Section 2.2 – Arithmetic Combinations of Functions

- Much like we can combine Real Numbers using addition, subtraction, multiplication, and division, we can combine Function in the exact same manner

Example1: Given the two function $f(x) = 3x + 2$ and $g(x) = x^2 - 4$ find the sum, difference, product, and quotient

Solution 1:

Sum	$f(x) + g(x) \quad \rightarrow \quad (3x + 2) + (x^2 - 4) = x^2 + 3x - 2$ *Brackets help distinguish between the two Functions*
Difference	$f(x) - g(x) \quad \rightarrow \quad (3x + 2) - (x^2 - 4) = -x^2 + 3x + 6$ *Do not forget to WATERBOMB the negative, this is why **brackets are important***
Product	$f(x)g(x) \quad \rightarrow \quad (3x + 2)(x^2 - 4) = 3x^3 + 2x^2 - 12x - 8$ *Distribute or use FOIL*
Quotient	$\frac{f(x)}{g(x)} = \frac{3x + 2}{x^2 - 4}, x \neq \pm 2$ *Identify any Domain Restrictions, what x causes the denominator to be zero*

- The Domain $(x - values)$ of the combination of functions $f \ and \ g$ is the set of real numbers that are common to both $f \ and \ g$. So we can consider the following notation.

Sum, Difference, Product, and Quotients of Functions	
1. ***Sum***	$(\boldsymbol{f} + \boldsymbol{g})(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})$
2. ***Difference***	$(\boldsymbol{f} - \boldsymbol{g})(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{x})$
3. ***Product***	$(\boldsymbol{fg})(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}) \cdot \boldsymbol{g}(\boldsymbol{x})$
4. ***Quotient***	$\left(\frac{\boldsymbol{f}}{\boldsymbol{g}}\right)(\boldsymbol{x}) = \frac{\boldsymbol{f}(\boldsymbol{x})}{\boldsymbol{g}(\boldsymbol{x})}, \qquad \boldsymbol{g}(\boldsymbol{x}) \neq \boldsymbol{0}$

Example 2: Compute the given expressions, given the functions provided.

$$f(x) = 2x + 1, \quad g(x) = x^2 - 2x + 1, \quad h(x) = x^3, \quad k(x) = 2$$

a) $(f + g)(x)$
b) $(h - k)(x)$
c) $\frac{kg}{h}(3)$
d) $(fk)(1) - (hg)(2)$
e) $[h \cdot (f + g)](x)$

Solution 2:

a) $\boldsymbol{(f + g)(x) = f(x) + g(x) = (2x + 1) + \left(x^2 - 2x + 1\right) = x^2 + 2}$

b) $(h - k)(x) = h(x) - k(x) = x^3 - 2$

c) $\left(\frac{kg}{h}\right)(3) = \frac{k(3) \cdot g(3)}{h(3)} = \frac{2 \cdot (3^2 - 2(3) + 1)}{3^3} = \frac{2 \cdot 4}{27} = \frac{8}{27}$

