Section 2.2 – Practice Problems

1. Use the following functions $f, g, h, i, j, and\ k$, to find:

$f(x) = 2x^2 + 5x + 3,\quad g(x) = 2x - 1,\quad h(x) = 3,\quad i(x) = \frac{1}{x},\quad j(x) = x^2 - 1,\quad k(x) = \frac{2}{x+2}$

a) $(g + j)(2)$ $\rightarrow g(2) + j(2)$

$g(2) = 2(2) - 1 = 3$ $\quad j(2) = 2^2 - 1 = 3$

$(g+j)(2) = 3 + 3 = \boxed{6}$

b) $(f - k)(-2)$ Algebraically is hard

$(f-k)(-2) \rightarrow f(-2) - k(-2)$ Solve piecewise

$f(-2) = 2(-2)^2 + 5(-2) + 3 \rightarrow 8 - 10 + 3 = 1$

$k(-2) = \frac{2}{-2+2} = \frac{2}{0}$ ← undefined

so $\boxed{(f-k)(-2) \text{ is undefined}}$

c) $(hi)(3)$ Algebraically

$h(x) \cdot i(x) = 3 \cdot \frac{1}{x} \rightarrow \frac{3}{x}$

$(hi)(3) = \frac{3}{3} = \boxed{1}$

d) $(jk)(-3)$ Algebraically

$j(x) \cdot k(x) \rightarrow (x^2-1) \cdot \frac{2}{x+2} = \frac{2(x^2-1)}{x+2}$

$(jk)(-3) \rightarrow \frac{2((-3)^2-1)}{-3+2} \rightarrow \frac{2(8)}{-1} = \frac{16}{-1} = \boxed{-16}$

e) $\left(\frac{g}{f}\right)(4)$ Piecewise

$g(4) = 2(4) - 1 \rightarrow 8 - 1 = 7$

$f(4) = 2(4)^2 + 5(4) + 3 \rightarrow 32 + 20 + 3 = 55$

$\left(\frac{g}{f}\right)(4) = \frac{7}{55}$

f) $\left(\frac{i}{k}\right)(-4)$

$i(-4) = \frac{1}{-4} = -\frac{1}{4}$

$k(-4) = \frac{2}{-4+2} = \frac{2}{-2} = -1$

$\left(\frac{i}{k}\right)(-4) = \frac{-\frac{1}{4}}{-1} = \boxed{\frac{1}{4}}$

g) $\left(\frac{h}{j}\right)(-2)$

$h(-2) = 3$ $\quad j(-2) = (-2)^2 - 1$

$= 4 - 1 = 3$

$\left(\frac{h}{j}\right)(-2) = \frac{3}{3} = \boxed{1}$

h) $(k - i)(7)$ Piecewise

$k(7) - i(7)$

$\frac{2}{x+2} - \frac{1}{x} \rightarrow \frac{2}{7+2} - \frac{1}{7} \rightarrow \frac{2}{9} - \frac{1}{7}$

$\frac{14}{63} - \frac{9}{63} = \boxed{\frac{5}{63}}$

2. Use the following functions $f, g, h, i, j,$ *and* k, to find the function and its Domain:

$f(x) = 2x^2 + 5x + 3, \quad g(x) = 2x - 1, \quad h(x) = 3, \quad i(x) = \frac{1}{x}, \quad j(x) = x^2 - 1, \quad k(x) = \frac{2}{x+2}$

a) $(f - g)(x)$

$f(x) - g(x) = 2x^2 + 5x + 3 - (2x - 1)$

$= 2x^2 + 5x + 3 - 2x + 1$

$= \boxed{2x^2 + 3x + 4}$

$\boxed{\text{D: All Real \#'s}}$

b) $(j + i)(x)$

$j(x) + i(x) \rightarrow x^2 - 1 + \frac{1}{x}$

$\frac{x(x^2-1)}{x} + \frac{1}{x} = \boxed{\frac{x^3 - x + 1}{x}}$

common denominator

$\boxed{\text{Domain: } x \neq 0}$

c) $\left(\frac{i}{h}\right)(x)$

$\frac{i(x)}{h(x)} = \frac{\frac{1}{x}}{3} \rightarrow \frac{1}{x} \cdot \frac{1}{3}$

