Section 1.2 – Practice Problems

1. Determine if the sequence is geometric. If so, find r.

a) $4, 12, 36, 72, \ldots$

$\frac{t_2}{t_1}$ $\frac{12}{4} = 3$ but $\frac{72}{36} \neq 3$ NO

$\frac{36}{12} = 3$

b) $3, 12, 48, 142, \ldots$

$\frac{12}{3} = 4$ but $\frac{142}{48} \neq 4$ NO

$\frac{48}{12} = 4$

c) $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$

$\frac{-\frac{1}{2}}{1} = -\frac{1}{2}$ Yes $r = -\frac{1}{2}$

$\frac{1}{4} \div -\frac{1}{2} = -\frac{1}{2}$

d) $1, -1, 1, -1, \ldots$

$\frac{-1}{1} = -1$ Yes $r = -1$

$\frac{1}{-1} = -1$

e) $3, -6, -12, 24, \ldots$

$\frac{-6}{3} = -2$ NO

$\frac{-12}{-6} = 2$

f) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

$\frac{\frac{1}{2}}{1} = \frac{1}{2}$ NO

$\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$

g) $\frac{1}{4}, \frac{1}{6}, \frac{1}{9}, \frac{2}{27}, \ldots$

$\frac{\frac{1}{6}}{\frac{1}{4}} = \frac{4}{6} = \frac{2}{3}$ $\frac{\frac{1}{9}}{\frac{1}{6}} = \frac{6}{9} = \frac{2}{3}$ Yes $r = \frac{2}{3}$

h) $\frac{2}{5}, -\frac{2}{3}, \frac{10}{9}, -\frac{50}{27}, \ldots$

$\frac{-\frac{2}{3}}{\frac{2}{5}} = -\frac{5}{3}$ $\frac{\frac{10}{9}}{-\frac{2}{3}} = -\frac{10}{9} \cdot \frac{3}{2} = -\frac{5}{3}$ Yes $r = -\frac{5}{3}$

i) $3x^2, 12x^4y^3, 48x^6y^6, \ldots$

$\frac{12x^4y^3}{3x^2} = 4x^2y^3$ Yes $r = 4x^2y^3$

$\frac{48x^6y^6}{12x^4y^3} = 4x^2y^3$

j) $\sqrt{2}, \sqrt{6}, 3\sqrt{2}, 3\sqrt{6}, \ldots$

$\frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$ Yes $r = \sqrt{3}$

$\frac{3\sqrt{2}}{\sqrt{6}} = \frac{\sqrt{18}}{\sqrt{6}} = \sqrt{3}$

2. Write the first five terms of the Geometric Sequence

a) $1, 4, \_\_, \_\_, \_\_$

$r = 4$

$1, 4, 16, 64, 256$

b) $1, \_\_, 4, \_\_, \_\_$

$1r^2 = 4$

$r^2 = 4$

$r = \pm 2$

option 1: $1, 2, 4, 8, 16$

option 2: $1, -2, 4, -8, 16$

c) $\frac{1}{2}, \_\_, \_\_, \frac{1}{16}, \_\_$

$\frac{1}{2}r^3 = \frac{1}{16}$

$r^3 = \frac{1}{8}$ $r = \frac{1}{2}$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$

