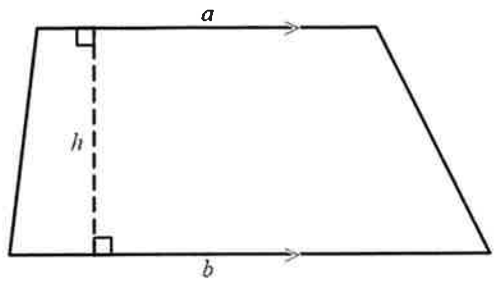
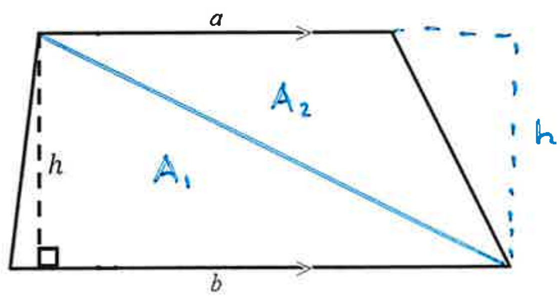
Review and Preview to Chapter 10

Area of a Trapezoid

A **trapezoid** is a quadrilateral with one pair of opposite sides parallel. The trapezoid shown below has parallel sides of lengths a and b. The perpendicular distance between these parallel sides is the height h of the trapezoid.



The area of the trapezoid can be calculated by separating the trapezoid into two triangles and adding their areas together.



$A_2 = \frac{1}{2}ah$

$A_1 = \frac{1}{2}bh$

$A_1 + A_2 = \frac{1}{2}ah + \frac{1}{2}bh$

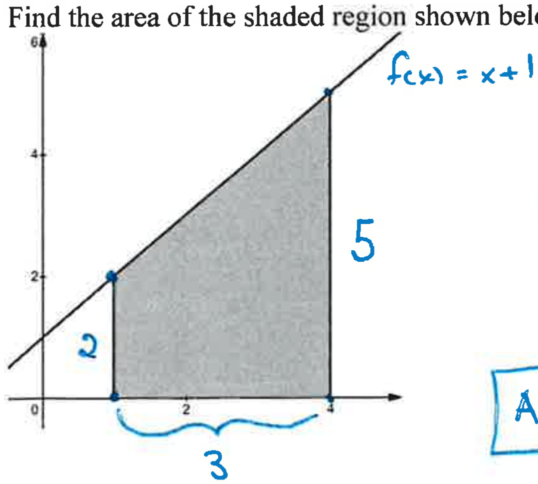
$= \frac{1}{2}h(a+b)$

Area of a Trapezoid

$$A = \frac{1}{2}(a+b)h$$

Ex. 1

Find the area of the shaded region shown below.



$A = \frac{1}{2}(3)(2+5)$

$= \frac{1}{2}(3)(7)$

$\boxed{A = 10.5}$

Sigma Notation

Recall that a series is the sum of a sequence. A series can be written using sigma notation.

$$\boxed{\sum_{i=1}^{n} t_i = t_1 + t_2 + t_3 + \cdots + t_n}$$

i = t

Ex. 1 i = 1 2 3 4 5

Express the series $1 + 3 + 5 + 7 + 9$ in sigma notation.

Consecutive odd integers

Each term given by: $2i - 1$, $i \in \mathbb{N}$

therefore: $\sum_{i=1}^{5} (2i-1)$

Basic Properties of Sigma Notation

(1) $\sum_{i=1}^{n} c = c + c + c + \cdots + c = nc$, c is a constant

(2) $\sum_{i=1}^{n} ct_i = c\sum_{i=1}^{n} t_i$, c is a constant

(3) $\sum_{i=1}^{n} (t_i + s_i) = \sum_{i=1}^{n} t_i + \sum_{i=1}^{n} s_i$

Ex. 2

Use the basic properties of sigma notation to express the following series in terms of monomial summations.

$$\sum_{i=1}^{n} (3i-2)^2 \leftarrow \text{FOIL}$$

$$\sum_{i=1}^{n} (3i-2)(3i-2) \Rightarrow \sum_{i=1}^{n} 9i^2 - 12i + 4$$

$$\sum_{i=1}^{n} 9i^2 - \sum_{i=1}^{n} 12i + \sum_{i=1}^{n} 4 \quad \text{(Property 3)}$$

$$\boxed{9\sum_{i=1}^{n} i^2 - 12\sum_{i=1}^{n} i + 4n} \quad \text{(Property 2)}$$

Sum of a Series

We will need to know the sums of some special series which are listed below.

(1) Sum of an **arithmetic series**:

$$a + (a + d) + (a + 2d) + \cdots + [a + (n - 1)d] = \frac{n}{2}[2a + (n - 1)d]$$

(2) Sum of a **geometric series**:

$$a + ar + ar^2 + \cdots + a^{n-1} = \frac{a(r^n - 1)}{r - 1}$$

(3) Sum of the **natural numbers**:

$$\sum_{i=1}^{n} i = \frac{n(n + 1)}{2}$$

(4) Sum of the **squares** of the **natural numbers**:

$$\sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}$$

(5) Sum of the **cubes** of the **natural numbers**:

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n + 1)^2}{4}$$

Ex. 1

Evaluate the following sum.

$$\sum_{i=1}^{n} (3i^2 - 2i)$$

$$\sum_{i=1}^{n} 3i^2 - \sum_{n=1}^{n} 2i \rightarrow 3\sum_{i=1}^{n} i^2 - 2\sum_{i=1}^{n} i$$

(4) (3)

$$\rightarrow 3\left(\frac{n(n+1)(2n+1)}{6}\right) - 2\left(\frac{n(n+1)}{2}\right)$$

$$\rightarrow \frac{3n(n+1)(2n+1)}{6} - \frac{6n(n+1)}{6} = \frac{3n[(n+1)(2n+1) - 2(n+1)]}{6}$$

$$= \frac{n(n+1)[(2n+1) - 2]}{2} = \boxed{\frac{n(n+1)(2n-1)}{2}}$$

Homework Assignment

- p. 448 #1 omit 1b