$\boxed{\frac{1}{3x}}$

$\boxed{\text{Domain: } x \neq 0}$

d) $\left(\frac{h}{i}\right)(x)$

$\frac{h(x)}{i(x)} = \frac{3}{\frac{1}{x}} \rightarrow \boxed{3x}$

$\boxed{\text{Domain: } x \neq 0}$

Due to initial condition

e) $(gk)(x)$

$g(x)\cdot k(x) \rightarrow (2x-1)\left(\frac{2}{x+2}\right)$

$\frac{2(2x-1)}{x+2} \rightarrow \boxed{\frac{4x-2}{x+2}}$

$\boxed{D: x \neq -2}$

f) $\left(\frac{g}{k}\right)(x)$

$\frac{g(x)}{k(x)} = \frac{2x-1}{\frac{2}{x+2}} \rightarrow (2x-1)\cdot\frac{(x+2)}{2}$

Foil Top

$\boxed{\frac{2x^2+3x-2}{2}} \leftarrow \frac{2x^2+4x-x-2}{2}$

$\boxed{D: x \neq -2}$

g) $\left(\frac{f}{j}\right)(x)$

$\frac{f(x)}{j(x)} = \frac{2x^2+5x+3}{x^2-1} \rightarrow \frac{(2x+3)\cancel{(x+1)}}{\cancel{(x+1)}(x-1)}$

$\boxed{\frac{2x+3}{x-1}}$

still both even though it cancelled

$\boxed{D: x \neq \pm 1}$

h) $(gj)(x)$

$g(x) = 2x-1$
$j(x) = x^2-1$

$g(x)\cdot j(x) = (2x-1)(x^2-1)$

$= 2x^3-2x-x^2+1$

$= \boxed{2x^3-x^2-2x+1}$

$\boxed{D: \text{All Real \#'s}}$

3. Find each expression, given that the function of $f, g, h, k, and\ l$, are defined as follows:

$f(x) = 2x+1, \quad g(x) = 2x^2-x-1, \quad h(x) = x^3, \quad k(x) = 3, \quad l(x) = x^2-1$

a) $\left(\frac{f}{l}\right)(x) - \left(\frac{l}{f}\right)(x)$

$\frac{2x+1}{x^2-1} - \frac{x^2-1}{2x+1}$ need common denominator

$\frac{(2x+1)(2x+1)-(x^2-1)(x^2-1)}{(x^2-1)(2x+1)}$

$\frac{4x^2+2x+2x+1-[x^4-x^2-x^2+1]}{(x^2-1)(2x+1)}$

$\boxed{D: x \neq \pm 1,\ x \neq -\frac{1}{2}}$

$\frac{4x^2+4x+1-x^4+2x^2-1}{(x^2-1)(2x+1)} \rightarrow \boxed{\frac{-x^4+6x^2+4x}{(x^2-1)(2x+1)}}$

b) $\left(\frac{f}{l}\right)(0) - \left(\frac{l}{f}\right)(0)$

Take solution from 3a and plug in 0

$\frac{-0^4+6(0)^2+4(0)}{(0-1)(2(0)+1)} = \frac{0}{(-1)(1)} = \frac{0}{1}$

$\boxed{0}$

could also solve piecewise

c) $[h(f+l)](x)$

$(f+l) \rightarrow (2x+1)+(x^2-1)$

$= x^2+2x$

$h(f+l) = x^3(x^2+2x)$

$= \boxed{x^5+2x^4}$

d) $(hf)(x)+(hl)(x)$

$(h\cdot f)(x) \rightarrow x^3(2x+1) = 2x^4+x^3$

$(h\cdot l)(x) \rightarrow x^3(x^2-1) = x^5-x^3$

$2x^4+x^3+x^5-x^3$

$\boxed{x^5+2x^4}$

e) $[l(k-h)](x)$

$(k-h)(x) \rightarrow 3-x^3$

$l(k-h)(x) \rightarrow (x^2-1)(3-x^3)$

FOIL

$3x^2-x^5-3+x^3$

$\boxed{-x^5+x^3+3x^2-3}$

f) $lk(x)-lh(x)$

$lk(x) \rightarrow (x^2-1)3 = 3x^2-3$

$lh(x) \rightarrow (x^2-1)x^3 = x^5-x^3$

$lk(x)-lh(x)$

$3x^2-3-(x^5-x^3)$

$3x^2-3-x^5+x^3 \longrightarrow \boxed{-x^5+x^3+3x^2-3}$

g) $(g+g)(x)$

$2x^2-x-1+2x^2-x-1$

$\boxed{4x^2-2x-2}$

h) $(g-g)(x)$

$2x^2-x-1-(2x^2-x-1)$

$2x^2-x-1-2x^2+x+1$

$\boxed{0}$

i) $(kg)(x)$

$3(2x^2-x-1)$

$\boxed{6x^2-3x-3}$

j) $(g+g)(-2)-(kg)(-2)$

$g(x) = 2x^2-x-1$

$g(-2) = 2(-2)^2-(-2)-1$
$= 8+2-1$
$= 9$

$g(-2)+g(-2) = 9+9$
$= 18$

$k(-2) = 3$
$g(-2) = 9$

$kg(-2) = 27$

$18-27$

$= \boxed{-9}$

4. Find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, $(ff)(x)$, $\left(\frac{f}{g}\right)(x)$ and $\left(\frac{g}{f}\right)(x)$, if:

