Section 10.1 – Practice Problems

1. Find the area under the given curve from a to b (Drawing the scenario helps)

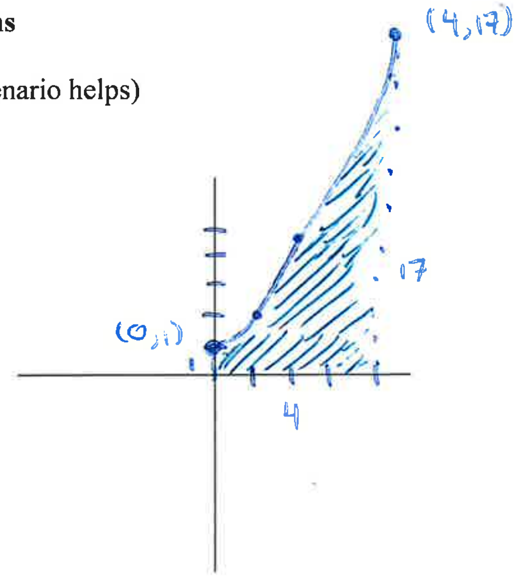
a) $y = x^2 + 1$ from 0 to 4

$A'(x) = x^2 + 1$

$A(x) = \frac{1}{3}x^3 + x$ from $0 \to 4$

$A(4) = F(4) - F(0)$

$= \frac{64}{3} + 4 - 0 \Rightarrow \frac{64}{3} + \frac{12}{3} = \boxed{\frac{76}{3}}$



b) $y = -x^2 + 1$ from $-1/2$ to $1/4$

$A'(x) = -x^2 + 1$

$A(x) = -\frac{1}{3}x^3 + x$

$F(\frac{1}{4}) - F(-\frac{1}{2}) = A(\frac{1}{4})$

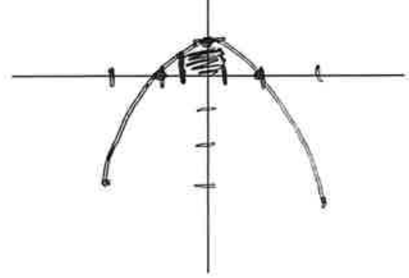
$-\frac{1}{3}\left(\frac{1}{4}\right)^3 + \frac{1}{4} - \left[-\frac{1}{3}\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)\right]$

