Section 1.1 – Factoring Polynomials $x^2 + bx + c$

This booklet belongs to:______________________Block:_____

Factoring Quadratics (Polynomials of degree 2): $x^2 + bx + c$

Consider this: $(x + a)(x + b) = x^2 + bx + ax + ab$

$$x^2 + (b + a)x + ab$$

- By looking at this we see that:
  - The **first term** is the **product** of x and x
  - The **coefficient** of the **middle term** is the **sum** of a and b
  - The **last term** is the **product** of a and b
- This leads us to the **general rule**:

When factoring $x^2 + bx + c$, look for **two factors of c**, that **multiply** to the **coefficient** of the **last term**, and **add** to the **coefficient** of the middle term.

Example: Factor $x^2 + 7x + 12$

Solution: What two numbers **add to 7** and **multiply to 12**?

- Integers that multiply to 12: $(1, 12)$ $(2, 6)$ $(3, 4)$ $(-1, -12)$ $(-2, -6)$ $(-3, -4)$
- Only integers $+3$ and $+4$ add to 7
- Therefore $x^2 + 7x + 12 = (x + 3)(x + 4)$
- We can **check our answer** using ***FOIL***: $(x + 3)(x + 4)$

$$= x^2 + 3x + 4x + 12$$
$$= \mathbf{x^2 + 7x + 12}$$

Example: Factor $x^2 + 8 - 6x$

Solution: First **arrange** the polynomial in **descending order** of powers

- $x^2 + 8 - 6x = \mathbf{x^2 - 6x + 8}$
- -4 and -2 add to -6 and multiply to $+8$
- Therefore: $x^2 - 6x + 8 = (x - 4)(x - 2)$
- We can **check using FOIL**

Example: Factor $5x^2 + 35x + 60$

Solution: Always look for a **common factor** first. The **largest common factor** is **5**

- Therefore: $5x^2 + 35 + 60 = 5(x^2 + 7 + 12)$
- Now we can factor the Quadratic like we did previous:
- Two numbers that **multiply** to $+12$ and **add** to $+7$
- $+4$ and $+3$ get the job done
- So $5(x^2 + 7 + 12) = 5(x + 4)(x + 3)$
- **Check** your answer **using FOIL**

Example: Factor $-x^2 + 5x + 6$

Solution: First factor out -1, so that the **coefficient** of x^2 becomes $+1$.

- So $-x^2 + 5x + 6$ becomes $-(x^2 - 5x - 6)$, now factor $(x^2 - 5x - 6)$
- -6 and 1 **multiply** to -6 and **add** to -5
- Therefore $-x^2 + 5x + 6 = -(x^2 - 5x - 6) \quad = -(x - 6)(x + 1)$
- **Note the factors are: $(\boldsymbol{x - 6})(\boldsymbol{x + 1})$ and $\boldsymbol{-1}$**

Example: Factor $-3x^4 - 18x^3 - 27x^2$

Solution: First look for a common factor. The largest here is $-3x^2$, factor it out

- So $-3x^4 - 18x^3 - 27x^2$ becomes $-3x^2(x^2 + 6x + 9)$, now factor $(x^2 + 6x + 9)$
- $+3$ and $+3$ **multiply** to $+9$ and **add** to $+6$
- Therefore $-3x^4 - 18x^3 - 27x^2 = -3x^2(x^2 + 6x + 9) = -3x^2(x + 3)^2$
- **Note the factors are: $(\boldsymbol{x + 3})(\boldsymbol{x + 3})$ and $\boldsymbol{-3x^2}$**

SUMMARY OF FACTORING QUADRATICS

1. Arrange the polynomial in **descending order** of powers

2. When:
   - The **last term** is **positive**, the **factors of c** are **both positive**, **or both negative**.
   - If the **middle term** is **positive**, **both integers** are **positive**.
   - If the **middle term** is **negative**, **both integers** are **negative**.