a) $f(x)=x^2-4,\ g(x)=x+2$

$(f+g)(x) \rightarrow (x^2-4)+(x+2)$
$\rightarrow x^2-4+x+2 = \boxed{x^2+x-2}$

$(f-g)(x) \rightarrow (x^2-4)-(x+2)$
$\rightarrow x^2-4-x-2 = \boxed{x^2-x-6}$

$(fg)(x) \rightarrow (x^2-4)(x+2)$
$\rightarrow \boxed{x^3+2x^2-4x-8}$

$(ff)(x) \rightarrow (x^2-4)(x^2-4)$
$\rightarrow x^4-4x^2-4x^2+16 = \boxed{x^4-8x^2+16}$

$\left(\frac{f}{g}\right)(x) \rightarrow \frac{x^2-4}{x+2} \rightarrow \frac{(x+2)(x-2)}{(x+2)} = (x-2)$
$x \neq -2$

$\left(\frac{g}{f}\right)(x) \rightarrow \frac{(x+2)}{x^2-4} \rightarrow \frac{(x+2)}{(x+2)(x-2)} = \frac{1}{(x-2)}$
$x \neq \pm 2$

b) $f(x)=2x^2-x-3,\ g(x)=x+1$

$(f+g)(x) \rightarrow 2x^2-x-3+(x+1)$
$\rightarrow 2x^2-x-3+x+1 = \boxed{2x^2-2}$

$(f-g)(x) \rightarrow 2x^2-x-3-(x+1)$
$\rightarrow 2x^2-x-3-x-1 = \boxed{2x^2-2x-4}$

$(fg)(x) \rightarrow (2x^2-x-3)(x+1)$
$2x^3+2x^2-x^2-x-3x-3$
$= \boxed{2x^3+x^2-4x-3}$

$(ff)(x) = (2x^2-x-3)(2x^2-x-3)$
$= 4x^4-2x^3-6x^2-2x^3+x^2+3x-6x^2+3x+9$
$= \boxed{4x^4-4x^3-11x^2+6x+9}$

$\left(\frac{f}{g}\right)(x) = \frac{2x^2-x-3}{(x+1)} \rightarrow \frac{(2x-3)(x+1)}{(x+1)} = \boxed{2x-3}$
$x \neq -1$

$\left(\frac{g}{f}\right)(x) = \frac{(x+1)}{2x^2-x-3} \rightarrow \frac{(x+1)}{(2x-3)(x+1)} = \frac{1}{2x-3}$ $x \neq \frac{3}{2}$ $x \neq -$

c) $f(x)=\sqrt{x},\ g(x)=\frac{1}{x}$

$(f+g)(x) \rightarrow \sqrt{x}+\frac{1}{x} \rightarrow \frac{x\sqrt{x}}{x}+\frac{1}{x}$

$\boxed{\frac{x\sqrt{x}+1}{x}}$ $\boxed{x>0 \quad x\neq 0}$

$(f-g)(x) \rightarrow \sqrt{x}-\frac{1}{x} \rightarrow \frac{x\sqrt{x}}{x}-\frac{1}{x}$

$\boxed{\frac{x\sqrt{x}-1}{x}}$ $\boxed{x>0 \quad x\neq 0}$

$(fg)(x) \rightarrow \sqrt{x}\cdot\frac{1}{x} \rightarrow \frac{\sqrt{x}}{x} \rightarrow \boxed{\frac{1}{\sqrt{x}}}$ $\boxed{x>0}$

$(ff)(x) \rightarrow \sqrt{x}\cdot\sqrt{x} = \boxed{x}$ $\boxed{x\geq 0}$

$\left(\frac{f}{g}\right)(x) \rightarrow \frac{\sqrt{x}}{\frac{1}{x}} \rightarrow \boxed{x\sqrt{x}}$ $\boxed{\begin{matrix}x>0\\ x\neq 0\end{matrix}}$

