Section 2.1 – Radicals and their Properties

This booklet belongs to:_____________________________Block: ______

n^{th} Root

- In order to really understand radicals, it helps to think about it in terms of rational exponents

> ***n^{th} Root***
>
> - If $a^n = x$ then x is the n^{th} root of a $\qquad x^n = a \quad \rightarrow \quad \sqrt[n]{a} = x$

Points about Roots

1. **If a is positive** and **n is even**, then there are **two real n^{th} roots**
   Example:
   $x^2 = 9 \rightarrow x = \sqrt{9} = 3$ *or* $x = -\sqrt{9} = -3$; Since $(-3)^2 = 3^2 = 9$
   $x^2 = 6 \rightarrow x = \sqrt{6}$ *or* $x = -\sqrt{6}$; Since $(-\sqrt{6})^2 = (\sqrt{6})^2 = 6$
   $x^4 = 3 \rightarrow x = \sqrt[4]{3}$ *or* $x = -\sqrt[4]{3}$; Since $(-\sqrt[4]{3})^4 = (\sqrt[4]{3})^4 = 3$

2. **If a is negative** and **n is even**, then there are **no real number solutions**
   Example:
   $x^2 = -9 \rightarrow x = \emptyset$; *No Solution*
   $x^4 = -5 \rightarrow x = \emptyset$; *No Solution*

3. **If n is odd,** then there is **one real n^{th} root of a**
   Example:
   $x^3 = 8 \rightarrow x = \sqrt[3]{8} = 2$; because $2^3 = 8$
   $x^3 = -8 \rightarrow x = \sqrt[3]{-8} = -2$; because $(-2)^3 = -8$
   $x^5 = 4 \rightarrow x = \sqrt[5]{4}$; because $\left(\sqrt[5]{4}\right)^5 = 4$

4. **If a is zero,** there is **one real n^{th} root of a**
   Example: $x^5 = 0 \rightarrow x = 0$

> **Domain Restrictions**
>
> For variable under the radical:
>
> If the root index is even $\sqrt{x}$, $\quad x \geq 0$ *x must be positive*
>
> If the root index is odd $\sqrt[3]{x}$, $\quad$ *No Restrictions*

Properties of Radicals

- For any ***positive integer n***:

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$$

$$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}} = \frac{1}{\left(\sqrt[n]{a}\right)^m}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Section 2.1 – Practice Problems

Solve for x

1. $x^2 = 4$	2. $x^2 = -4$	3. $x^3 = 8$
4. $x^3 = -8$	5. $x^4 = 16$	6. $x^4 = -16$
7. $x^5 = 32$	8. $x^7 = -128$	9. $x^9 = 0$

Solve for x

10. $x^2 = 3$	11. $x^2 = -3$	12. $x^3 = 3$
13. $x^3 = -3$	14. $x^4 = 3$	15. $x^4 = -4$
16. $x^6 = 3$	17. $x^7 = -3$	18. $x^7 = 3$

Simplify, assume all answers are positive whole numbers, using exponent notation instead of radicals, include restrictions for variables.

19. $\sqrt{9x^2} =$	20. $\sqrt{x^5} =$	21. $\sqrt{x^6y^4} =$
22. $\sqrt{xy^4} =$	23. $\sqrt{x^2y} =$	24. $\sqrt{xy^2} =$

25. $\sqrt{x^5y^2} =$	26. $\sqrt{16x^6y^8} =$	27. $\sqrt{4x^9} =$

Simplify. Assume the variables are positive numbers.

28. $\sqrt[3]{27} =$	29. $\sqrt[3]{-27} =$	30. $\sqrt[3]{x^3} =$
31. $\sqrt[3]{-x^3} =$	32. $\sqrt[3]{x^6} =$	33. $\sqrt[3]{-x^6} =$
34. $\sqrt[4]{x^4y^6} =$	35. $\sqrt[4]{-16}$	36. $\sqrt[4]{x^5y^7} =$

Rewrite as a single radical, use exponent notation to simplify things.

37. $\sqrt[3]{\sqrt[4]{x}}$	38. $\sqrt{\sqrt[3]{x^2y}}$	39. $\sqrt[4]{\sqrt[3]{\sqrt{x}}}$
40. $\sqrt{\sqrt[3]{xy^2z^3}}$	41. $\sqrt{\sqrt{\sqrt{xy^2z^3}}}$	42. $\sqrt[4]{\sqrt[3]{\sqrt{xy^2z^3}}}$

Answer Key – Section 2.1

1. $x = \pm 2$
2. $Does\ Not\ Exist$
3. $x = 2$
4. $x = -2$
5. $x = \pm 2$
6. $Does\ Not\ Exist$
7. $x = 2$
8. $x = -2$
9. $x = 0$
10. $x = \pm\sqrt{3}$
11. $Does\ Not\ Exist$
12. $x = \sqrt[3]{3}$
13. $x = \sqrt[3]{-3}$
14. $x = \pm\sqrt[4]{3}$
15. $Does\ Not\ Exist$
16. $x = \sqrt[6]{3}$
17. $x = \sqrt[7]{-3}$
18. $x = \sqrt[7]{3}$
19. $3x$
20. $x^2\sqrt{x}; x \geq 0$
21. x^3y^2

22. $y^2\sqrt{x}; x \geq 0$
23. $x\sqrt{y}; y \geq 0$
24. $y\sqrt{x}; x \geq 0$
25. $x^2y\sqrt{x}; x \geq 0$
26. $4x^3y^4$
27. $2 \cdot x^4 \cdot \sqrt{x}; x \geq 0$
28. 3
29. -3
30. x
31. $-x$
32. x^2
33. $-x^2$
34. $xy\sqrt{y^2}$
35. $Does\ Not\ Exist$
36. $xy \cdot \sqrt[4]{xy^3}$
37. $\sqrt[12]{x}$
38. $\sqrt[6]{x^2y}$
39. $\sqrt[24]{x}$
40. $\sqrt[6]{xy^2z^3}$
41. $\sqrt[8]{xy^2z^3}$
42. $\sqrt[24]{xy^2z^3}$

Extra Work Space