Section 1.4 – Interest and Compound Interest – Savings

Simple Interest

- Interest money that is added to the PRINCIPAL (money invested or borrowed)
- It is decided based on a given percentage rate
- Low Interest Rates
    - Awesome for a borrower, you pay less to the lender
    - Bad if you are trying to grow interest on savings
- High Interest Rates
    - Brutal/Crippling for a borrower, you pay more to the lender
    - Great if it is interest earned in savings

- **Simple Interest** is calculated this way: $\boldsymbol{I = Prt}$

- I: $is\ the\ amount\ of\ interest\ calculated$
- P: $is\ the\ Principal\ (the\ amount\ of\ money\ borrowed\ or\ saved)$
- r: $is\ the\ Percentage\ Rate, expressed\ as\ a\ decimal\ (25\% = 0.25)$
- t: $is\ Time, in\ years$

Example: How much interest is paid over a $3\ year$ period on $\$5000$ at a APR (Annual Percentage Rate) of 3%?

Solution: $I = Prt$

Here: $P = \$5000$ $t = 3$ $r = 3\% = 0.03$

$I = (5000)(0.03)(3)$

$I = 450$

So in 5 years without touching the money, you earn an extra $\$450$

- You can simply flip the scenario to borrowing and if you borrowed $5000 you would owe an extra $450 back on top of the $5000
- You can also use algebra to manipulate the equation to solve for $P, r, or\ t$

Example: What Principal at 4.75% will earn interest of $\$27.15$ in 9 months?

Solution: Since $\boldsymbol{I = Prt}$, We need $\boldsymbol{P}$ **alone,** and doing algebra give us:

$$\frac{I}{rt} = P$$

and since $\boldsymbol{Time\ in\ in\ years}$, **9 months means 9 out of 12 months**,

$$\frac{9}{12}\ of\ a\ year \qquad \text{Or....} \qquad \frac{3}{4} years$$

So…

$$P = \frac{I}{rt} \quad \rightarrow \quad \frac{27.15}{(0.0475)(\frac{3}{4})} \quad \rightarrow \quad \$762.11$$

Compound Interest

- **Compound Interest** is much more complicated. You build/owe on the $Principal\ +\ the\ Interest$ earned in a compounding period
  - It can be used to your benefit when savings
  - It can drown you when it's used against your debt

- **Compound Interest** is calculated this way: $\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{n(t)}}$

  - A: $is\ the\ final\ amount\ earned$
  - P: $is\ the\ Principal\ (the\ initial\ amount\ of\ money\ borrowed\ or\ saved)$
  - r: $is\ the\ \boldsymbol{Yearly}\ Percentage\ Rate, expressed\ as\ a\ decimal\ (25\% = 0.25)$
  - n: is the number of times yearly interest is compounded per year
  - t: $is\ time, in\ years$

  - Compounding periods means the number times the interest is calculated in a year
    - Yearly: $n = 1$
    - Quarterly: $n = 4$
    - Monthly: $n = 12$
    - Daily: $n = 365$

Example: Find the interest earned if $\$6500$ is deposited in an account paying 6% compounded monthly for $5\ years$

Solution: $\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{n(t)}}$ so we sub in for the information given to fins the solution

$$\boldsymbol{A = 6500\left(1 + \frac{0.06}{12}\right)^{12(5)}} \quad \rightarrow \quad \boldsymbol{A = 6500(1.005)^{60}} \quad \rightarrow \quad \boldsymbol{A = \$8767.53}$$

- The interest is: $\boldsymbol{I = A - P}$ $\rightarrow$ $\boldsymbol{\$8767.53 - \$6500 = \$2267.53}$

Example: What is the initial investment needed to become a millionaire in 25 years if interest is 12% compounded quarterly?

Solution: $\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{n(t)}}$ so we calculated what we can then use algebra to find the unknown

$$\boldsymbol{1\,000\,000 = P\left(1 + \frac{0.12}{4}\right)^{4(25)}} \quad \rightarrow \quad \boldsymbol{1\,000\,000 = P(1.03)^{100}} \quad \rightarrow \quad \boldsymbol{P = \frac{\$1\,000\,000}{(1.03)^{100}}}$$

$\boldsymbol{P = \$52\,032.84}$ You only need to invest $\boldsymbol{\$52\,032.84}$ to be a millionaire in 25 years

Example: What is the difference in earnings on an investment of $\$8000$ for 10 years at 6% compounded yearly, monthly, and daily?

Solution:

Yearly	Monthly	Daily
$\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{n(t)}}$	$\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{n(t)}}$	$\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{n(t)}}$
$\boldsymbol{A = 8000\left(1 + \frac{0.06}{1}\right)^{1(10)}}$	$\boldsymbol{A = 8000\left(1 + \frac{0.06}{12}\right)^{12(10)}}$	$\boldsymbol{A = 8000\left(1 + \frac{0.06}{365}\right)^{365(10)}}$
$\boldsymbol{A = \$14\,326.78}$	$\boldsymbol{A = \$14\,555.17}$	$\boldsymbol{A = \$14\,576.23}$

Section 1.4 – Practice Problems

Given the information provided, solve for Interest

1. $P = \$4500 \quad r = 12\% \quad t = 3\ yrs \quad I = ?$

2. $P = \$12\ 250 \quad r = 6.2\% \quad t = 6\ mnths \quad I = ?$

3. $P = \$47\ 200 \quad r = 3\% \quad t = 5\ yrs \quad I = ?$

4. $P = \$200 \quad r = 28\% \quad t = 6\ yrs \quad I = ?$

Given the information provided, solve for the missing value

5. $P = ? \quad r = 7.5\% \quad t = 4\ yrs \quad I = \$150.30$

6. $P = \$4800 \quad r = ? \quad t = 4\ mnths \quad I = \$12$

7. $P = \$2500 \quad r = ? \quad t = 5\ yrs \quad I = \$675$

8. $P = \$1\ 250\ 000 \quad r = 8\% \quad I = \$400\ 000 \quad t = ?$

9. What amount will an account have after 4 years, if $\$7500$ is invested at an annual rate of 8% compounded daily?

10. An investment opportunity of $\$50\ 000$ for 10 years has two options: the first pays 11% compounded quarterly, the second pays 9% compounded monthly. Which is the belter investment, and by how much?

11. John started an RRSP on January 1st, 2013, with a deposit of $2500. He added $1500 on January 1st, 2014, and $2000 on January 1st, 2015. What is the accumulated value of his account on January 1st, 2016, if the interest is 6% compounded quarterly?

Section 1.4 – Answer Key

1. $1620
2. $379.75
3. $7080
4. $336
5. $501
6. 0.75%
7. 5.4%
8. $4 yrs$
9. $10 328.10
10. 11% *one is better by* $25425.85
11. $6801.51