d) $4, \_\_, \_\_, -13.5, \_\_$

$4r^3 = -13.5$

$r^3 = -3.375$

$r = -1.5$

$4, -6, 9, -13.5, 20.25$

e) $\_\_, 54, 18, \_\_, \_\_$

$r = \frac{18}{54} = \frac{1}{3}$

$162, 54, 18, 6, 2$

f) $1, \_\_, 3, \_\_, \_\_$

$1r^2 = 3$

$r^2 = 3$

$r = \pm\sqrt{3}$

option 1: $1, \sqrt{3}, 3, 3\sqrt{3}, 9$

option 2: $1, -\sqrt{3}, 3, -3\sqrt{3}, 9$

g) $3, \_\_, 3^{2x+1}, \_\_, \_\_$

$3r^2 = 3^{2x+1}$

$3r^2 = 3^{2x} \cdot 3$

$r^2 = 3^{2x}$

$r^2 = (3^x)^2$

$r = \pm 3^x$

i) $3, 3^{x+1}, 3^{2x+1}, 3^{3x+1}, 3^{4x+1}$

ii) $3, -3^{x+1}, -3^{2x+1}, 3^{3x+1}, -3^{4x+1}$

h) $1, \_\_, x^4, \_\_, \_\_$

$1r^2 = x^4$

$r^2 = x^4$

$r = \pm x^2$

i) $1, x^2, x^4, x^6, x^8$

ii) $1, -x^2, x^4, -x^6, x^8$

i) $5, 5^{2x-1}, \_\_, \_\_, \_\_$

$5r = 5^{2x-1}$

$5r = \frac{5^{2x}}{5}$

$r = \frac{5^{2x}}{5^2}$

$r = 5^{2x-2}$

$5, 5^{2x-1}, 5^{4x-3}, 5^{6x-5}, 5^{8x-7}$

j) $1, -\frac{x}{3}, \_\_, \_\_, \_\_$

$r = -\frac{x}{3}$

$1, -\frac{x}{3}, \frac{x^2}{9}, -\frac{x^3}{27}, \frac{x^4}{81}$

3. Find all possible values of r for a Geometric Sequence with the two given terms.

a) $a_5 = 5, a_7 = 25$

2 ratio between

$5r^2 = 25$

$r^2 = 5$

$r = \pm\sqrt{5}$

b) $a_2 = 4, a_6 = \frac{1}{4}$

4 ratio between

$4r^4 = \frac{1}{4}$

$r^4 = \frac{1}{16}$

$r = \pm\frac{1}{2}$

c) $a_4 = 2\sqrt{2}, a_7 = 8$

3 ratio between

$2\sqrt{2}\,r^3 = 8$

$r^3 = \frac{8}{2\sqrt{2}}$

$r^3 = \frac{4}{\sqrt{2}}$

$r^3 = \frac{2\sqrt{2}\sqrt{2}}{\sqrt{2}}$

$r^3 = 2\sqrt{2}$

$r^3 = \sqrt{2}^3$

$r = \sqrt{2}$

d) $a_3 = 1, a_6 = \sqrt{2}$

3 ratio between

$r^3 = \sqrt{2}$

$r^3 = 2^{\frac{1}{2}}$

$(r^3)^{\frac{1}{3}} = (2^{\frac{1}{2}})^{\frac{1}{3}}$

$r = 2^{\frac{1}{6}}$

$r = \sqrt[6]{2}$

$t_n = ar^{n-1}$ or $a_n = ar^{n-1}$

4. Find the desired information.

a) a_{11}, *if* $a_1 = \frac{1}{128}, r = 2$

$a_{11} = \frac{1}{128}(2)^{11-1}$

$a_{11} = \frac{1}{128} \cdot 2^{10}$ $\quad 128 = 2^7$

$a_{11} = \frac{2^{10}}{2^7} = 2^3 = \boxed{8}$

b) a_9, *if* $a_1 = 3, a_2 = \sqrt{3}$

need r

$\frac{\sqrt{3}}{3} = r$

$\frac{\sqrt{3}}{\sqrt{3}\sqrt{3}} = r$

$r = \frac{1}{\sqrt{3}}$

$r = 3^{-\frac{1}{2}}$

$a_9 = 3 \cdot \left(3^{-\frac{1}{2}}\right)^8$

$a_9 = 3 \cdot 3^{-4}$

$= 3^{-3}$

$= \frac{1}{3^3}$

$= \boxed{\frac{1}{27}}$

c) a_{42}, *if* $a_{40} = 9, a_{41} = 36$

$r = \frac{36}{9} = 4$

$a_{42} = a_{41} \cdot r$

$a_{42} = 36 \cdot 4$

$\boxed{a_{42} = 144}$

d) a_9, *if* $a_4 = 5, a_6 = 20$

$r \Rightarrow 5r^2 = 20$

$r^2 = 4$

$r = \pm 2$

opt. a 1:

$a_9 = \frac{5}{8}(2)^8$

$= \frac{5}{2^3}(2)^8$

$= 5 \cdot 2^5$

$= \boxed{160}$

$a_1 \rightarrow a_4 = a_1 r^{4-1}$

opt 1: $5 = a_1 2^3$

$a_1 = 5/8$

opt 2: $5 = a_1 (-2)^3$

$a_1 = -5/8$

opt 2:

$-\frac{5}{8} \cdot (-2)^8$

$-\frac{5}{8} \cdot 2^8$

$= \boxed{-160}$

$a_n = a_1 r^{n-1}$

e) $n,\ if\ a_1 = 729, a_2 = 243, l = \frac{1}{9}$

$r = \frac{243}{729} = \frac{1}{3}$

a_n is the n unknown

$a_n = 729\left(\frac{1}{3}\right)^{n-1}$

$\frac{1}{9} = 729\left(\frac{1}{3}\right)^{n-1}$

$\frac{1}{6561} = \frac{1}{3^{n-1}}$ → so $6561 = 3^{n-1}$

$3^8 = 3^{n-1}$

$8 = n-1$

$n = 9$

f) $n,\ if\ a_1 = 2048, a_2 = 1024, l = 1$

$r = \frac{1024}{2048} = \frac{1}{2}$

$a_n = 2048 r^{n-1}$

$1 = 2048\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{2048} = \frac{1}{2^{n-1}}$

$2048 = 2^{n-1}$

$2^{11} = 2^{n-1}$

$11 = n-1$

$n = 12$

g) $a_1,\ if\ a_5 = 27, r = 3$

$a_n = a_1 r^{n-1}$

$a_5 = 27$

$n = 5$

$r = 3$

$27 = a_1(3)^{5-1}$

$27 = a_1(3)^4$

$27 = a_1 \cdot 81$

$a_1 = \frac{27}{81} = \frac{1}{3}$

h) $a_1,\ if\ a_7 = 128, r = 4$

$a_7 = 128$

$n = 7$

$r = 4$

$a_7 = a_1 r^{n-1}$

$128 = a_1(4)^{7-1}$

$128 = a_1 4^6$

$128 = a_1 \cdot 4096$

$\frac{128}{4096} = a_1$

$a_1 = \frac{1}{32}$

$a_n = a_1 r^{n-1}$

i) r, *if* $a_{10} = 25,\ a_{12} = 225$

$a_{10} r^2 = a_{12}$

$25 r^2 = 225$

$r^2 = \frac{225}{25}$

$r^2 = 9$

$r = \pm 3$

j) r, *if* $a_{25} = 12,\ a_{31} = 96$

6 r applications

$a_{25} r^6 = a_{31}$

$12 r^6 = 96$

$r^6 = \frac{96}{12}$

$r^6 = 8$

$r = \pm \sqrt[6]{8}$

$\pm 8^{1/6}$

$\Rightarrow \pm (2^3)^{1/6}$

$\rightarrow \pm 2^{1/2}$

$= \pm \sqrt{2}$

k) a_8, *if* $a_n = 3a_{n-1}, a_1 = \frac{1}{27}$

Recursive

$a_2 = 3 \cdot a_1$

$= 3 \cdot \frac{1}{27}$

$= \frac{1}{9}$

$r = \frac{\frac{1}{9}}{\frac{1}{27}}$

$r = \frac{27}{9} = 3$

$a_8 = \frac{1}{27} \cdot 3^{8-1}$

$a_8 = \frac{3^7}{3^3}$

$a_8 = 3^4$

$a_8 = 81$

l) a_6, *if* $a_n = 0.1a_{n-1}, a_1 = 1000$

Recursive

$r = \frac{100}{1000} = \frac{1}{10}$

$a_2 = 0.1 \cdot a_1$

$= 0.1 \cdot 1000$

$= 100$

$a_6 = 1000 \cdot \left(\frac{1}{10}\right)^5$

$= 10^3 \cdot 10^{-5}$

$= 10^{-2}$

$= \frac{1}{100}$

$a_6 = 0.01$

5. Insert two Geometric Means between a *and* b.

need r

$ar^3 = b$

$r^3 = \frac{b}{a}$

$r = \sqrt[3]{\frac{b}{a}}$

$a, \underset{r}{x}, \underset{r}{y}, \underset{r}{b}$

$x = ar = a \cdot \frac{b^{1/3}}{a^{1/3}}$

$x = \frac{a^{3/3} \cdot b^{1/3}}{a^{1/3}} = a^{2/3} b^{1/3}$

$\boxed{x = \sqrt[3]{a^2 b}}$

$y = a^{2/3} b^{1/3} \cdot \frac{b^{1/3}}{a^{1/3}}$

$y = a^{1/3} b^{2/3}$

$\boxed{y = \sqrt[3]{ab^2}}$

6. Given the geometric sequence $a, \frac{a}{b}, \frac{a}{b_2}, \ldots$ determine an expression for $t_n - t_{n-1}, n > 2$