d) $(fk)(1) - (hg)(2) = f(1) \cdot k(1) - h(2) \cdot g(2)$

$$= [2(1) + 1](2) - 2^3 \cdot (2^2 - 2(2) + 1)$$

$$= (3)(2) - 8(1) = 6 - 8 = -2$$

e) $[h \cdot (f + g)](x) = h(x)[f(x) + g(x)] = x^3[(2x + 1) + (x^2 - 2x + 1)]$

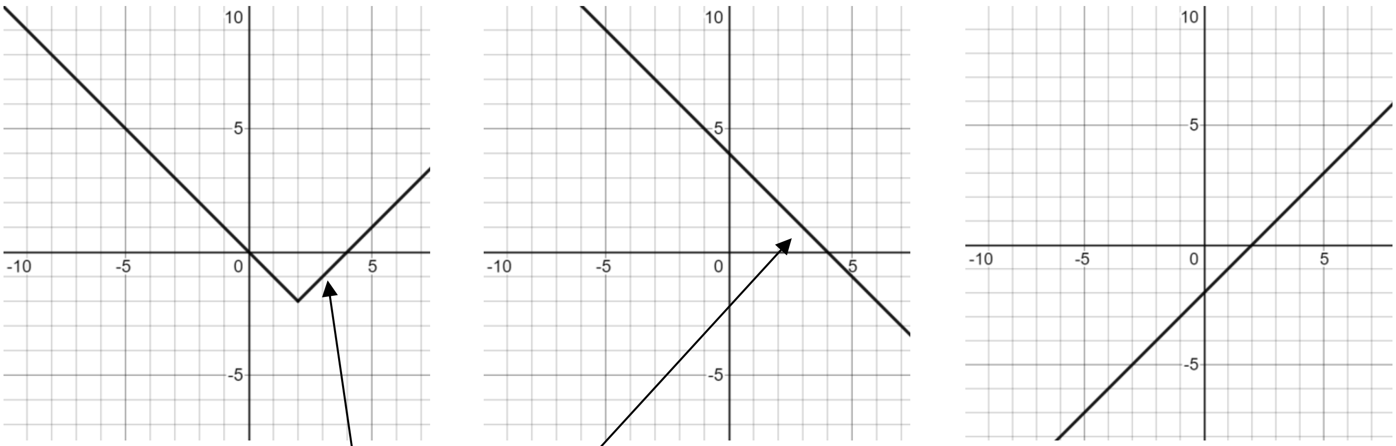
$$= x^3(x^2 + 2)$$

$$= x^5 + 2x^3$$

- These types of questions can involve very minor details – brackets, exponents laws, etc.
- Take your time, stay organized, and keep track of your process.

Example 3: Use the graphs provided below to evaluate the following

$$a)\ (f+g)(3), \qquad b)\ \left(\frac{h}{g}\right)(5), \qquad c)\ (fgh)(1), \qquad d)\ Graph: (f-h)(x)$$



$y = f(x)$ $\qquad$ $y = g(x)$ $\qquad$ $y = h(x)$

Solution 3:

a) $(f+g)(3) = f(3) + g(3)$ What is the output ($y - value$) when $x\ is\ 3$ in either function?

$$f(3) + g(3) = \quad \mathbf{-1 + 1 = 0}$$

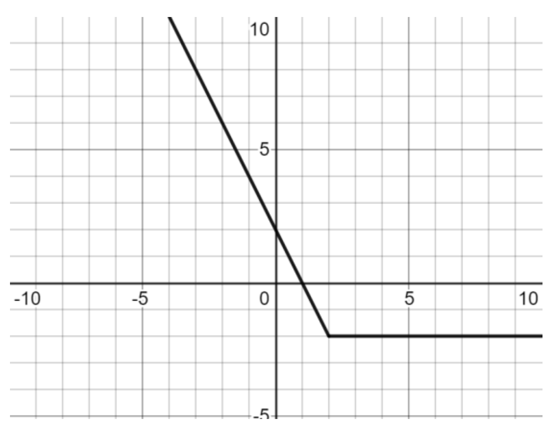
b) $\left(\frac{h}{g}\right)(5) = \frac{h(5)}{g(5)}$ What Is the output ($y - value$) when $x\ is\ 5$ in either function?

$$\frac{h(5)}{g(5)} = \quad \frac{\mathbf{3}}{\mathbf{-1}} = \mathbf{-3}$$

c) $(fgh)(1) = f(1)g(1)h(1) = (-1)(3)(-1) = \mathbf{3}$

d) $(\boldsymbol{f} - \boldsymbol{h})(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{h}(\boldsymbol{x})$

$\boldsymbol{x}$	$\boldsymbol{f(x) - h(x)}$
-2	$2-(-4) = \mathbf{6}$
0	$0-(-2) = \mathbf{2}$
1	$-1-(-1) = \mathbf{0}$
2	$-2-0 = \mathbf{-2}$
4	$0-2 = \mathbf{-2}$
6	$2-4 = \mathbf{-2}$