$-\frac{1}{192} + \frac{1}{4} - \left[\frac{1}{24} - \frac{1}{2}\right]$

$\boxed{45/64}$

$y = -\left(-\frac{1}{2}\right)^2 + 1 = -\frac{1}{4} + 1 = 3/4$

$y = -\left(\frac{1}{4}\right)^2 + 1 = -\frac{1}{16} + 1 = 5/16$



c) $y = x^2 - 1$ from -4 to -2

$A'(x) = x^2 - 1$

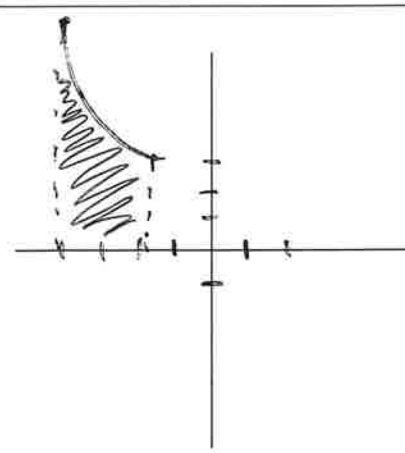
$A(x) = \frac{1}{3}x^3 - x$

$F(-2) - F(-4) = A(-2)$

$\frac{1}{3}(-2)^3 - (-2) - \left[\frac{1}{3}(-4)^3 - (-4)\right]$

$-\frac{8}{3} + 2 - \left[-\frac{64}{3} + 4\right]$

$= \boxed{50/3}$



d) $y = \frac{1}{x}$ from e to e^2

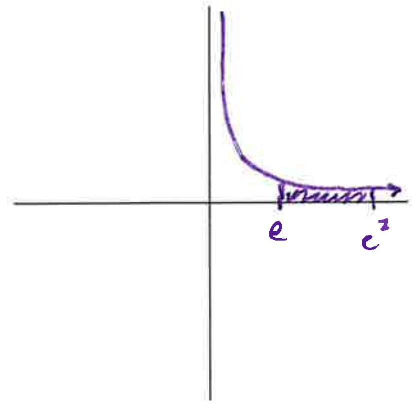
$A'(x) = \frac{1}{x}$

$A(x) = \ln x$

$F(e^2) - F(e) = A(e^2)$

$\ln e^2 - \ln e \rightarrow 2\ln e - \ln e \rightarrow \ln e(2-1)$

$\ln e = \boxed{1}$



e) $y = 2\cos x$ from $-\frac{\pi}{2}$ to 0

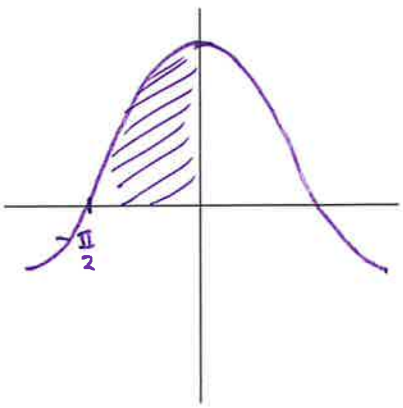
$A'(x) = 2\cos x$

$A(x) = 2\sin x$

$A(0) = F(0) - F(-\frac{\pi}{2})$

$= 2\sin 0 - 2\sin -\frac{\pi}{2} \rightarrow 2(0) - 2(-1)$

$\boxed{2}$



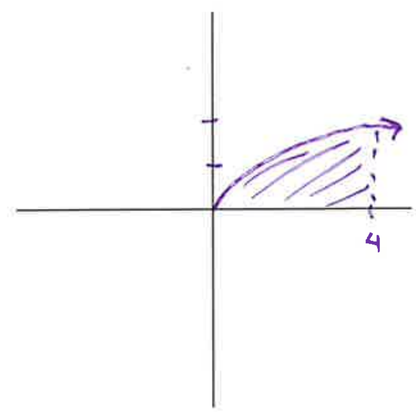
f) $y = \sqrt{x}$ from 0 to 4

$A'(x) = x^{\frac{1}{2}} \rightarrow A(x) = \frac{2}{3}x^{3/2}$

$A(4) = F(4) - F(0)$

$= \frac{2}{3}(4)^{3/2} - \frac{2}{3}(0)^{3/2}$

$= 16/3 - 0 = \boxed{16/3}$



g) $y = -\sin x$ from $-\pi$ to 0

$A'(x) = -\sin x$

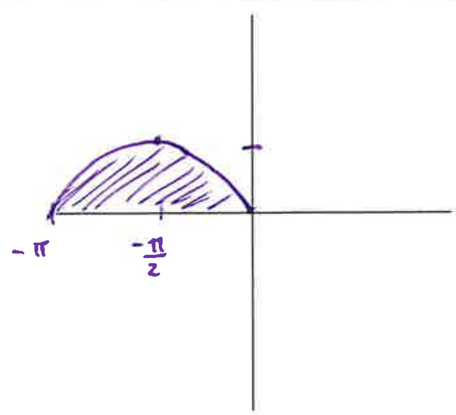
$A(x) = \cos x$

$A(0) = F(0) - F(-\pi)$

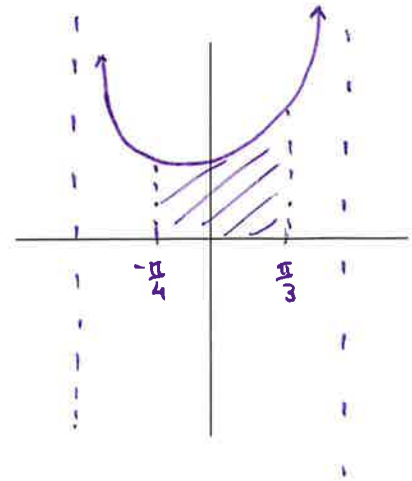
$A(0) = \cos 0 - \cos(-\pi)$

$= 1 - (-1)$

$= \boxed{2}$



h) $y = \sec^2 x$ from $-\frac{\pi}{4}$ to $\frac{\pi}{3}$



$A'(x) = \sec^2 x$

$A(x) = \tan x$

$A(\frac{\pi}{3}) = F(\frac{\pi}{3}) - F(-\frac{\pi}{4})$

$= \tan(\frac{\pi}{3}) - \tan -\frac{\pi}{4}$

$= \sqrt{3} - (-\frac{1}{1}) = \boxed{\sqrt{3}+1}$

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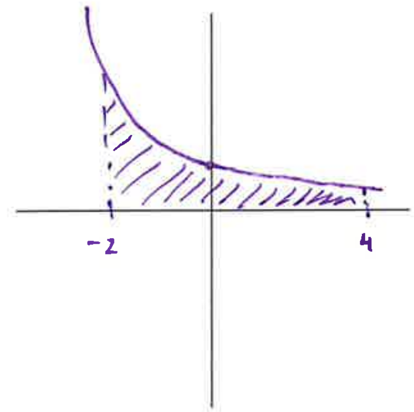
i) $y = e^{-x}$ from -2 to 4