$ar = \frac{a}{b}$

$r = \frac{1}{b}$

$t_n = a\left(\frac{1}{b}\right)^{n-1} = ab^{-n+1}$

$t_{n-1} = a\left(\frac{1}{b}\right)^{(n-1)-1}$

$= a\left(\frac{1}{b}\right)^{n-2} = ab^{-n+2}$

$t_n - t_{n-1} = ab^{-n+1} - ab^{-n+2}$

$= ab^{-n} \cdot b - ab^{-n} \cdot b^2$

$= ab\left(b^{-n} - b^{-n} \cdot b\right)$

$\boxed{= ab\left(\frac{1-b}{b^n}\right)}$

7. Find x so that $x - 1,\ x, and\ x + 2$ are consecutive (one after the other) terms of a Geometric Sequence.

$(x-1)r = x \rightarrow r = \frac{x}{x-1}$

$xr = (x+2) \rightarrow r = \frac{x+2}{x}$

$r = r$ so

$\frac{x}{x-1} = \frac{x+2}{x}$

$x^2 = (x+2)(x-1)$

$x^2 = x^2 + x - 2$

$0 = x - 2$

$\boxed{x = 2}$

$x \neq 1$ or 0

If $x=-1$: $-3, 6, \ldots$ $r=-2$ ✓

If $x=\frac{11}{4}$: $3/4, 9/4, \ldots$ $r=3$

8. Find the common ratio r for the geometric sequence: $x-2, 5-x, 5x-7, \ldots$

$(x-2)r=(5-x) \rightarrow r=\frac{5-x}{x-2}$

$(5-x)r=(5x-7) \rightarrow r=\frac{5x-7}{5-x}$

$\frac{5-x}{x-2}=\frac{5x-7}{5-x}$ → FOIL → $(5-x)(5-x)=(5x-7)(x-2)$

$25-10x+x^2=5x^2-17x+14$

$4x^2-7x-11=0$

AC method

$x^2-7x-44=0$

$(x-\frac{11}{4})(x+\frac{4}{4})=0$

$(4x-11)(x+1)=0$

$x=-1$ or $x=11/4$

$x \neq 2$ or 5

9. What number must be added to $-2, 4, 19$ so that the resulting numbers are three terms of a Geometric Sequence.

Let x be that number

$(-2+x)r=(4+x) \rightarrow r=\frac{4+x}{-2+x}$

$(4+x)r=(19+x) \rightarrow r=\frac{19+x}{4+x}$

NOTE: $-2+x \rightarrow x-2$; $19+x \rightarrow x+19$

$\frac{(4+x)}{(x-2)}=\frac{(x+19)}{(4+x)}$

$(4+x)(4+x)=(x+19)(x-2)$

$16+8x+x^2=x^2+17x-38$

$-9x=-54$

$x=6$

10. If the first two terms of a geometric sequence are $\sqrt{2}$, *and* $\sqrt[3]{2}$, what is the fourth term?

$\sqrt{2}, \sqrt[3]{2}, \ldots$

$r=\frac{2^{1/3}}{2^{1/2}} \rightarrow \frac{2^{2/6}}{2^{3/6}} \rightarrow 2^{-1/6}$

$2^{1/3} \cdot 2^{-1/6} = 2^{2/6} \cdot 2^{-1/6} = 2^{1/6}$ ← 3rd term

$2^{1/6} \cdot 2^{-1/6} = 2^0 = 1$

11. If the product of the first three terms of a Geometric Sequence is -8, and the sum is $\frac{14}{3}$, what is the common ratio of the sequence?

a, b, c

a, ar, ar^2

$a \cdot ar \cdot ar^2 = -8$

$a^3 r^3 = -8$

$ar^3 = -8 \rightarrow ar = -2$

$a = -2/r$

$a + ar + ar^2 = \frac{14}{3}$

$a + ar + ar \cdot r = \frac{14}{3}$

$a - 2 - 2r = \frac{14}{3}$

$\frac{-2}{r} - 2 - 2r = \frac{14}{3}$

$-2 - 2r - 2r^2 = \frac{14r}{3}$

$-6 - 6r - 6r^2 = 14r$

$-6r^2 - 20r - 6 = 0$

$3r^2 + 10r + 3 = 0$

$(3r + 1)(r + 3) = 0$

$r = -\frac{1}{3}$ or $r = -3$

12. In the sequence $3, x, y, 25$, the first three terms form an arithmetic sequence, and the last three terms form a geometric sequence. Find x *and* y.

