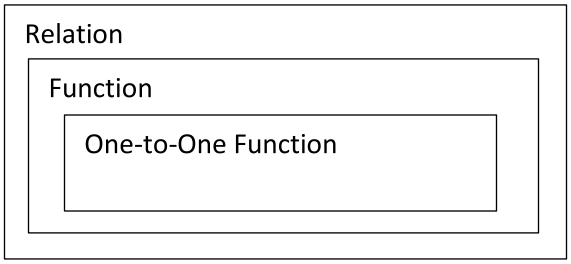
Section 2.1 – Functions and Relations Review

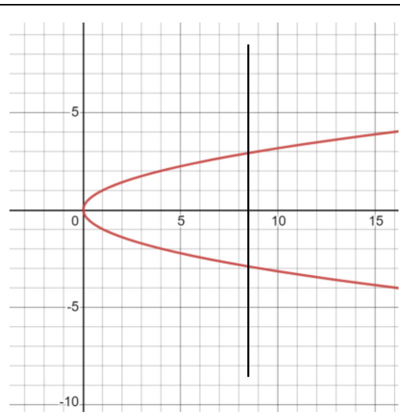
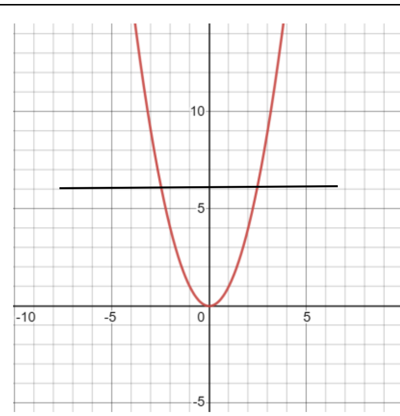
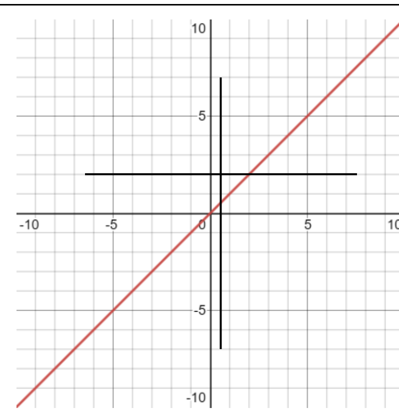
- In graph theory, we plot two-dimensional figures using (x, y) coordinate pairs
- There is a hierarchy of what it is we graph



- Everything that is graphed is known as a **Relation**
    - It is a set of ordered pairs (x, y)
    - A bunch of individual points or those points connected by a line

- In order to become **a Function** the Relation has to pass a specific test/qualification
    - For every value of **the Domain ($x - values$),** there is **one and only one value** for the **Range ($y - values$)**
    - It has to pass a **VERTICAL LINE TEST**
        - If you draw a **Vertical Line** through the graph**,** it should **only intersect it at one point**

- In order to become a **One-to-One Function,** the Function has to pass a specific test/qualification
    - For **every one value** of the **Domain** there is **only one value** in the **Range**
    - It has to pass the **Vertical Line Test first,** then it has to pass the **HORIZONTAL LINE TEST**
        - If you draw a **Vertical and Horizontal Line** through the graph, **each line only intersects the graph once**

Relation	Function	One-to-One Function
		
Does not pass the **Vertical Line Test**	**Passes the Vertical Line Test** but **Does Not Pass** the **Horizontal Line Test**	**Passes both Tests**

Function Notation

- Mathematical script can give us Function Notation $f(x) = x + 5$

- The notation $\boldsymbol{f(x)}$ is another way of writing $\boldsymbol{y\ as\ a\ function}$.
- $y = 2x - 4$ may be written as $f(x) = 2x - 4$, where $f(x)$ is read "$f\ of\ x$"

- Given $y = 2x - 4$, we could ask, "find y when $x = 5$"
- Using function notation, the same problem can be asked by writing

$f(x) = 2x - 4$, find $f(5)$. ← This is the input

- **The notation $\boldsymbol{f(5)}$ implies the value of $\boldsymbol{y\ when\ x\ is\ 5}$.**
- $f(5) = 2(5) - 4 = 6$, this **implies** that when $\boldsymbol{x\ is\ 5, y\ is\ 6}$. ← We get the output
- This gives us the point $(\boldsymbol{5, 6})$. [x is the input, $y\ or\ f(x) is\ the\ output$]

Example 1: Given $f(x) = 3x + 5$, determine the coordinates of one point on the line for $f(2)$.

