Section 10.2 – Practice Problems

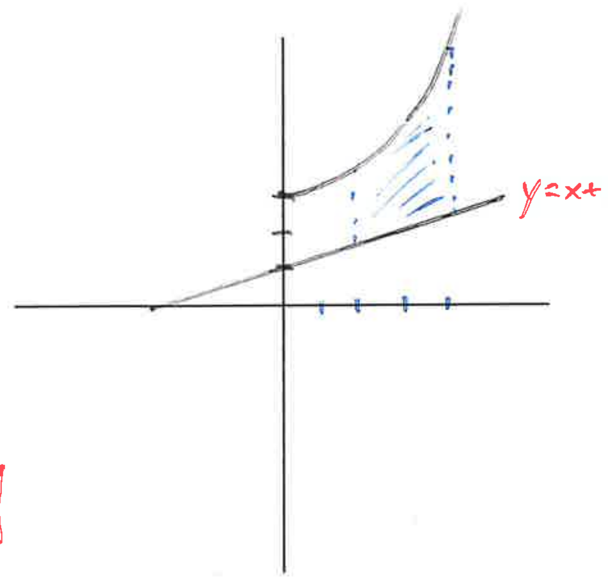
1. Find the area of the region between the given curves. Include a sketch of the region.

a) $y = x^2 + 3$ and $y = x + 1$ from 2 to 4

above below

$x^2+3-(x+1)$ $f(x) = x^2 - x + 2$

$A(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x$



$F(4) - F(2)$

$\frac{1}{3}(4)^3 - \frac{1}{2}(4)^2 + 2(4) - \left[\frac{1}{3}(2)^3 - \frac{1}{2}(2)^2 + 2(2)\right]$

$\frac{64}{3} - 8 + 8 - \frac{8}{3} + 2 - 4 = \boxed{50/3}$

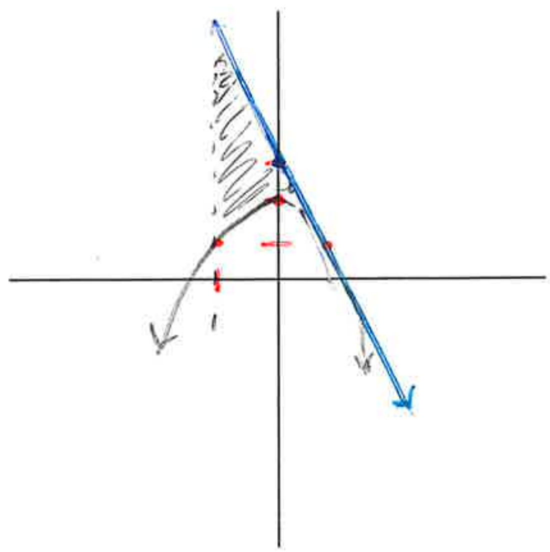
---

b) $y = 2 - x^2$ and $y = -2x + 3$ from -1 to 1

below above

$-2x+3-(2-x^2) \rightarrow f(x) = x^2 - 2x + 1$

$A(x) = \frac{1}{3}x^3 - x^2 + x$



$F(1) - F(-1)$

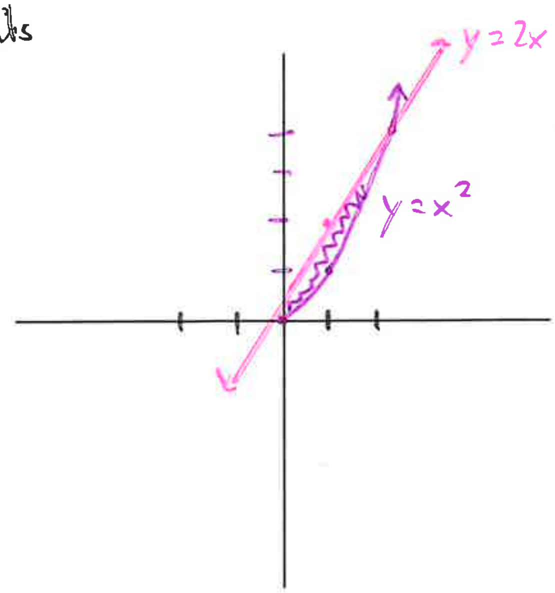
$\frac{1}{3} - 1 + 1 - \left[-\frac{1}{3} - 1 - 1\right]$