$A'(x) = e^{-x}$

$A(x) = e^{-x}(-1) = -e^{-x}$

$A(4) = F(4) - F(-2)$

$= -e^{-4} - (-e^{2})$

$= e^{2} - e^{-4}$

$e^2 - \frac{1}{e^4}$

$= \boxed{\frac{e^6 - 1}{e^4}}$

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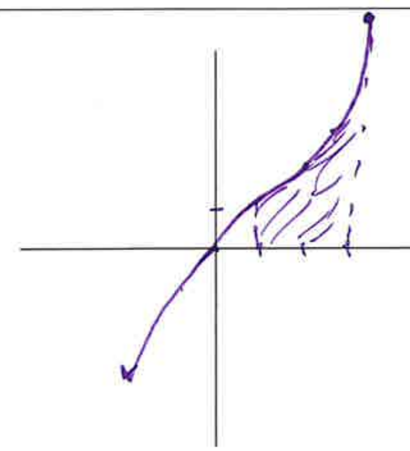
j) $y = x^3$ from 1 to 3



$A'(x) = x^3$

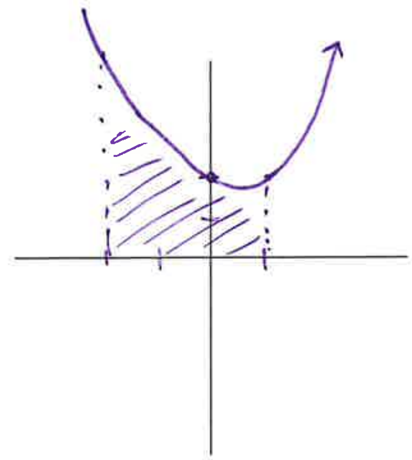
$A(x) = \frac{1}{4}x^4$

$A(3) = F(3) - F(1)$

$A(3) = \frac{1}{4}(3)^4 - \frac{1}{4}(1)^4$

$= \frac{81}{4} - \frac{1}{4} = \frac{80}{4} = \boxed{20}$

k) $y = x^2 - x + 2$ from -2 to 1



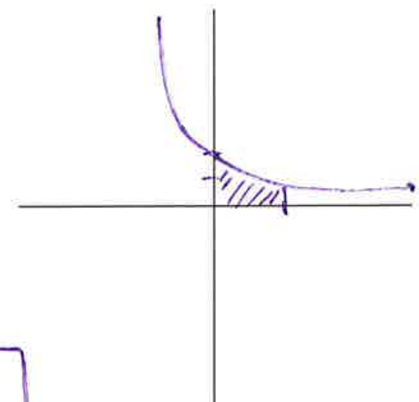
$A'(x) = x^2 - x + 2$

$A(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x$

$A(1) = F(1) - F(-2)$

$= \frac{1}{3} - \frac{1}{2} + 2 - \left[-\frac{8}{3} - 2 - 4\right] = \boxed{10.5}$

l) $y = 2e^{-2x}$ from 0 to 1



$A'(x) = 2e^{-2x}$

$A(x) = 2e^{-2x} \cdot -\frac{1}{2}$

$= -e^{-2x}$

$A(1) = F(1) - F(0)$

$-e^{-2(1)} - (-e^{-2(0)})$

$-e^{-2} + e^{0}$

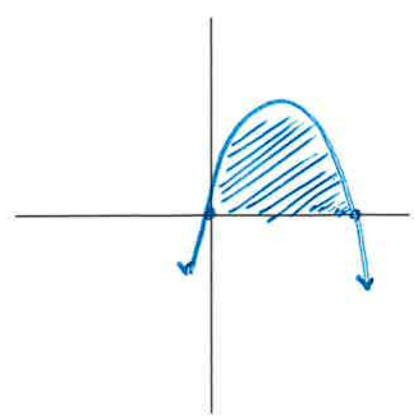
$1 - e^{-2}$

$= 1 - \frac{1}{e^2} = \boxed{\frac{e^2 - 1}{e^2}}$

2. Find the area below the given curve and above the x-axis.

a) $y = 4x - x^2$

x-int: $(0,0)$
$(4,0)$



$A'(x) = 4x - x^2$

$A(x) = \frac{4x^2}{2} - \frac{1}{3}x^3$

$= 2x^2 - \frac{1}{3}x^3$

$A(4) = F(4) - F(0)$

$= 2(4)^2 - \frac{1}{3}(4)^3 - 0$

$= 32 - \frac{64}{3} = \boxed{32/3}$

b) $y = 9 - x^2$ $\Rightarrow$ $(3-x)(3+x)$

x-ints: (3,0)
(−3,0)

