Section 1.2 – Factoring Polynomials $ax^2 + bx + c$

This booklet belongs to:______________________Block:_____

Factoring Quadratics: $ax^2 + bx + c$

In this section ***a*** will have an integer value greater than 1.

I'm going to show you two methods; it's called the **Factor by Grouping and The AC Method**.

The AC Method

The "AC" Method (Factoring Trinomials)

- The "AC" method is a technique used to factor trinomials
- A trinomial consists of three terms $(ax^2 + bx + c)$.

Example of "AC" method:

1. $\quad a \quad b \quad c$
   $6x^2 + 7x + 2$

2. Multiply the **coefficient of the $\boldsymbol{a\ term}$** with the **coefficient of the $\boldsymbol{c\ term}$** and rewrite the polynomial with the starting term now x^2

   So $6x^2 + 7x + 2$ becomes $x^2 + 7x + 12$

3. a) Find two numbers **that multiply** to $+12$ and **add** to $+7$

   These are: $+3\ and +4$

   b) Rewrite the **factored form** of the new version of the Polynomial

   $(x+3)(x+4)$

   c) Now **divide the two factors** by the $\boldsymbol{original\ a\ term}$ and **simplify the fractions**

   $(x+\frac{3}{6})(x+\frac{4}{6}) \rightarrow (x+\frac{1}{2})(x+\frac{2}{3})$

   d) If you are **not left with a denominator** then you are **done**. If a **denominator remains**, **rewrite** it **in front** of the $\boldsymbol{x\ term}$.

   Can't simplify so write the 2 in front of the x → $(x+\frac{1}{2})(x+\frac{2}{3})$ ← Can't simplify so write the 3 in front of the x

4. Rewrite as the factored from: $\boldsymbol{(2x+1)(3x+2)}$ This is the Factored Form.
5. **Check using FOIL**

Example: Factor $2x^2 + 7x - 4$

Solution: $2x^2 + 7x - 4$ is first changed to $x^2 + 7x - 8$

$(x + 8)(x - 1)$ *Factor*

$(x + \frac{8}{2})(x - \frac{1}{2})$ *Divide each constant by 2, the original coefficient of $2x^2$*

$(x + 4)(x - \frac{1}{2})$ *Simplify*

$(x + 4)(2x - 1)$ *If there is fraction left after division, it becomes the coefficient of x*

Check using FOIL: $(x + 4)(2x - 1) \quad = \quad 2x^2 + 7x - 4$

<u>Example:</u> $12x^2 - 5x - 2$

<u>Solution:</u> $12x^2 - 5x - 2$ is first changed to $x^2 - 5x - 24$

$(x - 8)(x + 3)$ — *Factor*

$(x - \frac{8}{12})(x + \frac{3}{12})$ — *Divide each constant by 12, the original coefficient of* $12x^2$

$(x - \frac{2}{3})(x + \frac{1}{4})$ — *Simplify*

$(3x - 2)(4x + 1)$ — *If there is fraction left after division, it becomes the coefficient of* x

Check using FOIL: $(3x - 2)(4x + 1) \quad = \quad 12x^2 - 5x - 24$

Factor by Grouping

- This method is similar to the AC method, but does not involve manipulating the initial equation
- We still need our **"multiply to this, add to this"** scenario, but with a minor tweak
- We are looking for $\boldsymbol{two\ numbers\ that\ add\ to\ the\ middle\ (b)\ term}$, but that $\boldsymbol{multiply\ to\ the\ product\ of\ the\ a\ and\ c\ term}$
- Then we **re-write the middle term** as the **sum of those two numbers**
- Now we **can group and factor**
- See the examples below

Example: Factor $2x^2+7x-4$ by grouping

Solution:

$2x^2+7x-4$

Identify two numbers that add to $+7$ and multiply to the product of the $a\ and\ c\ term$ $(2\ and -4)$

$\boldsymbol{So\ add\ to\ +7\ and\ multiply\ to\ -8}$

$2x^2-1x+8x-4$

In this case it is: $\boldsymbol{-1\ and\ 8}$

Split the $\boldsymbol{b\ term}$ $(+7x)$ into the $(-1x+8x)$

$x(2x-1)+4(2x-1)$

You need to be strategic to **factor out a common factor** so that you are **left with the same thing in the brackets**

$(2x-1)(x+4)$

Now each term had $(\boldsymbol{2x-1})$ **as a common factor** so factor it out.

You are left with $(2x-1)(x+4)$ to verify **imagine Waterbombing** the entire $(\boldsymbol{2x-1})$ **into the** $(\boldsymbol{x+4})$, it will all become more clear.

Example: $12x^2-5x-2$

Solution:

$$12x^2-5x-2$$

$$12x^2-8x+3x-2$$

Factor out a $4x$ | $4x(3x-2)+1(3x-2)$ | Factor out a $+1$

$(3x-2)(4x+1)$ | Factor out the $(3x-2)$

- Both Methods work great, it comes down to personal preference
- There is also the BOX METHOD (See additional Video) and FACTORING INTUITIVELY (Done in Class)
- Enjoy!

Section 1.2 – Practice Problems

Factor using Grouping or the AC Method

1. $2x^2 + 13x + 15$

2. $3x^2 + 8x + 4$

3. $10x^2 + 17x + 3$

4. $8y^2 - 18y + 9$

5. $21y^2 - 41y + 10$

6. $2y^2 - 7y + 5$

7. $20z^2 - 27z - 8$

8. $3z^2 - 20z - 63$

Factor Completely

9. $-3x^2 - x + 4$

10. $-2x^2 - 5xy - 2y^2$

11. $-6a^2 - 17ab + 3b^2$

12. $-4a^2b - 4ab^2 + 3b^3$

Factor Completely

13. $25x^2(a-1)^3 - 5x(a-1)^3 - 2(a-1)^3$

14. $9 - 10x^2 + x^4$

15. $8x^4 + 19x^2 - 27$

16. $9x^4 - 145x^2 + 16$

Answer Key – Section 1.2

1. $(x+5)(2x+3)$
2. $(x+2)(3x+2)$
3. $(2x+3)(5x+1)$
4. $(2y-3)(4y-3)$
5. $(3y-5)(7y-2)$
6. $(y-1)(2y-5)$
7. $(5z-8)(4z+1)$
8. $(z-9)(3z+7)$
9. $-(3x+4)(x-1)$
10. $-(x+2y)(2x+y)$
11. $-(a+3b)(6a-b)$
12. $-b(2a+3b)(2a-b)$
13. $(a-1)^3(5x-2)(5x+1)$
14. $(x+3)(x-3)(x+1)(x-1)$
15. $(8x^2+27)(x+1)(x-1)$
16. $(x+4)(x-4)(3x-1)(3x+1)$

Extra Work Space