Section 2.1b – Multiplication and Division of a Common Base

This booklet belongs to:_____ Block:_____

Multiplication of a Common Base

- When we start doing operations with exponents, ask a question…
- Do I have a **COMMON BASE**?
  - If the answer is **NO**, you are **done**
  - If the answer is **YES**, we can **continue**

Example 1:

$2^3 \cdot 2^4$ Do I have a **COMMON BASE**? **YUP!** It's 2

- What am I looking at then?

Remember from earlier that: $2^3 = 2 \cdot 2 \cdot 2$ and $2^4 = 2 \cdot 2 \cdot 2 \cdot 2$

So,

$$2^3 \cdot 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

What did I do? **I ADDED the Exponents**!

$$2^3 \cdot 2^4 = 2^{3+4} = 2^7$$

Example 2: Simplify the Following

i) $3^1 \cdot 3^6 = 3^{1+6} = 3^7$ $2^4 \cdot 2^4 = 2^{4+4} = 2^8$

ii) $5^5 \cdot 5^7 = 5^{5+7} = 5^{12}$ $7^9 \cdot 7^{12} = 7^{9+12} = 7^{21}$

Multiplication Rule

Must have a **COMMON BASE**

$$a^m \cdot a^n = a^{m+n}$$

Division of a Common Base

- Again, this only works with a **COMMON BASE**

Example 3:

$3^7 \div 3^5$ well we can **re-write** that as:

$$\frac{3^7}{3^5}$$

- It's a fraction and when we have the same number top and bottom we can **cancel things out**!

$$\frac{3^7}{3^5} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \frac{3 \cdot 3}{1} = 3 \cdot 3 = 3^2$$

In other words:

$$\frac{3^7}{3^5} = 3^{7-5} = 3^2$$

Example 4: Simplify the following

i) $12^5 \div 12^2 = 12^{5-2} = 12^3$ $6^8 \div 6^2 = 6^{8-2} = 6^6$

ii) $3^{54} \div 3^{51} = 3^{54-51} = 3^3$ $9^5 \div 9^7 = 9^{5-7} = 9^{-2}$

Division Rule

Must have a **COMMON BASE**

$$a^m \div a^n = a^{m-n}$$

Multiplication and Division with Negatives

- It gets tricky again when we bring **negatives** back into the fray
- We need to make sure we have a **COMMON BASE**
- Things are not always what they seem

Example 5:

$(-3)^2 \cdot (-3)^3$ Do we have a **COMMON BASE?**

- ✓ Since they are **both in brackets**, **YES, WE DO!**

So, we can do the same as we did previous:

$$(-3)^2 \cdot (-3)^3 = (-3)^{2+3} = (-3)^5$$

Example 6:

$-3^2 \cdot (-3)^3$ Do we have a **COMMON BASE**?

- ✓ Since they are **different with respect to brackets**, **NO WE DON'T**
- ✓ We need to look at how the **brackets** will **affect the result**
- ✓ Will they end up **POSITIVE** or **NEGATIVE**?

$$-3^2 \cdot (-3)^3$$

Since there are **NO** brackets, this will **ALWAYS** be negative

Since there are brackets, but the exponent is **ODD**, this will **END UP** being negative

So, we can re-write it like this:

$FOREVER\ NEGATIVE$ $FOREVER\ NEGATIVE$

$$-3^2 \cdot -3^3$$

From what we learned previously,

$$-3^2 \cdot -3^3 = (-1)3^2 \cdot (-1)3^3$$

And with some reshuffling, a now COMMON BASE and canceling out:

$$(-1)(-1)3^2 \cdot 3^3 = 3^{2+3} = 3^5$$

Example 7:

$$-4^3 \cdot (-4)^2 = \quad -4^3 \cdot 4^2 = \quad (-1)4^3 \cdot 4^2 = \quad (-1)4^{3+2} = \quad (-1)4^5 = \quad \mathbf{-4^5}$$

FOREVER NEGATIVE — FOREVER POSITIVE

Example 8:

$$-5^3 \cdot (-5)^2 \cdot (-5)^3 = \quad -5^3 \cdot 5^2 \cdot -5^3 = \quad (-1)5^3 \cdot 5^2 \cdot (-1)5^3 = \quad (-1)(-1)5^{3+2+3} = \quad \mathbf{5^8}$$