$A'(x) = f(x) = 9 - x^2$

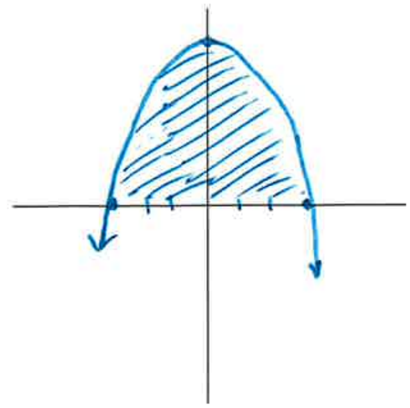
$A(x) = 9x - \frac{1}{3}x^3$

$[F(3) - F(0)]\,2$ → symmetry

$27 - \frac{1}{3}(27)$

$27 - 9 = 18$

$18 \cdot 2 = 36$



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c) $y = x^2 - x^3$ from -2 to 1

$y = x^2(1-x)$

x-int: (0,0)
(1,0)

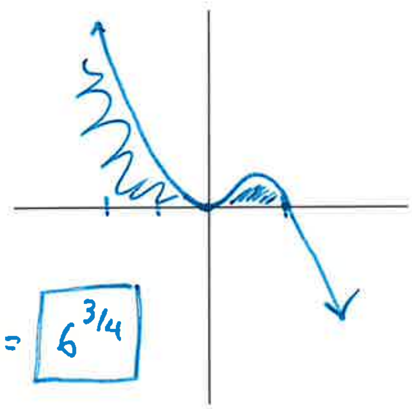
$A(x) = \frac{1}{3}x^3 - \frac{1}{4}x^4$

$F(1) - F(0) \rightarrow \frac{1}{3} - \frac{1}{4} - 0 = \frac{1}{12}$

+

$F(0) - F(-2) \rightarrow 0 - [-\frac{8}{3} - 4] = \frac{20}{3}$

$\frac{1}{12} + \frac{20}{3} = \boxed{6\,\frac{3}{4}}$



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d) $y = x^2 - x^4$

$= x^2(1 - x^2)$

$x^2(1-x)(1+x)$

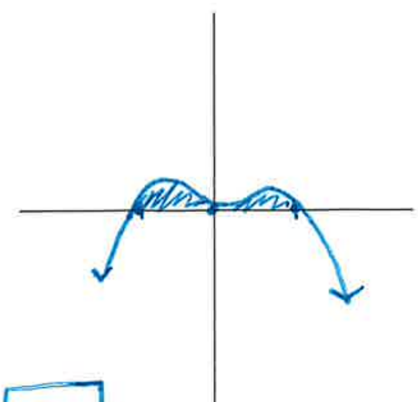
$A(x) = \frac{1}{3}x^3 - \frac{1}{5}x^5$

$F(1) - F(0) \rightarrow \frac{1}{3} - \frac{1}{5} - 0 = \frac{2}{15}$

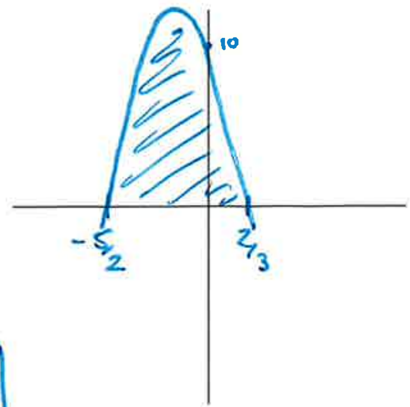
+

$F(0) - F(-1) \rightarrow 0 - [-\frac{1}{3} + \frac{1}{5}] = \frac{2}{15}$

$\boxed{\frac{4}{15}}$



e) $y = 10 - 11x - 6x^2$ $\rightarrow$ $(5+2x)(2-3x)$



$A(x) = 10x - \frac{11}{2}x^2 - 2x^3$

$F(2/3) - F(-5/2)$

$\frac{20}{3} - \frac{44}{18} - \frac{16}{27} - \left[-25 + \frac{275}{8} + \frac{125}{4}\right]$

