{5} = \mathbf{2\sqrt{5}}$$

- An alternate approach is to **break $\mathbf{20}$** down into prime **factors**
  - Then look for **pairs of factors**
  - Each **pair of like factors** produces **one factor outside** the radical.

$$\sqrt{20} = \sqrt{2 \cdot 2 \cdot 5} \quad \rightarrow \quad \sqrt{2 \cdot 2} \cdot \sqrt{5} = \mathbf{2\sqrt{5}}$$

One pair inside becomes one on the outside!

Example: Simplify $\sqrt[3]{24}$

Solution: Since it's a cube root, we need perfect cube factors, or a triplet of like factors

$\sqrt[3]{24} = \sqrt[3]{8 \cdot 3} \quad \rightarrow \quad \sqrt[3]{8} \cdot \sqrt[3]{3} = \mathbf{2\sqrt[3]{3}}$

$\sqrt[3]{24} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3} \quad \rightarrow \quad \sqrt[3]{2 \cdot 2 \cdot 2} \cdot \sqrt[3]{3} = \mathbf{2\sqrt[3]{3}}$

Example: Simplify $\sqrt{180x^6y^3}$

Solution: If the numerical coefficient is big, factor it into prime factors too

$$\sqrt{180x^6y^3} = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot x^6 \cdot y^3}$$

$$\sqrt{(2 \cdot 2)(3 \cdot 3) \cdot x^2 \cdot x^2 \cdot x^2 \cdot y^2} \cdot \sqrt{5y}$$

$$6x^3y\sqrt{5y}$$

x^2 becomes x when it comes out of the radical

Example: Simplify $\sqrt[3]{\frac{x^{12}}{64}}$

Solution: Notice that 64 is a perfect cube.

$$\sqrt[3]{\frac{x^{12}}{64}} = \frac{\sqrt[3]{x^{12}}}{\sqrt[3]{64}} \quad \rightarrow \quad \frac{\sqrt[3]{(x^4)^3}}{\sqrt[3]{64}} = \frac{x^4}{4}$$

Changing Mixed Radicals to Entire Radicals

- This is simply the reverse of the simplification process

Example: Write $3\sqrt{5}$ as an entire radical

Solution: $3\sqrt{5} = \sqrt{(3 \cdot 3) \cdot 5} = \sqrt{45}$

Example: Write $-4x^2\sqrt{5x^3}, x > 0$ as an entire radical

Solution: $-\sqrt{(4x^2 \cdot 4x^2) \cdot 5x^3} = -\sqrt{4 \cdot 4 \cdot 5 \cdot x^2 \cdot x^2 \cdot x^3}$

$$-\sqrt{80x^7}$$

This cannot go back inside! **No negatives in EVEN index roots**!

<u>Example:</u> Write $2xy\sqrt[3]{4x^2y^3}$, $x, y \geq 0$ as an entire radical

<u>Solution:</u> $2xy\sqrt[3]{4x^2y^3} = \sqrt[3]{(2xy)(2xy)(2xy) \cdot 4x^2y^3} \rightarrow \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 4 \cdot xxxx^2yyyy^3} = \sqrt[3]{32x^5y^6}$

<u>Example:</u> Write $\frac{3x^2y}{5}\sqrt[3]{2xy^2}$, $x, y \geq 0$ as an entire radical

<u>Solution:</u> $\frac{3x^2y}{5}\sqrt[3]{2xy^2} = \sqrt[3]{\frac{3x^2y}{5} \cdot \frac{3x^2y}{5} \cdot \frac{3x^2y}{5} \cdot (2xy^2)} \rightarrow \sqrt[3]{\frac{3 \cdot 3 \cdot 3 \cdot 2}{5 \cdot 5 \cdot 5}(x^2 \cdot x^2 \cdot x^2 \cdot x)(y \cdot y \cdot y \cdot y^2)} = \sqrt[3]{\frac{54x^7y^5}{125}}$

<u>Example:</u> Write $-2x\sqrt[5]{4x}$ as an entire radical

<u>Solution:</u> Since we have an **odd indexed root**, we can **multiply the negative in!**

$$-2x\sqrt[5]{4x} = \sqrt[5]{(-2x)(-2x)(-2x)(-2x)(-2x)4x}$$

$$\sqrt[5]{(-2)(-2)(-2)(-2)(-2)(4)(x^5 \cdot x)} = \sqrt[5]{\mathbf{-128x^6}}$$

Be careful with when you can and cannot multiply the negative in, it comes down to the index of the root!!

