Section 1.2 – Multiplication and Division of Fractions

This booklet belongs to: _____ **Block:** _____

Multiplication of Fractions

- It is simply **TOPS with TOPS** and **BOTTOMS with BOTTOMS**

$$\frac{Numerator \cdot Numerator}{Denominator \cdot Denominator}$$

Example: $\dfrac{2}{3} \cdot \dfrac{5}{7} = \dfrac{2 \cdot 5}{3 \cdot 7} = \dfrac{10}{21}$

Example: $\dfrac{-5}{9} \cdot \dfrac{1}{4} = \dfrac{-5 \cdot 1}{9 \cdot 4} = \dfrac{-5}{36} = -\dfrac{5}{36}$

Example: $\dfrac{4}{-7} \cdot \dfrac{-3}{5} = \dfrac{4 \cdot -3}{-7 \cdot 5} = \dfrac{-12}{-35} = \dfrac{12}{35}$

Example: $-\dfrac{1}{5} \cdot \dfrac{6}{11} = \dfrac{-1 \cdot 6}{5 \cdot 11} = \dfrac{-6}{55} = -\dfrac{6}{55}$

Now, what we can do though is **SIMPLIFY BEFORE WE MULTIPLY** the question first by **identifying the Common Factors**, just like when we **simplified individual fractions**.

Example:

$\dfrac{14}{49}$ can be written as: $\dfrac{2 \cdot 7}{7 \cdot 7}$ and since $\dfrac{7}{7}$ is equal to 1 what we have left is:

$\dfrac{2}{7} \cdot 1 = \dfrac{2}{7}$ see how we **cancelled out the common factors**

We can do the same steps before we multiply

Example 1: $\dfrac{2}{7} \cdot \dfrac{5}{8}$

Solution 1:

$$\dfrac{2}{7} \cdot \dfrac{5}{8} \quad \rightarrow \quad \dfrac{2}{7} \cdot \dfrac{5}{2 \cdot 4} \quad \rightarrow \quad \dfrac{2 \cdot 5}{2 \cdot 4 \cdot 7} \quad \rightarrow \quad \dfrac{\cancel{2} \cdot 5}{\cancel{2} \cdot 4 \cdot 7} \quad \rightarrow \quad \dfrac{5}{4 \cdot 7} = \dfrac{5}{28}$$

Example 2: $\dfrac{5}{12} \cdot \dfrac{3}{20}$

Solution 2:

$$\dfrac{5}{12} \cdot \dfrac{3}{20} \quad \rightarrow \quad \dfrac{5}{3 \cdot 4} \cdot \dfrac{3}{4 \cdot 5} \quad \rightarrow \quad \dfrac{5 \cdot 3}{3 \cdot 4 \cdot 4 \cdot 5} \quad \rightarrow \quad \dfrac{\cancel{5} \cdot \cancel{3}}{\cancel{3} \cdot 4 \cdot 4 \cdot \cancel{5}} \quad \rightarrow \quad \dfrac{1}{4 \cdot 4} = \dfrac{1}{16}$$

Example 3: $-\dfrac{2}{3} \cdot \dfrac{9}{14}$ Remember $(-2) = (-1) \cdot 2$

Solution 3:

$$\dfrac{-2}{3} \cdot \dfrac{9}{14} \quad \rightarrow \quad \dfrac{-2}{3} \cdot \dfrac{3 \cdot 3}{2 \cdot 7} \quad \rightarrow \quad \dfrac{(-1)2 \cdot 3 \cdot 3}{3 \cdot 2 \cdot 7} \quad \rightarrow \quad \dfrac{(-1)\cancel{2} \cdot \cancel{3} \cdot 3}{\cancel{3} \cdot \cancel{2} \cdot 7} \quad \rightarrow \quad \dfrac{(-1) \cdot 3}{7} = \dfrac{-3}{7} = -\dfrac{3}{7}$$