Example: $x^2 + 7x + 12 = (x + 4)(x + 3)$

- The **last term is positive**, and the **middle term is positive**, therefore the **factors of 12** are **both positive**.
- **Opposite** if the **middle term** was **negative** and the **last positive**.

3. When:
   - The **last term** is **negative**, the **factors of c** have **opposite signs**.
   - The **larger numeric value** takes **the sign** of the coefficient of the **middle term**.

Example: $x^2 - x - 6 = (x - 3)(x + 2)$

- The **last term is negative**, therefore the **signs of the factor** of 6 are **opposite of each other**
- Since the **middle term is negative** the **larger numeric value** has a **negative sign**.

Example: $x^2 + 2x - 15 = (x + 5)(x - 3)$

- The **last term is negative**, therefore the **signs of the factor** of 15 are **opposite of each other**
- Since the **middle term is positive** the **larger numeric value** has a **positive sign**.

Special Factors

For trinomial $ax^2 + bx + c$ to be a perfect square:

a) The **last term** must be a **positive**, and a **perfect square**
b) The **first term** must be a **perfect square**
c) The **coefficient of the middle term** is the **square root of the first term multiplied** by the **square root** of the **coefficient of the last term**, then **doubled.**

Example: $x^2 + 8x + 16 = (x + 4)^2$

Example: $x^2 - 8x + 16 = (x - 4)^2$

Factoring Perfect Square Trinomials

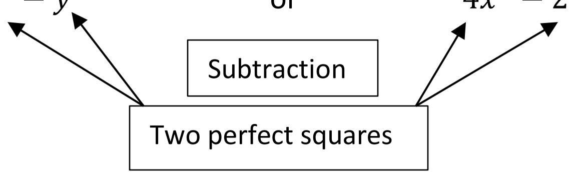
$\boldsymbol{a^2 + 2ab + b^2 = (a + b)^2}$

$\boldsymbol{a^2 - 2ab + b^2 = (a - b)^2}$

Difference of Squares

- Whenever we **see subtraction**, **two square terms** only, and **no term with degree one**, we have the possibility of a **difference of squares**

Example: $x^2 - y^2$ or $4x^2 - 25$



Difference of Squares

$\boldsymbol{a^2 - b^2 = (a + b)(a - b)}$

Example:

$\boldsymbol{4x^2 - 25 = (2x + 5)(2x - 5)}$

Check using FOIL

The middle (degree 1 term), cancels out!

Section 1.1 – Practice Problems

Give four examples for b so that the following trinomials can be factored

1. $x^2 + bx + 6$ ____,____,____,____

2. $x^2 + bx + 4$ ____,____,____,____

3. $x^2 + bx - 8$ ____,____,____,____

4. $x^2 + bx - 6$ ____,____,____,____

Give positive and negative examples for c so that the following trinomials can be factored

5. $x^2 + 6x + c$ ____,____,____,____

6. $x^2 - 4x + c$ ____,____,____,____

7. $x^2 + x + c$

_____,_____,_____,_____

8. $x^2 - 5x + c$

_____,_____,_____,_____

9. A student factored $x^3 - 5x^2 - 14x$ into $(x - 7)(x + 2)$. Explain the error that was made.

Factor

10. $a^2 + 9a + 8$	11. $b^2 + 16b + 15$
12. $c^2 + 10c + 24$	13. $d^2 + 7d + 10$
14. $x^2 - 18x + 72$	15. $y^2 - 20y + 91$
16. $z^2 - 13z + 36$	17. $u^2 - 4u + 4$
18. $l^2 + 7l - 30$	19. $m^2 + 4m - 12$