$\left(\frac{g}{f}\right)(x) \rightarrow \frac{\frac{1}{x}}{\sqrt{x}} \rightarrow \boxed{\frac{1}{x\sqrt{x}} = \frac{\sqrt{x}}{x^2}}$ $\boxed{x>0}$

d) $f(x)=\sqrt{x},\ g(x)=x^2$

$(f+g)(x) \rightarrow \boxed{\sqrt{x}+x^2}$ $\boxed{x\geq 0}$

$(f-g)(x) \rightarrow \boxed{\sqrt{x}-x^2}$ $\boxed{x\geq 0}$

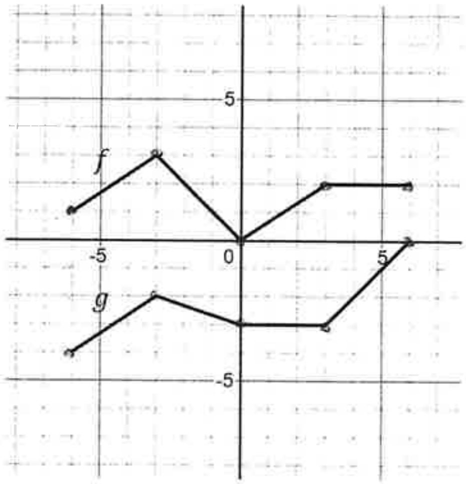
$(fg)(x) \rightarrow \sqrt{x}\cdot x^2 \rightarrow \boxed{x^2\sqrt{x}}$ $\boxed{x\geq 0}$

$(ff)(x) \rightarrow \sqrt{x}\sqrt{x} \rightarrow \boxed{x}$ $\boxed{x\geq 0}$

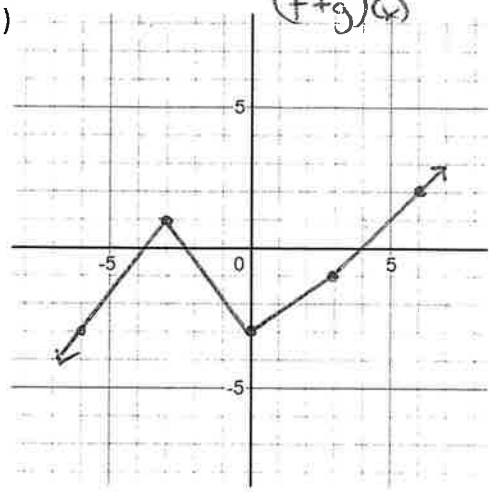
$\left(\frac{f}{g}\right)(x) \rightarrow \boxed{\frac{\sqrt{x}}{x^2}}$ $\boxed{x>0}$

$\left(\frac{g}{f}\right)(x) \rightarrow \frac{x^2}{\sqrt{x}} \rightarrow \boxed{\frac{x^2\sqrt{x}}{x}}$ $\boxed{x>0}$

5. Use the graphs below, to graph the following combinations



a) (f+g)(x)