$\frac{1}{3} - \left(-2\frac{1}{3}\right) = \boxed{2\,^2/_3}$

above

c) $y = x^2$ and $y = 2x$ need intersection points

$x^2 = 2x \rightarrow x^2 - 2x = 0$ $x = 0$
$x(x-2) = 0$ $x = 2$



From $x = 2$ to $x = 0$

$f(x) = 2x - x^2$

$F(x) = x^2 - \frac{1}{3}x^3$

$F(2) - F(0)$

$4 - 8/3 - 0 = \boxed{4/3}$

---

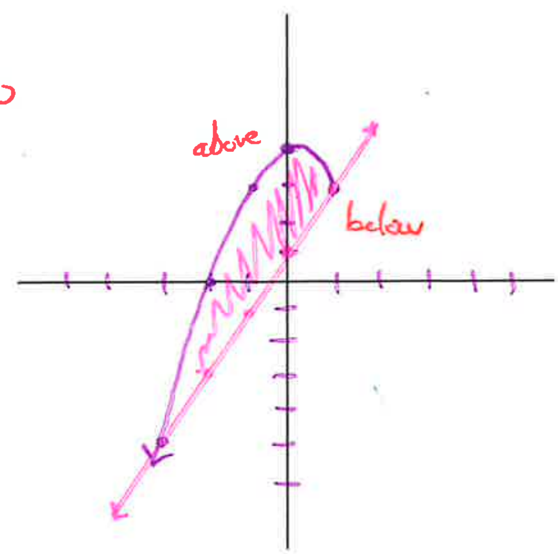
d) $y = 4 - x^2$ and $2x - y + 1 = 0$

$2x + 1 = y$

$4 - x^2 = 2x + 1 \longrightarrow x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

