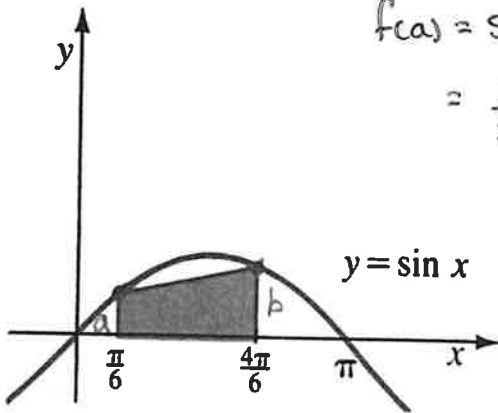


Section 10 – Review and Preview – Practice Problems

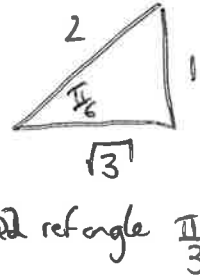
1. Calculate the area of the following shaded regions.

a)



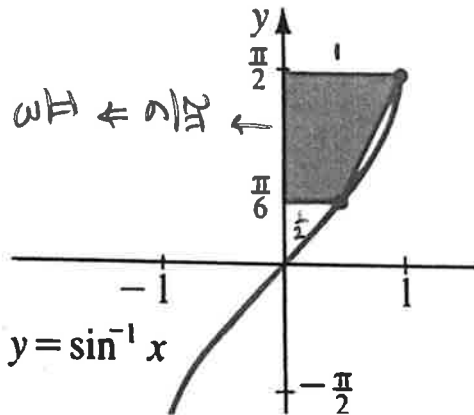
$$f(a) = \sin \frac{\pi}{6} = \frac{1}{2}$$

$$f(b) = \sin \frac{4\pi}{6} = \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$



$$A = \frac{1}{2} \left(\frac{\pi}{6} \right) \left(\frac{1}{2} + \frac{\sqrt{3}}{2} \right) \rightarrow \frac{\pi}{8} (1 + \sqrt{3})$$

b)

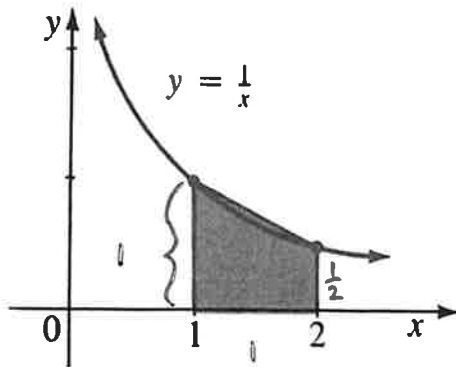


$$A = \frac{1}{2} \left(\frac{\pi}{3} \left(1 + \frac{1}{2} \right) \right)$$

$$= \frac{1}{2} \left(\frac{\pi}{3} \left(\frac{3}{2} \right) \right)$$

$$= \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{4}$$

c)



$$A = \frac{1}{2} \left(1 \cdot \left(1 + \frac{1}{2} \right) \right)$$

$$= \frac{1}{2} \left(\frac{3}{2} \right)$$

$$= \frac{3}{4}$$

2. Write each series in expanded form.

a)

$$\sum_{i=1}^5 (i^2 + 1) \Rightarrow (1^2 + 1) + (2^2 + 1) + (3^2 + 1) + (4^2 + 1) + (5^2 + 1)$$

$$= 2 + 5 + 10 + 17 + 26$$

$$= \boxed{60}$$

b)

$$\sum_{i=1}^4 \left(\frac{i}{4}\right) f(i) = \frac{1}{4}(f(1)) + \frac{1}{2}(f(2)) + \frac{3}{4}(f(3)) + f(4)$$

c)

$$\sum_{i=1}^n \frac{3}{n} f\left(1 + \frac{3}{4}i\right) \rightarrow \frac{3}{n} f\left(1 + \frac{3}{4}\right) + \frac{3}{n} f\left(1 + \frac{3}{2}\right) + \frac{3}{n} f\left(1 + \frac{9}{4}\right) + \dots + \frac{3}{n} f\left(1 + \frac{3}{4}n\right)$$

3. Evaluate the following

a) $3 + 7 + 11 + \dots + (4n - 1)$

sum of natural #'s

$$\sum_{i=1}^n 4n - 1 = 4 \sum_{i=1}^n n - \sum_{i=1}^n 1$$

$$\rightarrow \frac{4(n(n+1))}{2} - n$$

$$= 2n(n+1) - n$$

$$\rightarrow 2n^2 + 2n - n$$

$$= 2n^2 + n$$

$$= \boxed{n(2n+1)}$$

b) $1 + 3 + 9 + 27 + \dots + 3^{n-1}$

Geometric Series $r=3$
 $a=1$

$$\rightarrow \frac{1(3^n - 1)}{3 - 1} = \frac{3^n - 1}{2} = \boxed{\frac{1}{2}(3^n - 1)}$$