a) $(f+g)(x)$

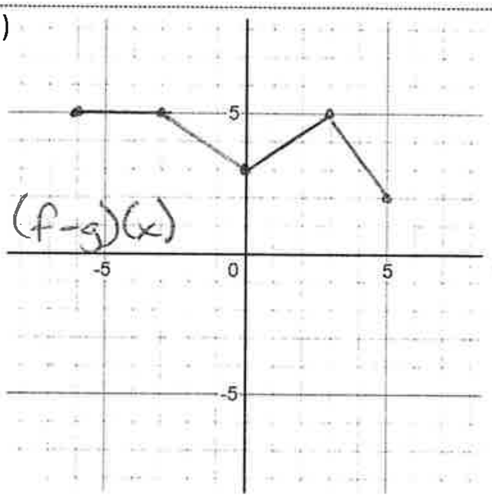
b) $(f-g)(x)$

c) $(g-f)(x)$

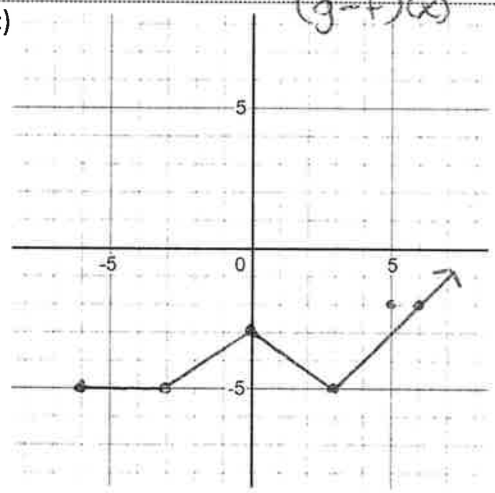
d) $(-f-g)(x)$

e) $\left(2f+\frac{1}{2}g\right)(x)$

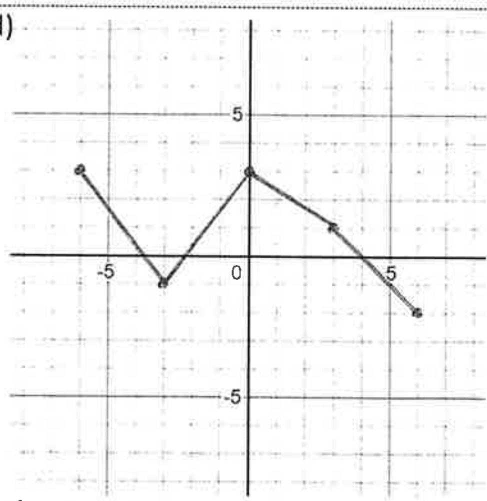
b)



c) (g-f)(x)

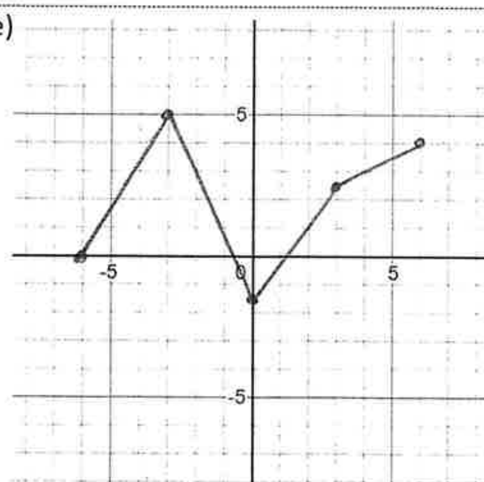


d)



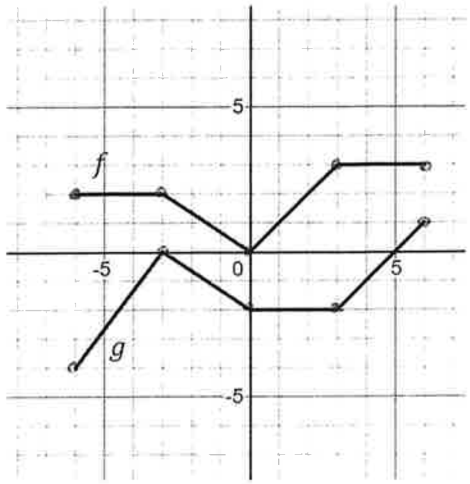
(-f-g)(x)

e)



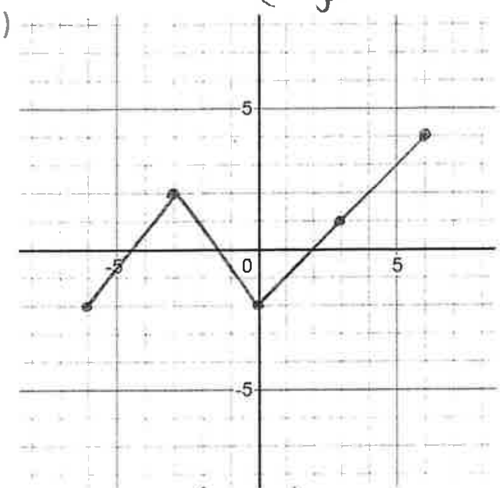
$\left(2f+\frac{1}{2}g\right)(x)$

6. Use the graphs below, to graph the following combinations



$(f+g)(x)$

a)