Intersection: $x = -3$
$x = 1$



$4 - x^2 - [2x + 1] = -x^2 - 2x + 3$

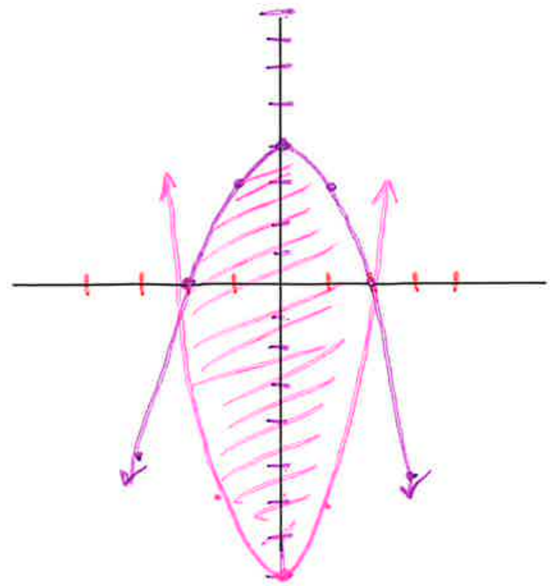
$A(x) = -\frac{1}{3}x^3 - x^2 + 3x$

$F(1) - F(-3)$

$-\frac{1}{3} - 1 + 3 - [9 - 9 - 9]$

$\boxed{32/3}$

above below
e) $y = 4 - x^2$ and $y = 2x^2 - 8$



Intersection: $4 - x^2 = 2x^2 - 8$

$12 = 3x^2$ $x = \pm 2$

$x^2 = 4$

$f(x) = 4 - x^2 - (2x^2 - 8)$

$= 12 - 3x^2$

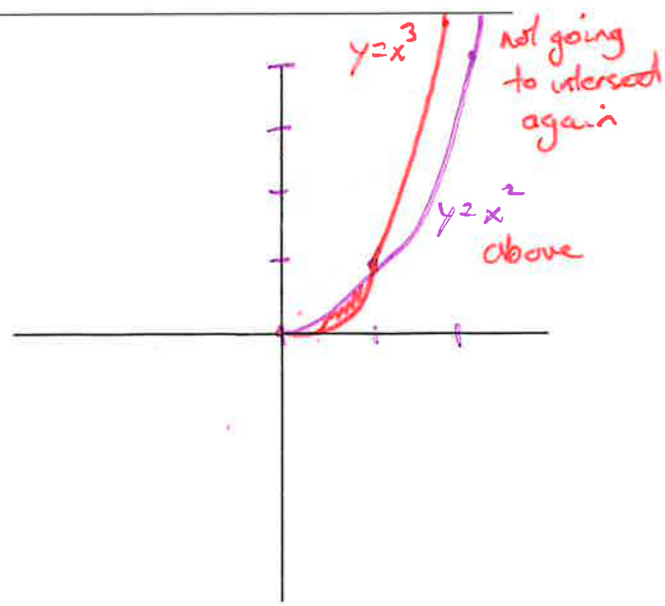
$A(x) = 12x - x^3$

$F(2) - F(-2)$

$24 - 8 - (-24 + 8)$

$48 - 16 = \boxed{32}$

f) $y = x^2$ and $y = x^3$



Intersect $x^2 = x^3$

$0 = x^3 - x^2$ → $0 = x^2(x - 1)$

$x = 0$

$x = 1$

$x^3 - x^2 = x^3 - x^2$

$f(x) = x^2 - x^3$

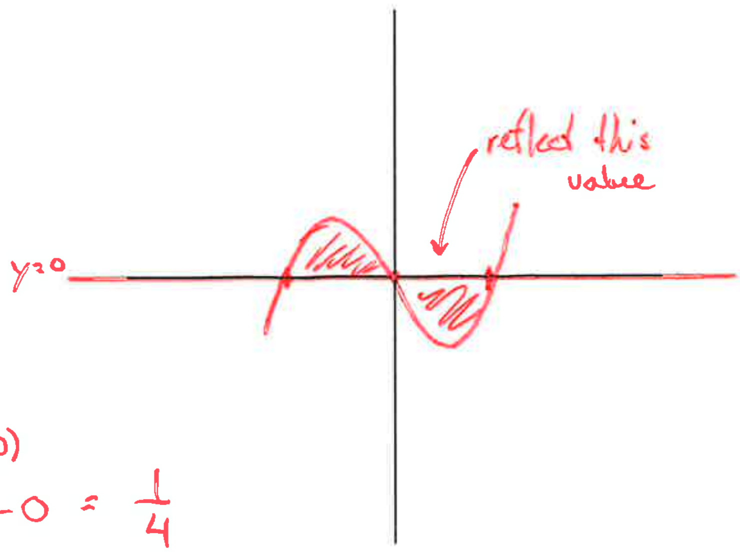
$A(x) = \frac{1}{3}x^3 - \frac{1}{4}x^4$

$F(1) - F(0)$

$\frac{1}{3} - \frac{1}{4} - 0 \Rightarrow \boxed{\frac{1}{12}}$

g) $y = x^3 - x$ and $y = 0$

$x(x^2-1)$ intersect at $x=0$, $x=\pm 1$



$f_1(x) = -(x^3-x)$

$f_2(x) = x^3 - x$

$A_1(x) = -\frac{1}{4}x^4 + \frac{1}{2}x^2 \rightarrow F_1(1) - F_1(0) = -\frac{1}{4} + \frac{1}{2} - 0 = \frac{1}{4}$

$A_2(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 \rightarrow F_2(1) - F_2(0)$

$\frac{1}{4} - \frac{1}{2} - 0 = \frac{1}{4}$

$\boxed{A(1) = \frac{1}{2}}$

h) $y = x^3 + 8$ and $y = 4x + 8$

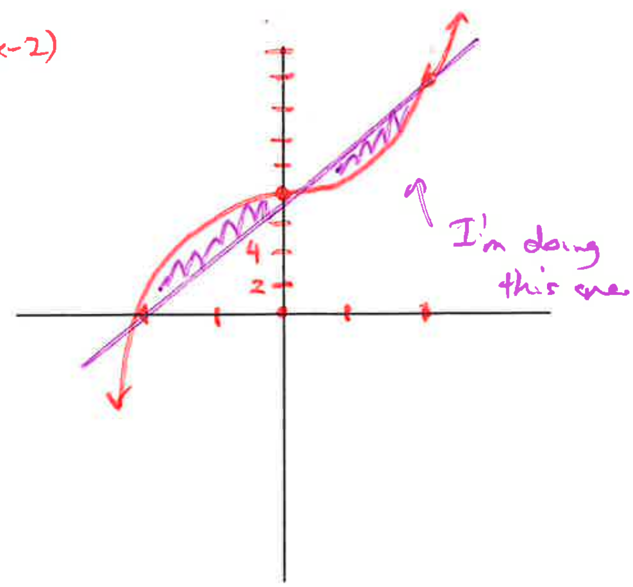
Intersection: $x^3+8 = 4x+8$

$x^3 - 4x = 0$

$x(x^2-4) = 0 \rightarrow x(x+2)(x-2)$

$x=0$

$x=\pm 2$



Like the previous question regions are symmetric about y-axis compute one and double it.

$4x+8-(x^3+8)$

$f(x) = -x^3+4x$

$A(x) = -\frac{1}{4}x^4 + 2x^2$

$4(2) = \boxed{8}$

$F(2) - F(0)$

$-4+8-0 = 4$