Section 2.2 – Practice Problems

1. Use the following functions $f, g, h, i, j, and\ k$, to find:

$f(x) = 2x^2 + 5x + 3, \quad g(x) = 2x - 1, \quad h(x) = 3, \quad i(x) = \frac{1}{x}, \quad j(x) = x^2 - 1, \quad k(x) = \frac{2}{x+2}$

a) $(g + j)(2)$

b) $(f - k)(-2)$

c) $(hi)(3)$

d) $(jk)(-3)$

e) $\left(\frac{g}{f}\right)(4)$

f) $\left(\frac{i}{k}\right)(-4)$

g) $\left(\frac{h}{j}\right)(-2)$

h) $(k-i)(7)$

2. Use the following functions $f, g, h, i, j, and\ k$, to find the function and its Domain:

$f(x) = 2x^2 + 5x + 3$, $g(x) = 2x - 1$, $h(x) = 3$, $i(x) = \frac{1}{x}$, $j(x) = x^2 - 1$, $k(x) = \frac{2}{x+2}$

a) $(f-g)(x)$

b) $(j+i)(x)$

c) $\left(\frac{i}{h}\right)(x)$

d) $\left(\frac{h}{i}\right)(x)$

e) $(gk)(x)$

f) $\left(\frac{g}{k}\right)(x)$

g) $\left(\frac{f}{j}\right)(x)$

h) $(gj)(x)$

3. Find each expression, given that the function of $f, g, h, k, and\ l$, are defined as follows:

$$f(x) = 2x + 1, \quad g(x) = 2x^2 - x - 1, \quad h(x) = x^3, \quad k(x) = 3, \quad l(x) = x^2 - 1$$

a) $\left(\frac{f}{l}\right)(x) - \left(\frac{l}{f}\right)(x)$

b) $\left(\frac{f}{l}\right)(0) - \left(\frac{l}{f}\right)(0)$

c) $[h(f+l)](x)$

d) $(hf)(x)+(hl)(x)$

e) $[l(k-h)](x)$

f) $lk(x)-lh(x)$

g) $(g+g)(x)$

h) $(g-g)(x)$

i) $(kg)(x)$

j) $(g+g)(-2)-(kg)(-2)$

4. Find $(f+g)(x)$, $(f-g)(x)$, $(fg)(x)$, $(ff)(x)$, $\left(\frac{f}{g}\right)(x)\ and\ \left(\frac{g}{f}\right)(x), if$:

a) $f(x)=x^2-4,\ g(x)=x+2$

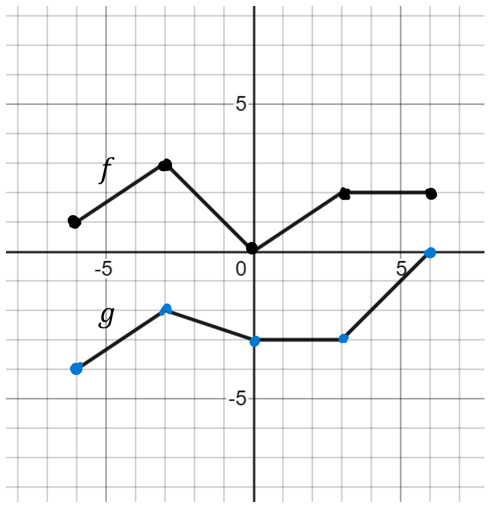
b) $f(x)=2x^2-x-3,\ g(x)=x+1$

c) $f(x) = \sqrt{x},\ g(x) = \frac{1}{x}$

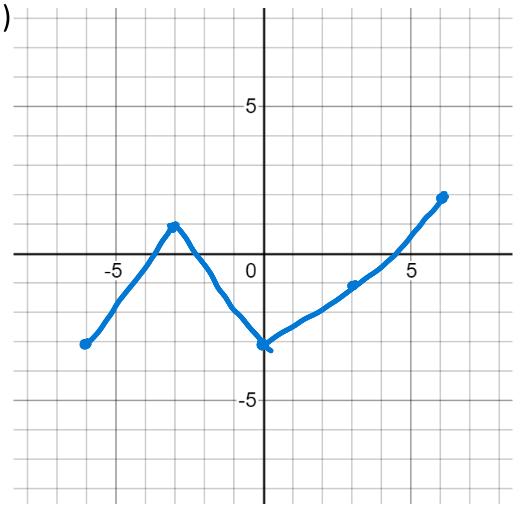
d) $f(x) = \sqrt{x},\ g(x) = x^2$

f(x) + g(x)