FOREVER NEGATIVE — FOREVER NEGATIVE — FOREVER POSITIVE

Division yields the same scenario

- We have to assess the BRACKET situation

Example 9:

$\frac{-5^5}{(-5)^2}$

Since there are **NO** brackets, this will **ALWAYS** be negative

Since there are **BRACKETS**, and an **EVEN** exponent, this will be positive

$$\frac{-5^5}{(-5)^2} \overset{FN}{=} \frac{-5^5}{5^2} = \quad \frac{(-1)5^5}{5^2} = \quad (-1)5^{5-2} = \quad (-1)5^3 = \quad \mathbf{-5^3}$$

Example 10:

$$\frac{2^4}{-2^2} \overset{FP}{=} \frac{2^4}{(-1)2^2} = \quad (-1)2^{4-2} = \quad (-1)2^2 = \quad \mathbf{-2^2}$$

Example 11:

$$\frac{(-3)^5}{-3^3} \overset{FN}{=} \frac{-3^5}{(-1)3^3} = \quad \frac{(-1)3^5}{(-1)3^3} = \quad (-1)(-1)3^{5-3} = \quad \mathbf{3^2}$$

Section 2.1b – Practice Questions

EMERGING LEVEL QUESTIONS

Simplify the following, leaves answer in **Exponential Form**.

1. $2^3 \cdot 2^4$

2. $3^2 \cdot 3^5$

3. $(-4)^2 \cdot (-4)^5$

4. $2^3 \cdot 2^2$

5. $3^2 \cdot 3^3$

6. $2^4 \cdot 3^2 \cdot 2^5 \cdot 3^6$

7. $2^{-2} \cdot 2^3$

8. $(4) \cdot (-4)^2 \cdot (-4^3)$

PROFICIENT LEVEL QUESTIONS

9. $3^4 \cdot -3^5 \cdot (-3)^2$

10. $(-2)^8 \cdot (-2)^{-3} \cdot (-2)^{-4}$

11. $(-5)^6 \cdot (-5)^4 \cdot (-5)^2 \cdot (-5)^3$

12. $(-3)^4 \cdot (-3)^5 \cdot (-3)^2 \cdot (-3)^6$

13. $-2^3 \cdot 2^4 \cdot -2^7 \cdot 2^3 \cdot 2^{-12}$

14. $5 \cdot -5^3 \cdot (-5)^7 \cdot 5^6 \cdot (-5)^3$

Simply the following, leave answer in Exponential Form

15. $2^7 \div 2^3$

16. $\frac{(-3)^{10}}{(-3)^2}$

17. $\frac{(7)^4}{(7)^1}$

18. $\frac{(6)^8}{(6)^8}$

19. $\frac{-5^4}{5^3}$

20. $\frac{(-2)^6}{(-2)^{-3}}$

21. $\frac{(-5)^8}{5^3}$

22. $\frac{8^{12}}{8^{-3}}$

EXTENDING LEVEL QUESTIONS

23. $2^{a+3} \cdot 2^{a-1}$

24. $\frac{5^{r+1}}{5^{r}}$

25. $3^{-a+4} \cdot 3^{a-3}$

26. $\frac{3^{2m}}{3^{m-1}}$

Answer Key – Section 2.1b

1.	2^7	2.	3^7	3.	$(-4)^7$ or -4^7
4.	2^5	5.	3^5	6.	$2^9 \cdot 3^8$
7.	2	8.	-4^6	9.	-3^{11}
10.	(-2) or -2	11.	$(-5)^{15}$ or -5^{15}	12.	$(-3)^{17}$ or -3^{17}
13.	2^5	14.	-5^{20}	15.	2^4
16.	$(-3)^8$ or 3^8	17.	7^3	18.	6^0 or 1
19.	-5	20.	$(-2)^9$ or -2^9	21.	5^5
22.	8^{15}	23.	2^{2a+2}	24.	5
25.	3	26.	3^{m+1}		

Extra Work Space