Solution 1: $f(2) = 3(2) + 5 = 6 + 5 = 11$ Therefore the point is $(\boldsymbol{2, 11})$.

Example 2: Given $f(x) = 3x + 5$, determine the coordinates of the point where $f(x) = -7$.

Solution 2: $\boldsymbol{f(x) = -7}$ is the same as saying $\boldsymbol{y = -7}$

$-7 = 3x + 5 \quad \rightarrow \quad -7 - 5 = 3x$

$-12 = 3x \quad \rightarrow \quad -\frac{12}{3} = x \quad \rightarrow \quad x = -4$

Therefore, the point is $(\boldsymbol{-4, -7})$.

[You can write (x, y) coordinates as $(x, f(x))$ if you think about it as a function]

Domain and Range Notation

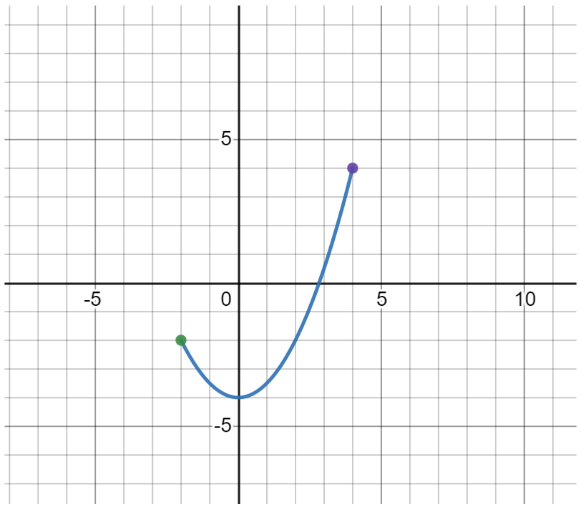
There are a number of syntax forms for Domain and Range, but I will use these primarily.

Domain	$x \geq 0$	This means x is any values greater than or equal to 0
	$x = All\ Real\ Numbers$	This means x is any value
	$-4 \leq x \leq 7$	This means x is any value between or equal to $-4\ and\ 7$
	$\{-1, 5, 7, 12\}$	This means the points are not connected, just $x - values$
Range	$Range\ values\ are\ written\ the\ same\ way, but\ with\ y\ instead\ of\ x$	
	$-4 \leq y \leq 7$	This means y is any value between or equal to $-4\ and\ 7$

Section 2.1 – Practice Problems

1. For each graph, identify the Domain and Range and whether it is a function or not

a)

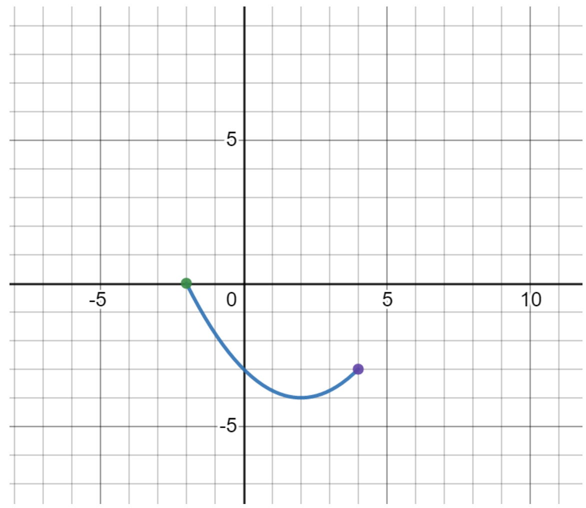


D:

R:

$Function$:

b)