5. Use the graphs below, to graph the following combinations



a)



a) $(f + g)(x)$

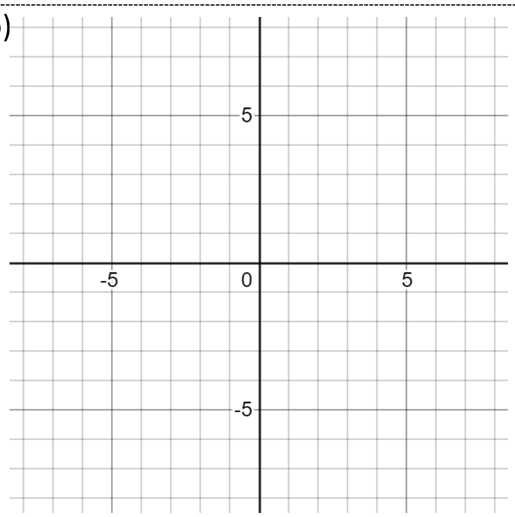
b) $(f - g)(x)$

c) $(g - f)(x)$

d) $(-f - g)(x)$

e) $\left(2f + \frac{1}{2}g\right)(x)$

b)



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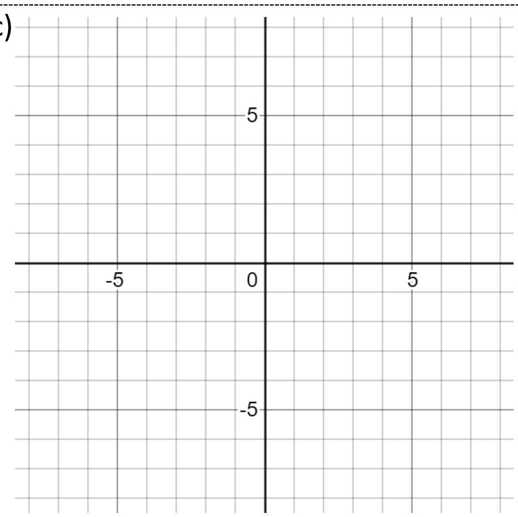
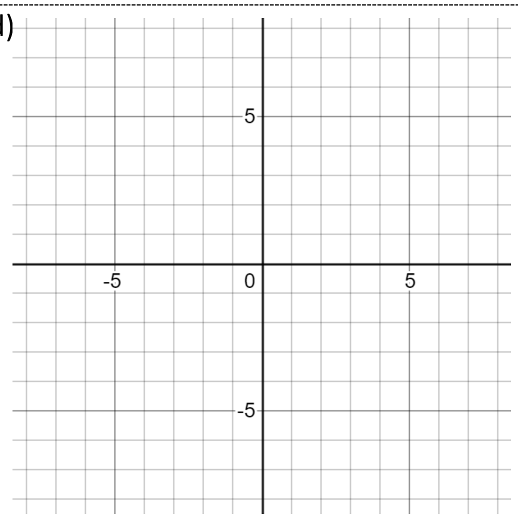
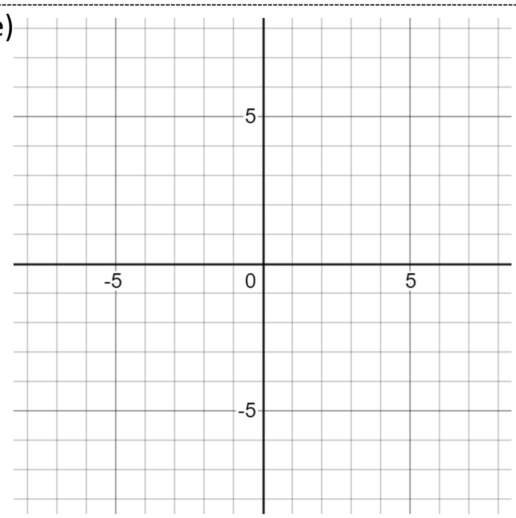