Example 4: $\dfrac{21}{36} \cdot \dfrac{42}{153}$

Solution 4:

$$\dfrac{21}{36} \cdot \dfrac{42}{153} \rightarrow \quad \dfrac{3 \cdot 7}{6 \cdot 6} \cdot \dfrac{6 \cdot 7}{3 \cdot 3 \cdot 17} \rightarrow \quad \dfrac{3 \cdot 7 \cdot 6 \cdot 7}{6 \cdot 6 \cdot 3 \cdot 3 \cdot 17} \rightarrow \quad \dfrac{\cancel{3} \cdot 7 \cdot \cancel{6} \cdot 7}{\cancel{6} \cdot 6 \cdot \cancel{3} \cdot 3 \cdot 17} \rightarrow \quad \dfrac{7 \cdot 7}{6 \cdot 3 \cdot 17} = \dfrac{49}{306}$$

Example 5: $\quad -\dfrac{6}{12} \cdot -\dfrac{2}{3}$

Solution 5:

$$\dfrac{-6}{12} \cdot \dfrac{-2}{3} \rightarrow \qquad \dfrac{(-1)\cdot 2 \cdot 3}{2 \cdot 2 \cdot 3} \cdot \dfrac{(-1)\cdot 2}{3} \rightarrow \qquad \dfrac{(-1)\cdot 2 \cdot 3 \cdot (-1) \cdot 2}{2 \cdot 2 \cdot 3 \cdot 3} \rightarrow \qquad \dfrac{(\cancel{-1})\cdot \cancel{2} \cdot \cancel{3} \cdot (\cancel{-1}) \cdot \cancel{2}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot 3}$$

$$= \dfrac{1}{3}$$

Division of Fractions

- First, I'll show you the somewhat complicated but quite gorgeous method.

You may have been told somewhere along the line that dividing fractions is **just flipping the second fraction** and **changing the division sign to multiplication**, how many of you heard this before?

Do you know why?

Here's why.

Example:

$\quad \dfrac{1}{2} \div \dfrac{2}{3} \ \ well\ the\ fraction\ bar\ essentially\ means\ division\ so\ we\ can\ rewrite\ this\ as\ \dots$

$$\dfrac{\frac{1}{2}}{\frac{2}{3}} \qquad yes\ it\ is\ one\ big\ fraction, made\ up\ of\ two\ fractions$$

- Now let's make this into an **equivalent fraction** with a denominator of one. Remember that in order for it to be equivalent we need to multiply the big fraction by 1.

$$\dfrac{\frac{1}{2}\cdot\frac{3}{2}}{\frac{2}{3}\cdot\frac{3}{2}} \qquad this\ second\ portion\ is\ equal\ to\ 1$$

So, what do we get…

$$\frac{\frac{1}{2} \cdot \frac{3}{2}}{\frac{6}{6}} = \frac{\frac{1}{2} \cdot \frac{3}{2}}{1} = \frac{1}{2} \cdot \frac{3}{2}$$

We ended up with,

$$\frac{1}{2} \cdot \frac{3}{2}$$

So, what has happened? The division symbol changed to multiplication and the fraction flipped.

And the result is:

$$\frac{1}{2} \cdot \frac{3}{2} \qquad = \qquad \frac{3}{4}$$

Now here is another method, the logic here is awesome…

Consider our starting point…

$$\frac{1}{2} \div \frac{2}{3} \quad how\ can\ I\ divide\ up\ pieces\ if\ they\ are\ the\ same\ size?$$

If I get a **COMMON DENOMINATOR:**   So, my equation now looks like:

$$\frac{1}{2} = \frac{3}{6} \quad and \quad \frac{2}{3} = \frac{4}{6}$$ $$\frac{3}{6} \div \frac{4}{6}$$

If you now divide the same sized pieces,

$$\frac{3 \div 4}{6 \div 6} = \frac{3 \div 4}{1} = 3 \div 4 \qquad = \qquad \frac{3}{4} \qquad BOOM!$$

Example 6: $\quad \dfrac{2}{3} \div \dfrac{5}{7}$

Solution 6:

Flip Method	Denominator Method
$\dfrac{2}{3} \div \dfrac{5}{7} = \dfrac{2}{3} \cdot \dfrac{7}{5} = \dfrac{14}{15}$	$\dfrac{2}{3} \div \dfrac{5}{7} = \dfrac{14}{21} \div \dfrac{15}{21} = \dfrac{14 \div 15}{21 \div 21} = \dfrac{14 \div 15}{1} = \dfrac{14}{15}$

