Section 1.3 – Perfect Squares, Cubes, and their Roots

This booklet belongs to: ___________________________ Block: ______

Squares and square roots

- To **square** a number is to raise the number to the **second** power
- A perfect Square then has **2 identical factors**

**Example:**

\[
4^2 = 4 \cdot 4 = 16 \\
9^2 = 9 \cdot 9 = 81
\]

- The identical factors are called the **square root** of a number
- The number with the rational square roots are called **perfect squares**
- We use the ‘radical’ or ‘house’ symbol \(\sqrt{}\) to indicate square roots

**Example:** Determine which of the following are perfect squares.

\[
a) \quad 49 \\
b) \quad \frac{4}{9} \\
c) \quad 7 \\
d) \quad \frac{4}{15}
\]

**Solution:**

- a) Yes, because \(7 \cdot 7 = 49\), **two identical factors**

- b) Yes, because \(\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}\), **two identical factors**

- c) No, because 7 cannot be written as the product of two identical factors

- d) No, because \(\frac{4}{15}\) cannot be written as the product of two identical factors
Determining square roots sans calculator

Using a Factor Tree

Example: Determine the square root of 196

\[
\begin{align*}
196 & \quad \text{or} \quad 196 \\
2 & \quad 98 \\
| & \quad | \\
2 & \quad 49 \\
| & \quad | \\
7 & \quad 7
\end{align*}
\]

\[
\sqrt{196} = \sqrt{2 \cdot 2 \cdot 7 \cdot 7} = \sqrt{2 \cdot 2} \cdot \sqrt{7 \cdot 7} = 2 \cdot 7 = 14
\]

NOTE: For whole numbers \( \sqrt{x^2} = \sqrt{x} \cdot x = x \)

Example: Determine the square root of 225

\[
\begin{align*}
225 & \quad 45 \\
5 & \quad 9 \\
| & \quad | \\
3 & \quad 3
\end{align*}
\]

\[
\sqrt{225} = \sqrt{3 \cdot 3 \cdot 5 \cdot 5} = \sqrt{3 \cdot 3} \cdot \sqrt{5 \cdot 5} = 3 \cdot 5 = 15
\]
Cubes and Cube Roots

- To cube a number is to raise the number to the third power

**Example:**

\[ 4^3 = 4 \cdot 4 \cdot 4 = 64 \]
\[ 7^3 = 7 \cdot 7 \cdot 7 = 343 \]

- Some numbers can be written as the product of three identical factors

  - \[ 27 = 3 \cdot 3 \cdot 3 \]
  - \[ 125 = 5 \cdot 5 \cdot 5 \]

- The identical factors are called the cube root of a number
- The number with the rational square roots are called perfect cubes
- We use the ‘radical’ or ‘house’ symbol \( \sqrt[3]{\phantom{x}} \) to indicate cube roots

**Example:** Determine which are perfect cubes.

\[ \begin{array}{cccc}
  a) 8 & b) \frac{27}{64} & c) 25 & d) \frac{8}{9} \\
\end{array} \]

**Solution:**

- a) Yes, because \( 2 \cdot 2 \cdot 2 = 8 \), three identical factors

- b) Yes, because \( \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64} \), three identical factors

- c) No, because 25 cannot be written as the product of three identical factors

- d) No, because \( \frac{8}{9} \) cannot be written as the product of three identical factors
Determining cube roots sans calculator

Using a Factor Tree

Example: Determine the cube root of 216

\[
\begin{array}{c}
\sqrt[3]{216} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3} = 2 \cdot 3 = 6
\end{array}
\]

Example: Determine the cube root of 512

\[
\begin{array}{c}
\sqrt[3]{512} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 8
\end{array}
\]

Note: For whole numbers $\sqrt[3]{x^3} = x$.

Note: In the expression $\sqrt[k]{a}$, we call $k$ the index, and assume $k \geq 2$. If the index is not written, the expression is assumed to be a square root, i.e. $k = 2$.

Example: $\sqrt[5]{32} = 2$ because $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$, five identical factors.
Section 1.3 – Practice Questions

1. Find the square root of the perfect squares without a calculator

   a) \( \sqrt{100} \)
   b) \( \sqrt{441} \)
   c) \( \sqrt{225} \)
   d) \( \sqrt{361} \)
   e) \( \sqrt{529} \)
   f) \( \sqrt{2890000} \)

2. Find the cube root of the perfect cubes without a calculator

   a) \( \sqrt[3]{27} \)
   b) \( \sqrt[3]{1000} \)
   c) \( \sqrt[3]{343} \)
   d) \( \sqrt[3]{1728} \)
3. Find the perfect square root, if it exists, without a calculator

a) 25  

b) 29  

c) 80  

d) 81  

e) 169  

f) 99  

g) 1600  

h) 900  

i) \( \frac{81}{400} \)  

j) \( \frac{8}{18} \)
4. Find the perfect cube root, if it exists, without a calculator

a) 8
b) 9
c) 64
d) 81
e) 100
f) 216
g) 1000
h) 144
i) 625
j) 729

5. The area of a rectangle with a length twice as long as the width is $1250\text{m}^2$. Determine the length and the width of the rectangle.
6. A cube has a volume of $216\,cm^3$. Determine the length of each side of the cube.

7. A rectangular solid has a length three times the width and a height twice its width. If the volume of the rectangle solid is $384\,in^3$, determine the dimensions of the rectangular solid.
## Answer Key

### Section 1.3

1.  
   a) 10  
   b) 21  
   c) 15  
   d) 19  
   e) 23  
   f) 1700

2.  
   a) 3  
   b) 10  
   c) 7  
   d) 12  
   e) 15  
   f) 20

3.  
   a) 5  
   b) \textit{DNE}  
   c) \textit{DNE}  
   d) 9  
   e) 13  
   f) \textit{DNE}  
   g) 40  
   h) 30  
   i) \frac{9}{20}  
   j) \frac{3}{5}

4.  
   a) 2  
   b) \textit{DNE}  
   c) 4  
   d) \textit{DNE}  
   e) \textit{DNE}  
   f) 6  
   g) 10  
   h) \textit{DNE}  
   i) \textit{DNE}  
   j) 9

5.  
   \[ l = 50m \]  
   \[ w = 25m \]

6.  
   6cm

7.  
   \[ l = 12\text{in} \]  
   \[ h = 8\text{in} \]  
   \[ w = \frac{9}{4}\text{in} \]