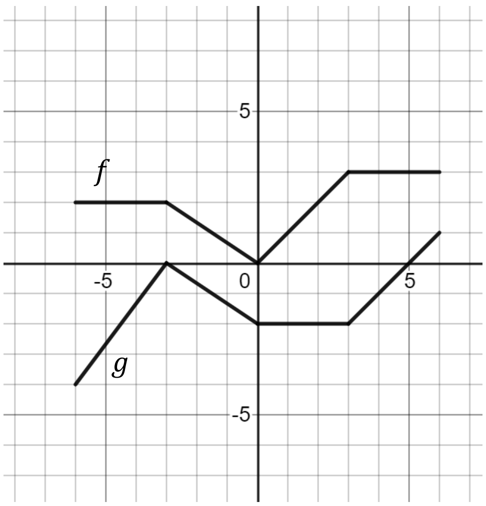
d)



e)



6. Use the graphs below, to graph the following combinations



a) $(f+g)(x)$

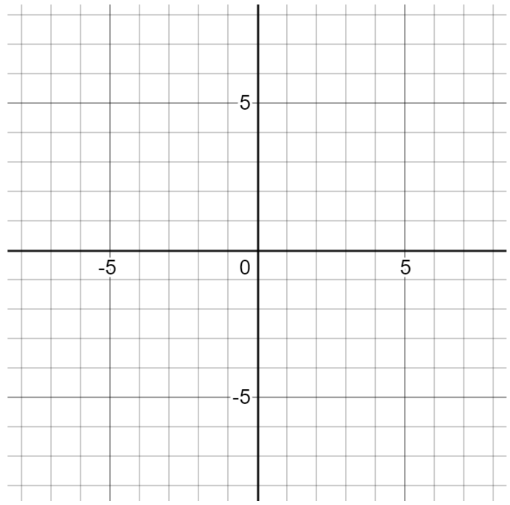
b) $(f-g)(x)$

c) $(g-f)(x)$

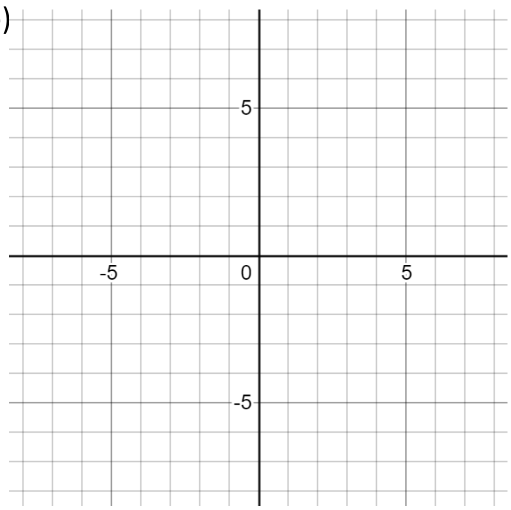
d) $(-f-g)(x)$

e) $\left(2f+\frac{1}{2}g\right)(x)$

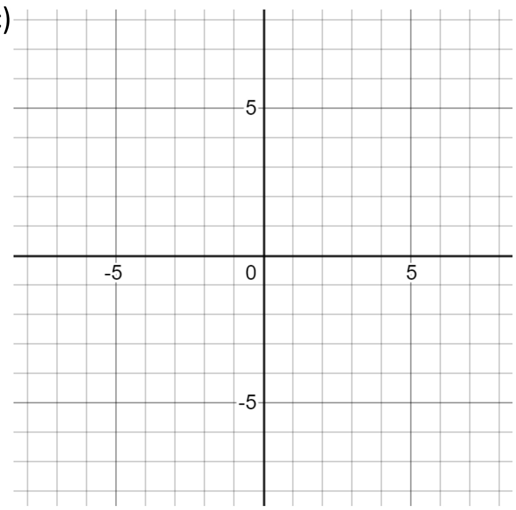
a)



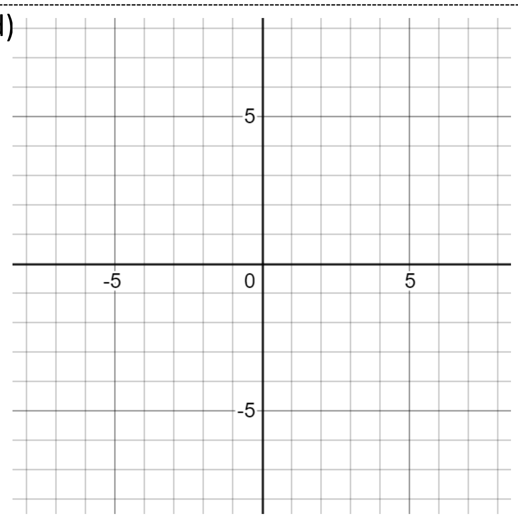
b)