Section 2.2 – Practice Problems

Simplify

1. $-\sqrt{16}$	2. $\sqrt{\frac{1}{4}}$
3. $\sqrt{0.16}$	4. $-\sqrt{144}$
5. $\sqrt[3]{-8}$	6. $\sqrt[5]{32}$
7. $\sqrt{169}$	8. $\sqrt[6]{0.000064}$

Solve for x

9. $x^2 = 25$	10. $x^2 = 81$
11. $x^2 = 0.04$	12. $x^2 = 121$
13. $x^3 = -8$	14. $x^3 = -64$

15. $x^4 = 16$	16. $x^4 = -16$
17. $x^5 = -32$	18. $x^6 = 64$

Change to simplest radical form

19. $\sqrt{32}$	20. $\sqrt{80}$
21. $\sqrt{75}$	22. $\sqrt[3]{54}$
23. $\sqrt[3]{16}$	24. $\sqrt[3]{-72}$
25. $3\sqrt{45}$	26. $-2\sqrt[3]{162}$

Express in simplest radical form

27. $\sqrt{x^3y^2}$	28. $\sqrt{49x^3y^5}$
29. $\sqrt{18x^6}$	30. $\sqrt{25x^6y^{11}}$

31. $\sqrt{5t^4}$

32. $\sqrt{32r^5t^7}$

33. $\sqrt[3]{40x^3}$

34. $\sqrt[3]{-8x^9z^3}$

35. $\sqrt[3]{216h^5}$

36. $\sqrt[3]{-64x^3y^6}$

Write as an entire radical

37. $2\sqrt{5}$

38. $-4\sqrt{3}$

39. $3\sqrt{4}$

40. $2\sqrt[3]{3}$

41. $3\sqrt[3]{4}$

42. $-4\sqrt[3]{5}$

43. $2\sqrt[4]{3}$

44. $2\sqrt[6]{3}$

45. Express the following in simplest radical form. $\left(2x\sqrt[3]{2y^4}\right)\left(x^2\sqrt[3]{4y^2}\right)$

46. A rectangular solid has a volume of $192cm^3$. If the height is twice the width and the length is three times the width, what are the dimensions of the rectangular solid?

Answer Key – Section 2.2

1.	-4
2.	$\frac{1}{2}$
3.	0.4
4.	-12
5.	-2
6.	2
7.	13
8.	0.2
9.	± 5
10.	± 9
11.	± 0.2
12.	± 11
13.	-2
14.	-4
15.	± 2
16.	*Does Not Exist*
17.	-2
18.	± 2
19.	$4\sqrt{2}$
20.	$4\sqrt{5}$
21.	$5\sqrt{3}$
22.	$3\sqrt[3]{2}$
23.	$2\sqrt[3]{2}$

24.	$-2\sqrt[3]{9}$
25.	$9\sqrt{5}$
26.	$-6\sqrt[3]{6}$
27.	$xy\sqrt{x}$
28.	$7xy^2\sqrt{xy}$
29.	$3x^3\sqrt{2}$
30.	$5x^3y^5\sqrt{y}$
31.	$t^2\sqrt{5}$
32.	$4r^2t^3\sqrt{2rt}$
33.	$2x\sqrt[3]{5}$
34.	$-2x^3z$
35.	$6h\sqrt[3]{h^2}$
36.	$-4xy^2$
37.	$\sqrt{20}$
38.	$-\sqrt{48}$
39.	$\sqrt{36}$
40.	$\sqrt[3]{24}$
41.	$\sqrt[3]{108}$
42.	$\sqrt[3]{-320}$
43.	$\sqrt[4]{48}$
44.	$\sqrt[6]{192}$
45.	$4x^5y^2$
46.	$w = 2\sqrt[3]{4}$ $\quad h = 4\sqrt[3]{4}$ $l = 6\sqrt[3]{4}$

Extra Work Space