$= \frac{20}{3} - \frac{44}{18} - \frac{16}{27} + 25 - \frac{275}{8} - \frac{125}{4}$ $\boxed{\frac{6859}{216}}$

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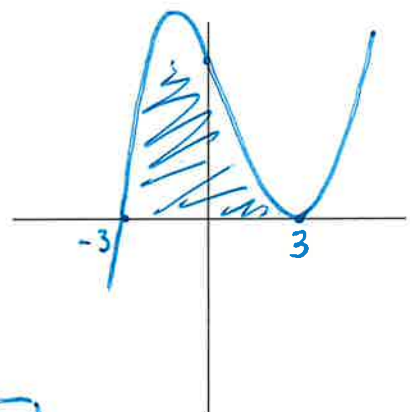
f) $y = x^3 - 3x^2 - 9x + 27$

$x^2(x-3) - 9(x-3)$

$(x^2-9)(x-3)$

$(x+3)(x-3)^2$



$A(x) = \frac{1}{4}x^4 - x^3 - \frac{9}{2}x^2 + 27x$

$F(3) - F(-3)$

$\frac{81}{4} - 27 - \frac{81}{2} + 81 - \left[\frac{81}{4} + 27 - \frac{81}{2} - 81\right] = \boxed{108}$

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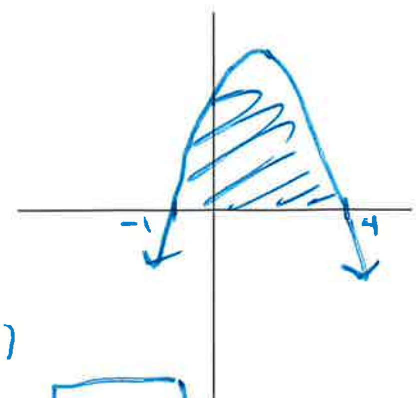
g) $y = 4 + 3x - x^2$

$= -(x^2 - 3x - 4)$

$-(x-4)(x+1)$



$A(x) = 4x + \frac{3}{2}x^2 - \frac{1}{3}x^3$

$F(4) = 16 + 24 - \frac{64}{3} = 56/3$

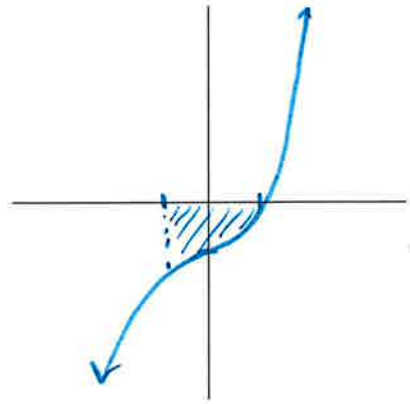
$F(-1) = -4 + \frac{3}{2} + \frac{1}{3} = -13/6$

$F(4) - F(-1)$

$\frac{56}{3} + 13/6 = \boxed{125/6}$

3. Find the area between $y = x^3 - 1$ and the x-axis from $x = -1$ to $x = 1$.

consider a reflected version of the area above the axis so use $-f(x)$



$-x^3+1$

$A(x) = -\frac{1}{4}x^4 + x$

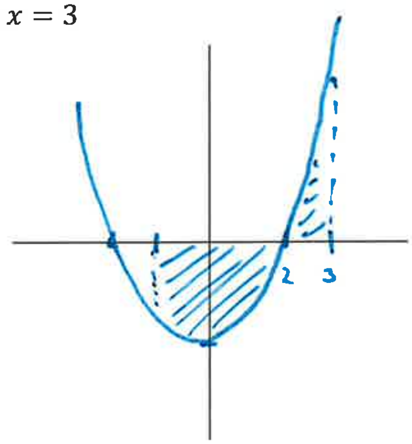
$F(1) - F(-1)$

$-\frac{1}{4} + 1 - \left[-\frac{1}{4} - 1\right]$

$= \boxed{2}$

4. Find the area between $y = x^2 - 4$ and the x-axis from $x = -1$ to $x = 3$

Reflect the chunk between -1 and 2



$A_1(x) = \frac{1}{3}x^3 - 4x$

$F_1(3) - F_1(2) = 9 - 12 - \left[8/3 - 8\right]$

$= 7/3$

$A_2'(x) = -x^2 + 4$

$A_2 = -\frac{1}{3}x^3 + 4x$

$F_2(2) - F_2(-1)$

$-8/3 + 8 - \left[\frac{1}{3} - 4\right]$

$= 9$

$9 + 7/3 = \boxed{11\,1/3}$