a) $(f+g)(x)$

b) $(f-g)(x)$

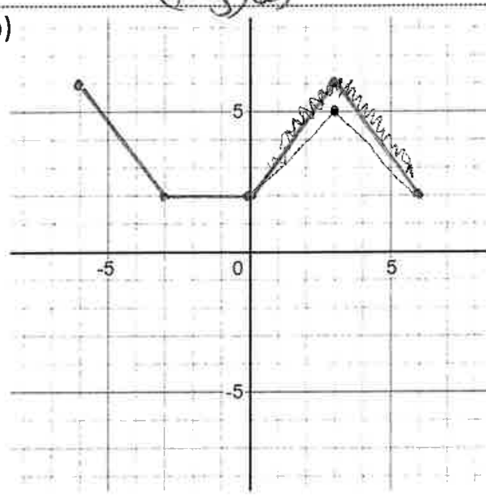
c) $(g-f)(x)$

d) $(-f-g)(x)$

e) $\left(2f+\frac{1}{2}g\right)(x)$

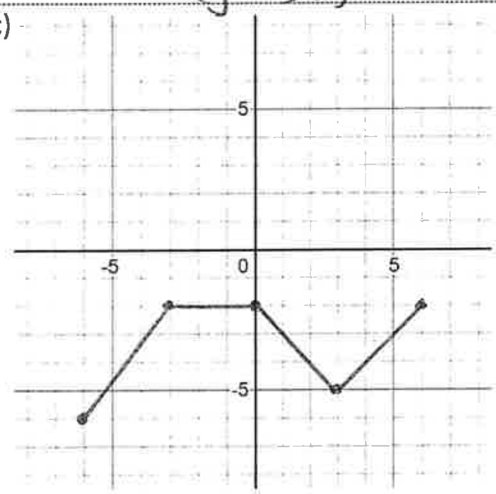
$(f-g)(x)$

b)

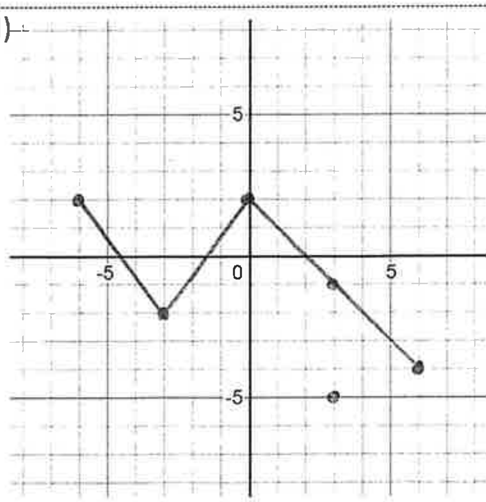


$(g-f)(x)$

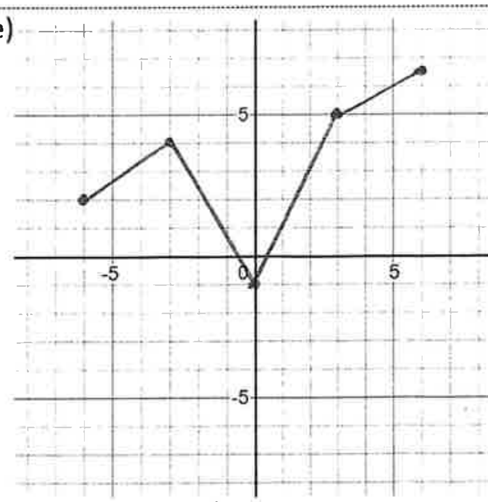
c)



d)



e)