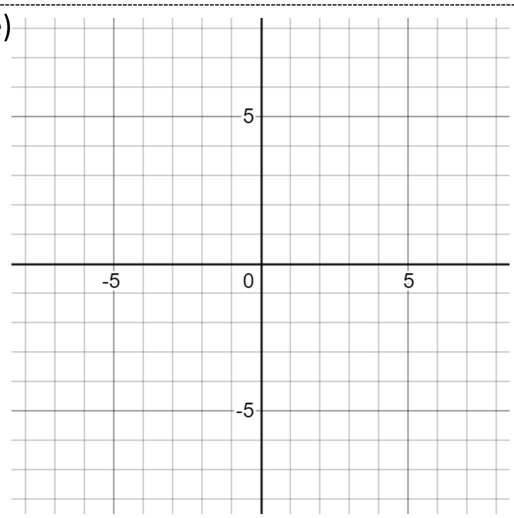
c)



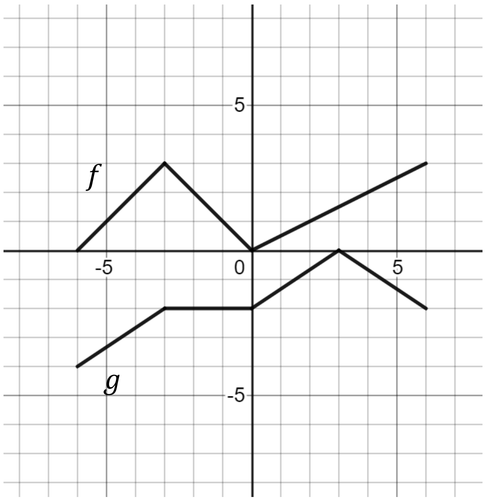
d)

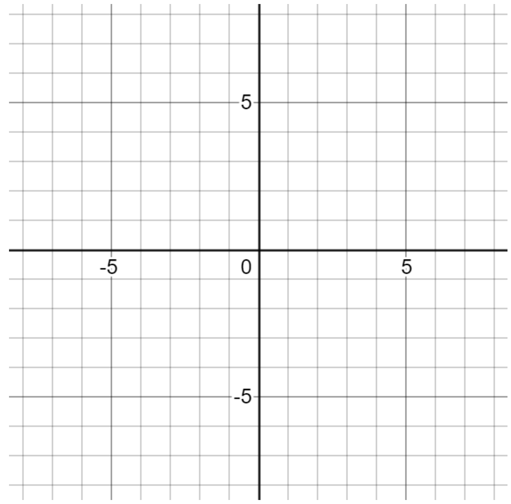


e)



7. Use the graphs below, to graph the following combinations



a)