$3 + d = x$

$3 + 2d = y$

$d = x - 3$

$3 + 2(x - 3) = y$

$3 + 2x - 6 = y$

$y = 2x - 3$

$xr = y \rightarrow xr = 2x - 3 \rightarrow \frac{2x-3}{x} = r$

$yr = 25 \rightarrow (2x - 3)r = 25 \rightarrow \frac{25}{2x-3} = r$

$\frac{2x-3}{x} = \frac{25}{2x-3}$

$(2x - 3)(2x - 3) = 25x$

$4x^2 - 12x + 9 = 25x$

$4x^2 - 37x + 9 = 0$

$4x^2 - 36x - x + 9 = 0$

$4x(x - 9) - 1(x - 9) = 0$

$(4x - 1)(x - 9) = 0$

if $x = \frac{1}{4}$, $y = -\frac{5}{2}$

if $x = 9$, $y = 15$

13. The enrolment at Vic High in Victoria was 400 *in* 1973. If the school's population has increased by 5% a year, how many students will be going to the school in 2010.

year diff is 38 years

$a_1 = 400$

$n = 38$

$r = 1.05$

↑ increase of 5%

$a_{37} = 400(1.05)^{38-1}$

$400(1.05)^{37}$

$= 2432.6$

$= \boxed{2433}$

remember $n - k + 1$

$\sum_{1973}^{2010} 1$

14. If a starting salary is $28 000 and you get an annual increase of 6%, what is your salary at the beginning of the eighth year of work?

$n = 8$

$t_1 = 28000$

$r = 1.06$

$t_n = a_1 r^{n-1}$

$t_8 = 28000(1.06)^7$

$= \$42\,101.65$

15. With each cycle, a vacuum pump removes 25% of the air in a glass container. What percent of the air has been removed after 10 *cycles*.

$n = 10$

$a_1 = 1$ ← Full

$r = 0.75$ ← removes 25%

$a_{10} = 1(0.75)^{10-1}$

$a_{10} = 1(0.75)^9$

$a_{10} = 0.075$ left

$1 - 0.075 = 0.925$ removed

92.5% air removed

16. A car costs a company $40 000. Each year, the car depreciates 16% of its value. What is the value of the care after five years.

first buy the car ← (0, 1, 2, 3, 4, 5)

$n = 6$

$a_1 = 40000$

$r = 84\%$ 0.84

$a_6 = 40000(0.84)^5$

$= \$16\,728.48$

17. Initially a pendulum swings through an arc of $45cm$. On each successive swing, the length of the arc is decreased by 2% of the previous length. What is the length of the arc after 12 *swings*.

initial swing starts at 0 so $n = 13$

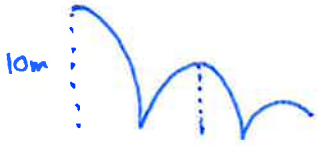
$r = 0.98$

$a_1 = 45$

$a_{13} = 45(0.98)^{12}$

$\boxed{a_{13} = 35.3\,cm}$

18. A ball is dropped from a height of 10 *meters*. Each time it strikes the ground it bounces up 75% of its previous height. How many bounces does the ball need before the bounce is less than 20 cm high (Watch your units)?



$10\,m = 1000\,cm$

$a_1 = 1000\,cm$

$a_n < 20\,cm$

$r = 0.75$

$n = ?$

$20 < 1000(0.75)^{n-1}$

$\frac{20}{1000} < 0.75^{n-1}$

$0.02 < 0.75^{n-1}$

trial and error with n here

when $n = 14$

$0.02 < 0.023$ need one more

$0.02 < 0.75^{15-1} \rightarrow 0.02 < 0.0178$! so $\rightarrow \boxed{n = 15}$

19. From farmer to consumer a shipment goes through a number of handlers: *Farmer* (1) → *Trucker* (2) → *Regional Market* (3) → *Trucker* (4) → *Wholesaler* (5) → *Trucker* (6) → *Retailer* (7) → *Consumer* (8). If the farmer get 75 *cents per kilogram*, and if each person in the chain makes a 20% *profit*, how much does the consumer pay?

$a_1 = 0.75$

$r = 1.2$

$n = 8$

$a_8 = a_1 r^{n-1}$

$a_8 = 0.75(1.2)^7$

$\boxed{a_8 = \$2.69}$

20. A truck radiator contains fifty litres of water. Five litres of water is removed and replaced with pure antifreeze, then five litres of mixture is removed and replaced. How much antifreeze is in the radiator after the process is repeated five times.

Consider how much water is removed r is the process

$n = 6$

$a_1 = 50$

$a_2 = 45$

$r = \frac{45}{50} = \frac{9}{10}$

$a_6 = 50\left(\frac{9}{10}\right)^5$

$= 29.5$

↑

29.5 water removed

so

$50 - 29.5 = 20.5$

↓

pure anti. freeze added.

See Website for Detailed Answer Key

Extra Work Space