Example 7: $\quad \dfrac{12}{13} \div \dfrac{6}{11}$

Solution 7:

Flip Method

$$\frac{12}{13} \div \frac{6}{11} = \frac{12}{13} \cdot \frac{11}{6} = \frac{2}{13} \cdot \frac{11}{1} = \mathbf{\frac{22}{13}}$$

Denominator Method

$$\frac{12}{13} \div \frac{6}{11} = \frac{132}{142} \div \frac{78}{142} = \frac{132 \div 78}{142 \div 142} = \frac{132 \div 78}{1}$$

$$= \frac{132}{78} = \frac{66}{39} = \mathbf{\frac{22}{13}}$$

Simplified both of these to get our final answer.

Section 1.2 – Practice Questions

Multiply the following, simplify before you multiply if desired, leave answer in simplified form

EMERGING LEVEL QUESTIONS

1. $\dfrac{1}{3} \cdot \dfrac{12}{7}$

2. $-\dfrac{8}{9} \cdot \dfrac{21}{16}$

3. $\dfrac{12}{14} \cdot \dfrac{7}{8}$

4. $\dfrac{8}{25} \cdot \dfrac{35}{4} \cdot \dfrac{2}{5}$

PROFICIENT LEVEL QUESTIONS

5. $\dfrac{5}{14} \cdot \left(-\dfrac{21}{10}\right) \cdot \dfrac{15}{7}$

6. $-\dfrac{7}{4} \cdot \dfrac{2}{21} \cdot \dfrac{14}{8}$

EMERGING LEVEL QUESTIONS

Divide the following fractions, simplify when you can, leave answer in simplified form

7. $\dfrac{2}{3} \div \dfrac{8}{9}$

8. $-\dfrac{3}{4} \div \dfrac{15}{8}$

PROFICIENT LEVEL QUESTIONS

9. $\dfrac{12}{5} \div 4$

10. $4 \div \dfrac{12}{15}$

11. $\dfrac{34}{121} \div \dfrac{17}{55}$

12. $-\dfrac{38}{27} \div \dfrac{57}{18}$

13. $-\dfrac{13}{17} \div \dfrac{39}{34}$

14. $-\dfrac{343}{125} \div \dfrac{49}{25}$

EXTENDING LEVEL QUESTIONS

Answer the following, leave answer as a simplified fraction, improper if applicable

15. $3\dfrac{1}{2} \cdot 2\dfrac{1}{3}$

16. $3\dfrac{1}{2} \div 2\dfrac{1}{3}$

17. $-5\frac{2}{5} \cdot 3\frac{1}{3}$

18. $-5\frac{2}{5} \div 3\frac{1}{3}$

19. $3\frac{3}{4} \div 1\frac{1}{8} \cdot 1\frac{2}{25}$

20. $3\frac{1}{4} \div 2\frac{7}{16} \cdot 1\frac{1}{8}$

Answer Key – Section 1.2

1. $\dfrac{4}{7}$	2. $-\dfrac{7}{6}$ or $-1\dfrac{1}{6}$	3. $\dfrac{3}{4}$	4. $\dfrac{28}{25}$ or $1\dfrac{3}{25}$
5. $-\dfrac{45}{28}$ or $-1\dfrac{17}{28}$	6. $-\dfrac{7}{24}$	7. $\dfrac{3}{4}$	8. $-\dfrac{2}{5}$
9. $\dfrac{3}{5}$	10. 5	11. $\dfrac{10}{11}$	12. $-\dfrac{4}{9}$
13. $-\dfrac{2}{3}$	14. $-\dfrac{7}{5}$ or $-1\dfrac{2}{5}$	15. $\dfrac{49}{6}$ or $8\dfrac{1}{6}$	16. $\dfrac{3}{2}$ or $1\dfrac{1}{2}$
17. -18	18. $-\dfrac{81}{50}$ or $-1\dfrac{31}{50}$	19. $\dfrac{18}{5}$ or $3\dfrac{3}{5}$	20. $\dfrac{3}{2}$ or $1\dfrac{1}{2}$

Extra Work Space