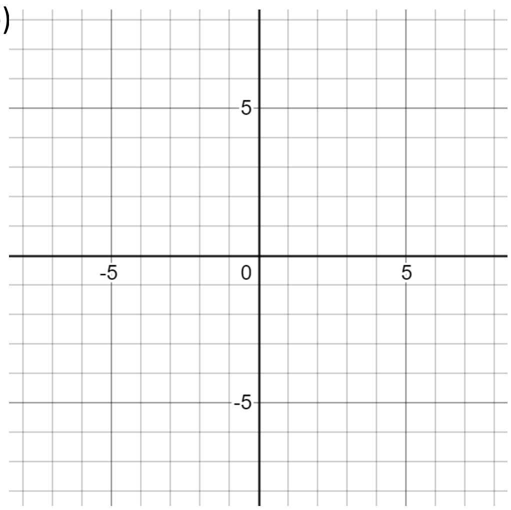
a) $(f+g)(x)$

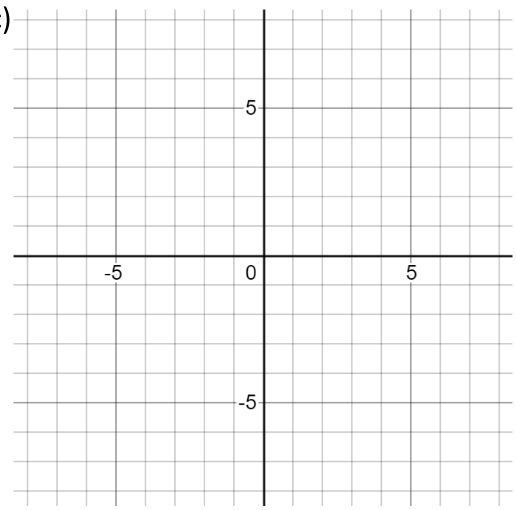
b) $(f-g)(x)$

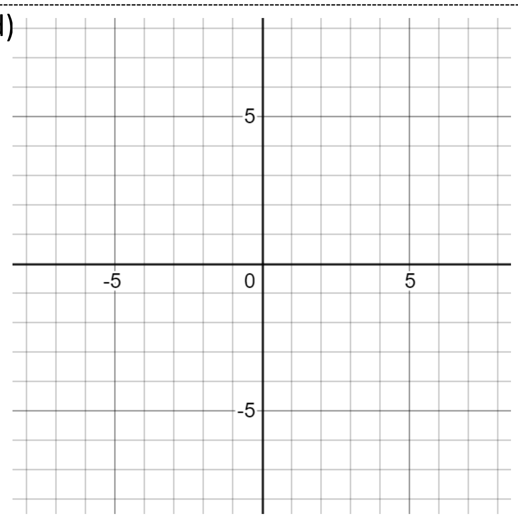
c) $(g-f)(x)$

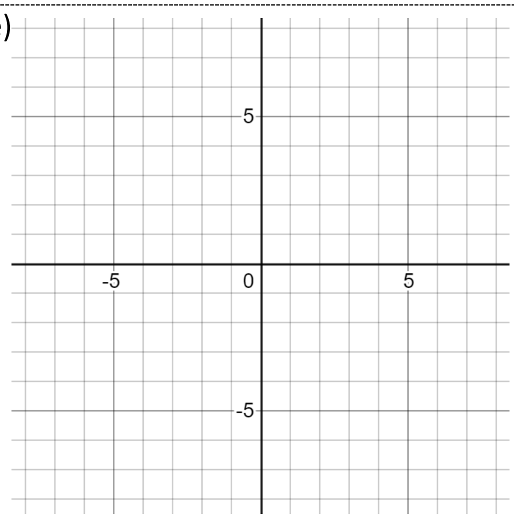
d) $(-f-g)(x)$

e) $\left(2f+\frac{1}{2}g\right)(x)$

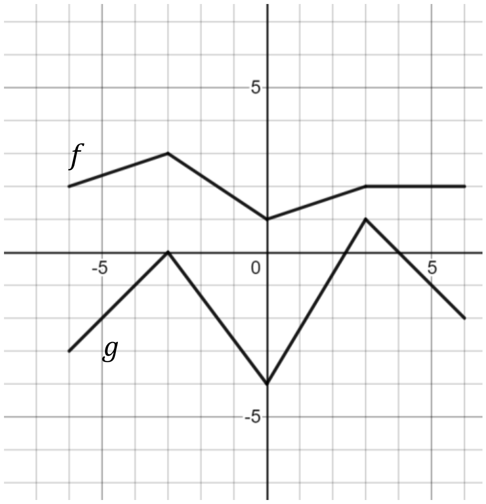
b)


c)


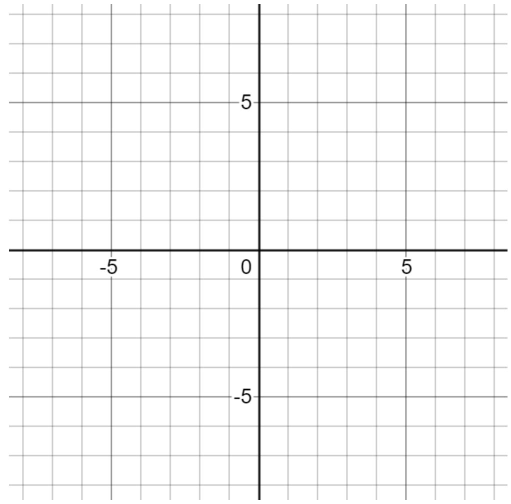
d)


e)


