# Section 1.3 – Inductive and Deductive Reasoning

**This booklet belongs to:_____ Block:_____**

## Inductive Reasoning

- ✓ **Inductive Reasoning** is when we reach **conclusions by observation**
- ✓ We try using inductive reasoning to establish a **GENERAL EQUATION** for different patterns

**Example:** Predict the $n^{th}$ **term** in the following pattern

$$3, 6, 12, 24, \ldots$$

**Solution**:

| 1st | 2nd | 3rd | 4th |
|---|---|---|---|
| 3 | 6 | 12 | 24 |

- Do you see a pattern?
- Let's try a couple ideas

Multiples of 3?

$3 * 1 = 3$

$3 * 2 = 6$

$3 * 3 = 9$

$3 * 4 = 12$

Not this pattern

Looks like the pattern above goes up by a **factor of 2!** So let's try this:

$3 * 1 = 3$

$3 * 2 = 6$

$3 * 2(2) = 12$

$3 * 2(2)(2) = 24$

This pattern looks good!

Think Exponents:

$2^{\mathbf{0}} = 1$ $\quad \boldsymbol{n - 1 = 0}$

$2^{\mathbf{1}} = 2$ $\quad \boldsymbol{n - 1 = 1}$

$2^{\mathbf{2}} = 2 * 2$ $\quad \boldsymbol{n - 1 = 2}$

$2^{\mathbf{3}} = 2 * 2 * 2$ $\quad \boldsymbol{n - 1 = 3}$

***n* is the term number!!**

So let's **generalize this**:

- ✓ If my pattern goes up by a factor of two, then we are talking exponents!

$$\mathbf{3 * 2^{n-1}}$$

$n$ is the **PLACE HOLDER for the term in the sequence**

**Example:** What is the $n^{th}$ tern of the pattern?

$$2, 8, 14, 20, \ldots$$

**Solution:**

Let's look at what we have (in multiple ways):

| $1st$ | $2nd$ | $3rd$ | $4th$ |
|---|---|---|---|
| $2$ | $8$ | $14$ | $20$ |
| $2$ | $2+6$ | $2+12$ | $2+18$ |
| $2$ | $2+6(1)$ | $2+6(2)$ | $2+6(3)$ |

So, looks like the **multiple of 6** is our $\boldsymbol{place\ holder\ n}$, but one less, so **n-1**

- That way our **general equation** is:

$$2+6(n-1)$$

- But if we do a bit of **algebra**

$$2+6n-6$$

This is:

$$6n-4$$

**Example:** Predict the $n^{th}$ term if the pattern $2, 6, 12, 20, 30, 42, \ldots$

**Solution:** Notice the pattern does not have a constant increase, so the $n^{th}$ term **isn't linear**

| $1st$ | $2nd$ | $3rd$ | $4th$ | $5th$ | $6th$ |
|---|---|---|---|---|---|
| $2$ | $6$ | $12$ | $20$ | $30$ | $42$ |
| $1\cdot 2$ | $2\cdot 3$ | $3\cdot 4$ | $4\cdot 5$ | $5\cdot 6$ | $6\cdot 7$ |

So here we see the $n^{th}$ term is: $n(n+1)$ or $n^2+n$.

## Counterexamples

- We can also **prove general statements false** by providing **Counterexamples**

**Example:**

All apples are green

**Counterexample:**

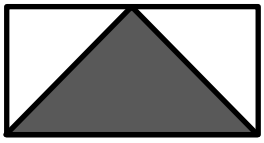
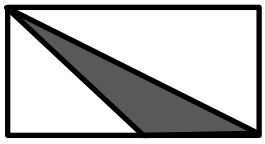
All I need to do is find an apple that is red, yellow, or any colour other than green.

**Find some Counterexamples**

i) Every prime Number is odd.