$(-f-g)(x)$

$\left(2f+\frac{1}{2}g\right)(x)$

7. Use the graphs below, to graph the following combinations

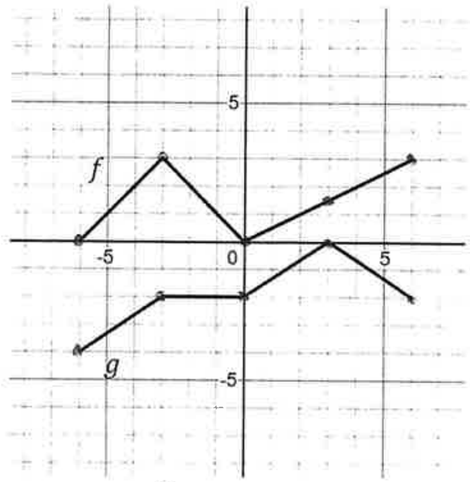
a) $(f+g)(x)$

b) $(f-g)(x)$

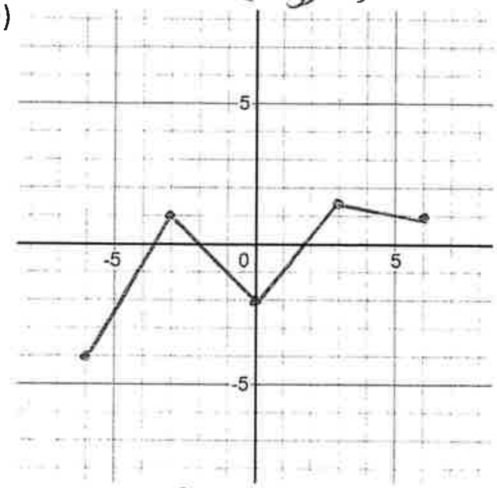
c) $(g-f)(x)$

d) $(-f-g)(x)$

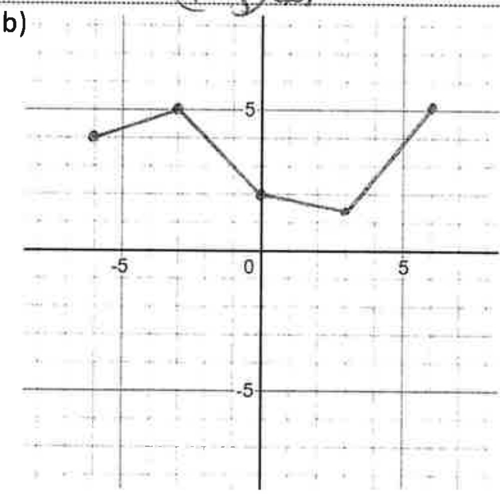
e) $\left(2f+\frac{1}{2}g\right)(x)$



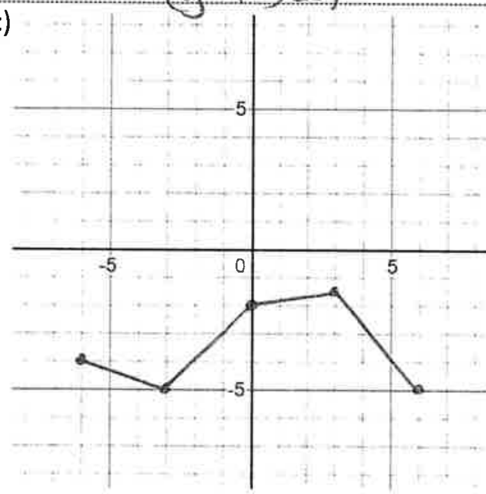
a) $(f+g)(x)$



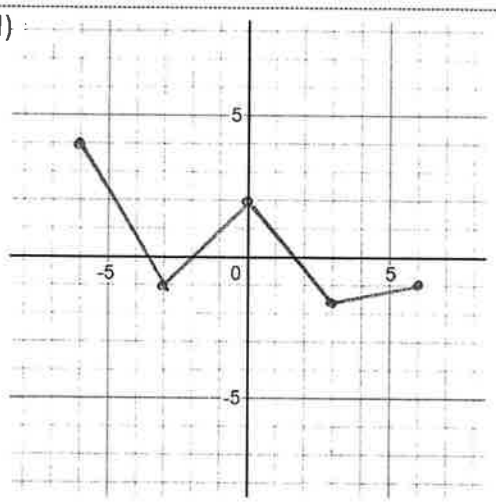
b) $(f-g)(x)$



c) $(g-f)(x)$

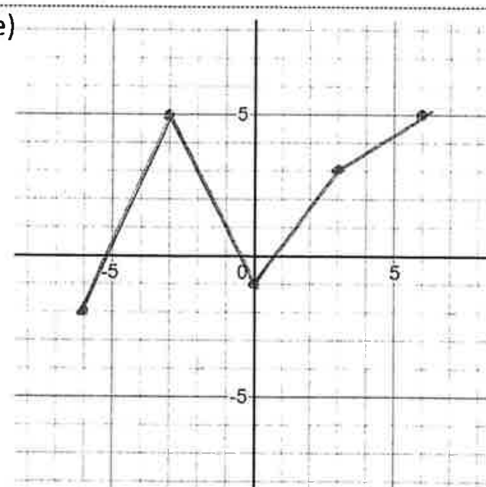


d)



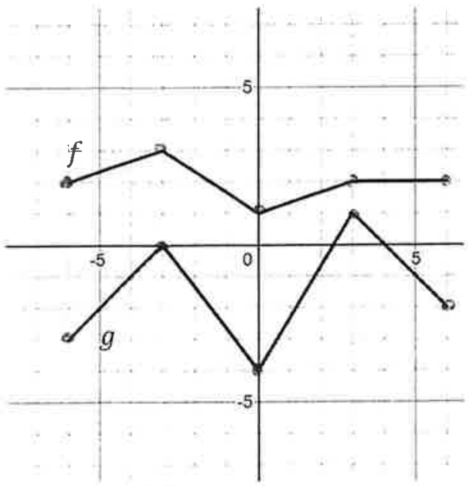
$(-f-g)(x)$

e)