8. Use the graphs below, to graph the following combinations



a)



f) $(f+g)(x)$

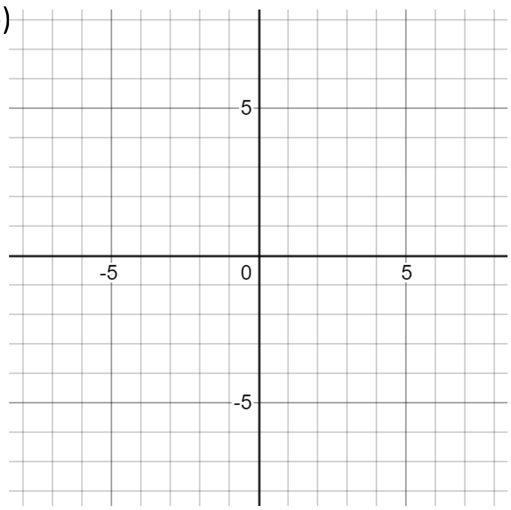
g) $(f-g)(x)$

h) $(g-f)(x)$

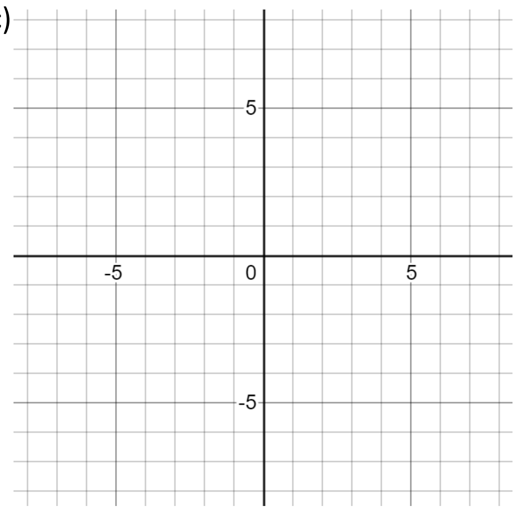
i) $(-f-g)(x)$

j) $\left(2f+\frac{1}{2}g\right)(x)$

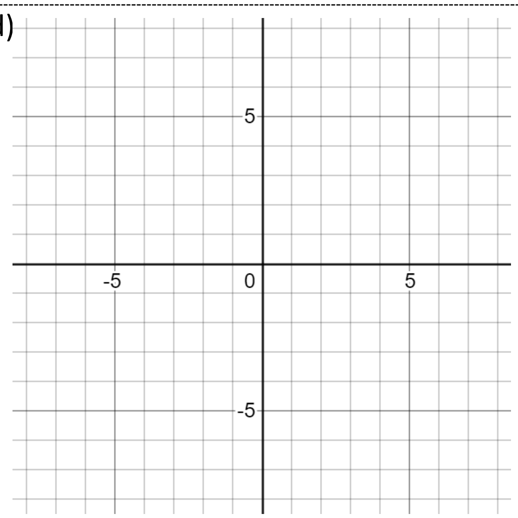
b)



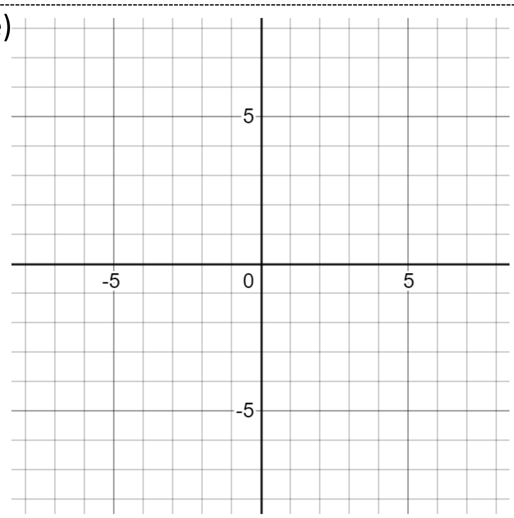
c)



d)



e)



See Website for Detailed Answer Key

Extra Work Space