- 2 is a prime number and is even

ii) A triangle drawn from two corners of a square is half the area of the square

- True 
- Not True 

iii) Multiplying leads to large numbers

- Multiplying by zero leads to zero
- Multiplying by 1 leads to the same number
- Multiplying by a proper fraction gives a smaller number

## Deductive Reasoning

- ✓ **Deductive Reasoning** is the method of **arriving at conclusions** from **accepted facts**
- ✓ Each step in Deductive Reasoning represents **conclusions** from the **statement the came before**
- ✓ If any steps **are in error**, then the **final solution is FALSE**

**Example:**

*Premise*

- ✓ All planets move around the sun in an elliptical orbit
- ✓ Saturn is a planet

*Conclusion*

- ✓ Saturn moves around the sun in an elliptical orbit

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*Premise*

- ✓ If $n$ is a prime number greater than $3$, then $(n+1)(n-1)$ is divisible by $24$
- ✓ $47$ is a prime number

*Conclusion*

- ✓ (48)(46) = 2208
- ✓ 2208 is divisible by 24

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*Premise*

- ✓ All English teachers like to read
- ✓ Sam does not like to read

*Conclusion? Sam is not an English teacher*

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*Premise*

- ✓ If a quadrilateral is a square, it is a regular polygon
- ✓ A regular polygon has all sides and angles equal

*Conclusion? A square has all sides and angles equal*

**Example:** Are the following statements true? If not provide a counterexample.

- ✓ Every even number divisible by 6 is divisible by 3.

  True!

- ✓ A number bigger than 12 is divisible by 12 if it is divisible by 2 and 3

  False! (18 is divisible by 2 and 3, but not 12)

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- We need to be careful; we also can't deduce information just because we have a statement.
- Be careful not to jump to conclusions.

*Premise*

- ✓ A person must be 16 years old to have a driver's license.
- ✓ What can we deduce about the following:
  - Fred has a drivers license
  - Mike drives a car
  - Kevin is 20
  - Aurora is 12
  - Phil does not drive a car

*Solution*

  - Fred is 16 years old or older
  - Nothing, just because he drives doesn't mean he has a license
  - Nothing
  - Aurora foes not have a drivers license
  - Nothing

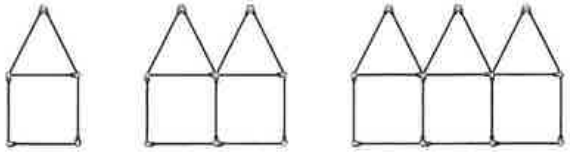
## Section 1.3 – Practice Questions

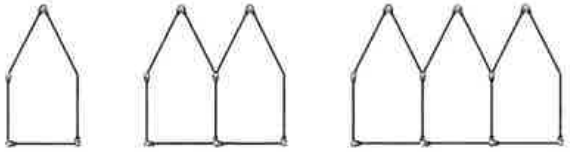
Study the pattern, predict the $n^{th}$ term.

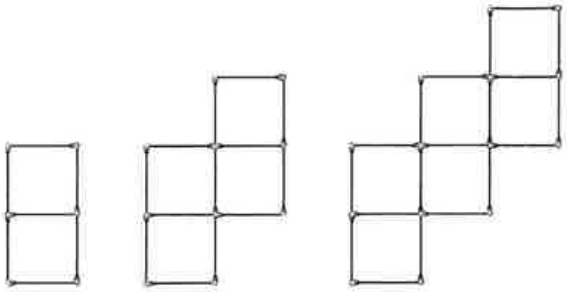
1. $1, 2, 3, \ldots,$ _____
2. $1, 3, 5, \ldots,$ _____
3. $2, 4, 6, \ldots,$ _____
4. $3, 7, 11, 15, \ldots,$ _____
5. $10, 17, 24, 31, \ldots,$ _____
6. $0, 6, 12, 18, \ldots,$ _____
7. $0, 2, 6, 12, \ldots,$ _____
8. $1, 6, 15, 28, \ldots,$ _____