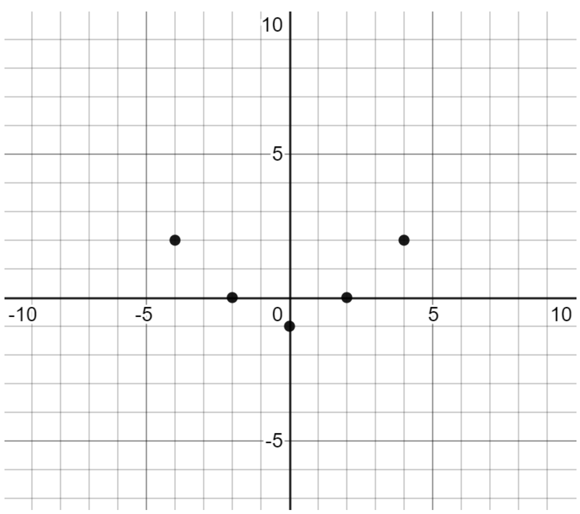


D:

R:

$Function$:

c)

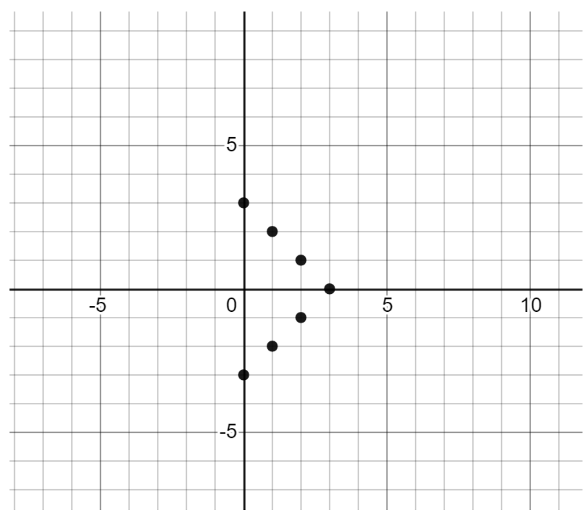


D:

R:

$Function$:

d)

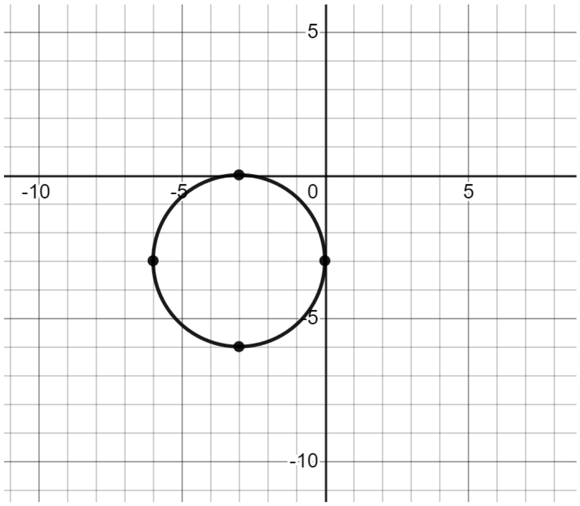


D:

R:

$Function$:

e)

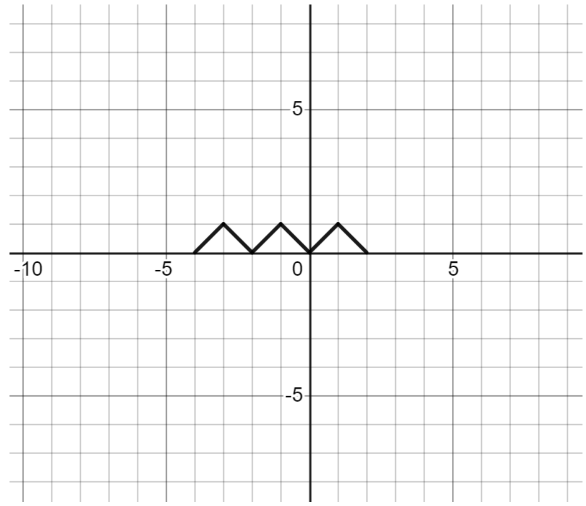


D:

R:

$Function$:

f)

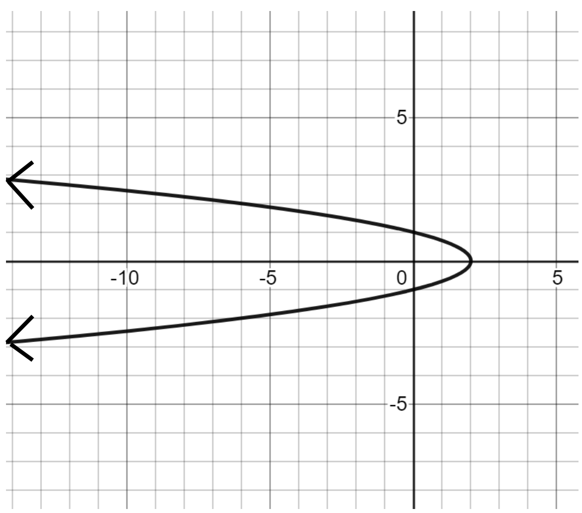


D:

R:

$Function$:

g)

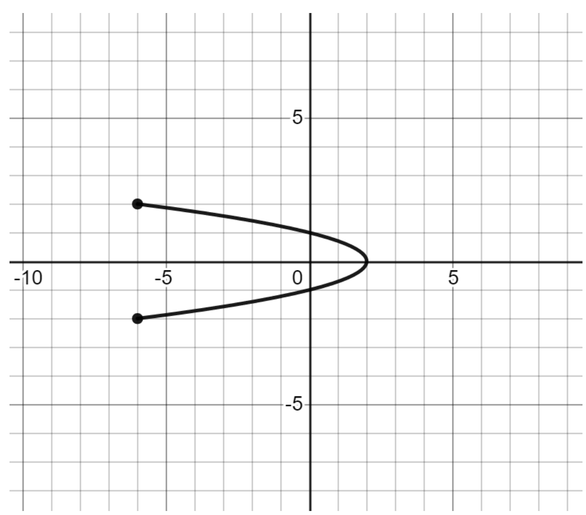


D:

R:

$Function$:

h)



D:

R:

$Function$:

For $f(x) = 3x - 2,\ find$:

2. $f(3)$

3. $f(-4)$

4. $f(k)$

5. $f(2x - 1)$

6. $f(x + h)$

7. $f(x) + f(h)$

For $f(x) = 4x + 5,\ find$:

8. $f(3)$

9. $f(-4)$

10. $f(k)$

11. $f(2x-1)$

12. $f(x+h)$

13. $f(x)+f(h)$

For $f(x)=-5x+2,\ \ find\ x\ when$:

14. $f(x)=-12$

15. $f(x)=7$

Answer Key

1.
a) $D: -2 \le x \le 4$ $R: -4 \le y \le 4$ $Function: Yes$
b) $D: -2 \le x \le 4$ $R: -4 \le y \le 0$ $Function: Yes$
c) $D: \{-4, -2, 0, 2, 4\}$ $R: \{-1, 0, 2\}$ $Function: Yes$
d) $D: \{0, 1, 2, 3\}$ $R: \{-3, -2, -1, 0, 1, 2, 3\}$ $Function: No$
e) $D: -6 \le x \le 0$ $R: -6 \le y \le 0$ $Function: No$
f) $D: -4 \le x \le 2$ $R: 0 \le y \le 1$ $Function: Yes$
g) $D: x \le 2$ $R: All\ Real\ Numbers$ $Function: No$
h) $D: -6 \le x \le 2$ $R: -2 \le y \le 2$ $Function: No$

2. 7
3. -14
4. $3k - 2$
5. $6x - 5$
6. $3x + 3h - 2$
7. $3x + 3h - 4$
8. 17
9. -11
10. $4k + 5$
11. $8x + 1$
12. $4x + 4h + 5$
13. $4x + 4h + 10$
14. $x = \frac{14}{5}$
15. $x = -1$

Extra Work Space