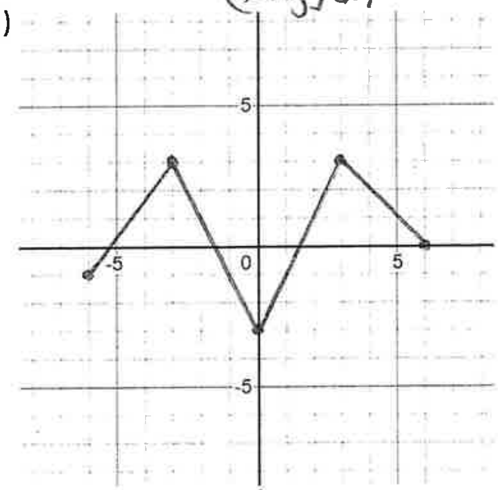
$(2f+\frac{1}{2}g)(x)$

8. Use the graphs below, to graph the following combinations

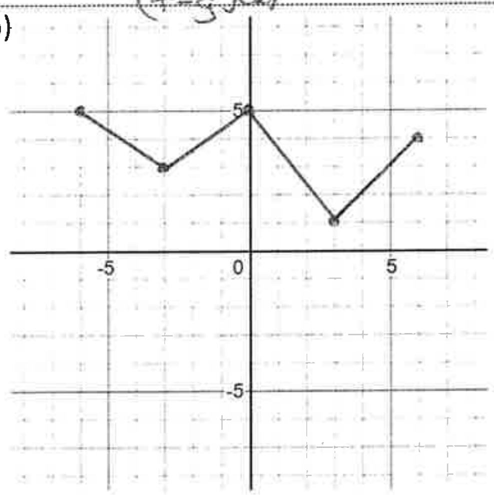


- f) $(f+g)(x)$
- g) $(f-g)(x)$
- h) $(g-f)(x)$
- i) $(-f-g)(x)$
- j) $\left(2f+\frac{1}{2}g\right)(x)$

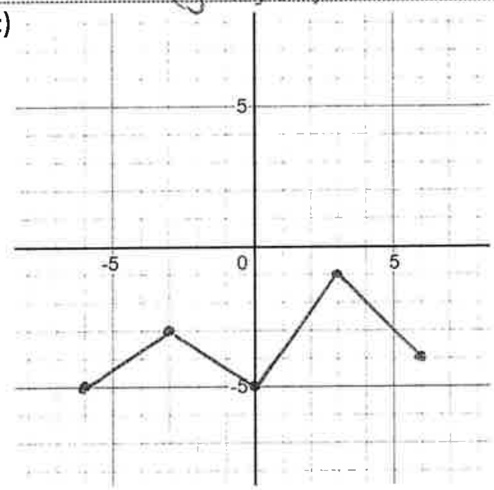
a) $(f+g)(x)$



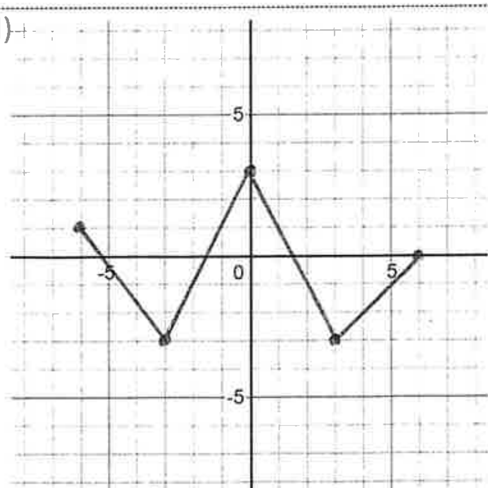
b) $(f-g)(x)$



c) $(g-f)(x)$

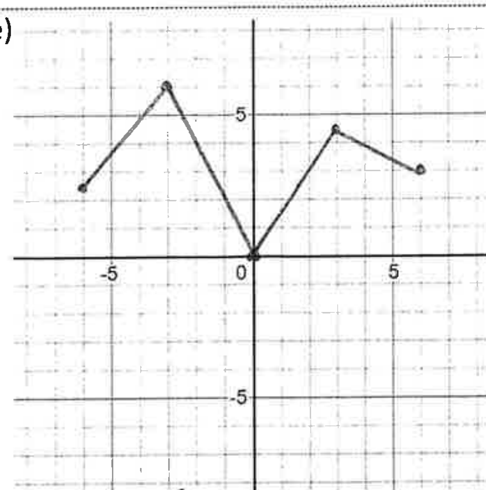


d)



$(-f-g)(x)$

e)



$\left(2f+\frac{1}{2}g\right)(x)$

Extra Work Space