Determine the number of matchsticks in the $n^{th}$ pattern

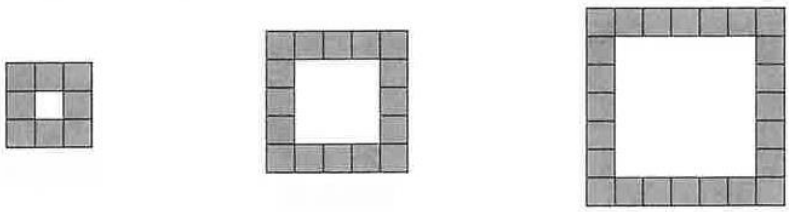
9.  _____

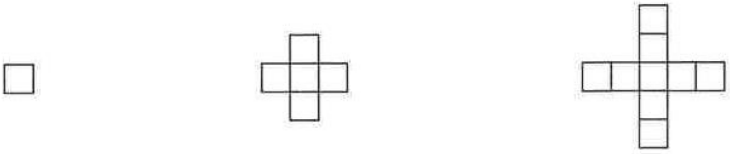
10.  _____

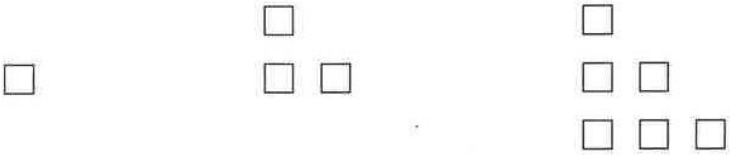
11.  _____

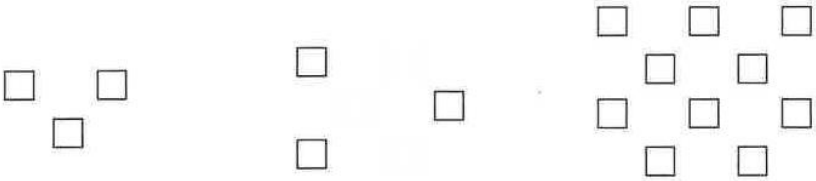
12.  _____

Study the pattern and predict the $n^{th}$ term

13. 

14. 

15. 

16. 

When possible, find a counter example. If not write 'true'

17. The acute angles in a right triangle are equal

18. A real number to the zero power is one

19. For any real number $x$, $x^2 > x$

20. The second power of any real number is positive

21. For any real number, $x$, $-x$, is a negative

22. An even number is any number which is not odd

Tell whether the statement is **true** or **false.** If false, give a counterexample

23. If a triangle has two equal sides, then it has equal angles

24. If two triangles have equal perimeters, then they have equal sides

25. If $x^2 > 0$, then $x > 0$

26. The diameter is the axis of symmetry of a circle

27. A number is divisible by 4 if the last digit is divisible by 4

28. A number is divisible by 12 if it is an even number divisible by 3.

29. A number is divisible by 15 if it is an odd number divisible by 5

30. A number is divisible by 18 if it is an even number divisible by 9

Reach a conclusion using the following assumptions

31. All citizens of Calgary are Albertans
All Albertans are Canadians

32. All Manitobans are fishermen
Sue is a Manitoban

33. All rectangles are quadrilaterals
All squares are rectangles

34. All whales are mammals
All mammals can swim

35. If you study for the exam you will pass
You study for the exam

36. $a$ is greater than $b$
$b$ is equal to $c$

Use deductive reasoning to reach a conclusion based on the given assumption of a triangle
(Every question represents an independent scenario)

37. One angle is $80°$

38. One angle is $80°$ and the other 2 angles are equal

39. All 3 angles are equal

40. All three angles are consecutive integers

41. The middle angle is $10°$ more than the smallest angle, which is half the amount of the largest angle

42. What is the sum of the angles in a pentagon? (5 sides)

43. All members of the volleyball team are over 6 feet tall. What, if anything, can you deduce with a certainty about each person?

a) Sue is on the Volleyball Team

b) Tom is over 6ft tall

c) Mary is $5'6"$ tall

d) Bert is not on the Volleyball Team

44. A person must be 12 years old or over to have a fishing license. What can be deduced with certainty about each person?

a) Sally has a fishing license

b) Bill went fishing

c) Lora is 15 years old

d) George is under 12 years old

e) Tim does not fish

## Answer Key – Section 1.3

| | | | | | |
|---|---|---|---|---|---|
| 1. $n$ | 2. $2n-1$ | 3. $2n$ | 4. $4n-1$ | 5. $7n+3$ | 6. $6n-6$ |
| 7. $n(n-1)$ | 8. $n(2n-1)$ | 9. $3n+1$ | 10. $5n+1$ | 11. $4n+1$ | 12. $6n+1$ |
| 13. $8n$ | 14. $4n-3$ | 15. $\frac{n(n+1)}{2}$ | 16. $\frac{(n+1)(n+2)}{2}$ | | |

For $17-44$: *See Website*

## Extra Work Space