Factor Completely

20. $3x^2 + 15x + 12$

21. $4y^2 + 20y + 24$

22. $-5x^2 + 25x - 20$

23. $-2y^2 + 58y - 200$

24. $-x^2 - 6x + 27$

25. $-x^2 + 7x + 44$

26. $x^3 + 8x^2 - 20x$

27. $-2x^4 - 4x^3 + 30x^2$

28. $-x^3y - x^2y^2 + 6xy^3$

29. $2x^4 - 16x^3y + 32x^2y^2$

30. $-x^3y^2 - 3x^2y^3 + 4xy^4$

31. $x^6 - 11x^5y + 28x^4y^2$

Factor Completely

32. $(2a+5)y^2 + 9(2a+5)y - 10(2a+5)$

33. $x^3(a+b) - 6x^2(a+b) + 8x(a+b)$

34. $(2a+b)x^2-12(2a+b)x+27(2a+b)$

35. $(3a-b)y^2-13(3a-b)y+40(3a-b)$

36. x^4+x^2+1

37. $(2x+3)^2+(2xz+3z)-20z^2$

38. $(x-2y)^2-8a(x-2y)+15a^2$

39. $(5x-y)^2+(10xz-2yz)-24z^2$

40. The volume of a rectangular solid is $(x^3 + 7x^2 + 12x)cm^3$. Determine its dimensions in terms of x.

41. A sheet of cardboard measuring $5in\ by\ 7in$ has squares $x\ inches$ wide cut from each corner. Then the sides are folded up to form an open top box. Express the volume of the box in factored form.

Factor each binomial completely

42. $x^2 - 1$

43. $4x^2 - 1$

44. $y^2 - 25$

45. $25y^2 - 9$

46. $4 - 9z^2$

47. $16 - 25y^2$

48. $16x^2 - 9y^2$

49. $25x^4 - 81y^6$

50. $16x^2y^8 - 4$

51. $20x^2 - 5y^2$

52. $(x + 1)^2 - y^2$

53. $4 - (x + 2)^2$

Factor each perfect square trinomial completely

54. $x^2 + 10x + 25$

55. $x^2 + 8x + 16$

56. $y^2 - 12y + 36$

57. $y^4 - 6y^2 + 9$

58. $2z^2 - 28z + 98$

59. $-9x^2 - 24xy - 16y^2$

60. $(x^2 + 6x + 9) - 4y^2$

61. $(x^6 - 4x^3y^3 + 4y^6) - (a^4 + 6a^2b^2 + 9b^4)$

Answer Key – Section 1.1

1. $7, 5, -7, -5$
2. $5, -5, 4, -4$
3. $7, -7, 2, -2$
4. $-5, 5, -1, 1$
5. $5, 8, -7, -16$
6. $4, 3, -5, -12$
7. $\frac{1}{4}, \frac{3}{16}, -2, -6$
8. $6, 4, -6, -14$
9. $x(x-7)(x+2)$
10. $(a+1)(a+8)$
11. $(b+15)(b+1)$
12. $(c+4)(c+6)$
13. $(d+2)(d+5)$
14. $(x-12)(x-6)$
15. $(y-7)(y-13)$
16. $(z-9)(z-4)$
17. $(u-2)(u-2)$
18. $(l+10)(l-3)$
19. $(m+6)(m-2)$
20. $3(x+1)(x+4)$
21. $4(y+2)(y+3)$
22. $-5(x-4)(x-1)$
23. $-2(y-25)(y-4)$
24. $-(x+9)(x-3)$
25. $-(x-11)(x+4)$
26. $x(x+10)(x-2)$
27. $-2x^2(x+5)(x-3)$
28. $-xy(x+3y)(x-2y)$
29. $2x^2(x-4y)(x-4y)$
30. $-xy^2(x+4y)(x-y)$
31. $x^4(x-7y)(x-4y)$
32. $(2a+5)(y+10)(y-1)$
33. $x(a+b)(x-4)(x-2)$
34. $(2a+b)(x-9)(x-3)$
35. $(3a-b)(y-8)(y-5)$
36. $(x^2+1-x)(x^2+1+x)$
37. $(2x+3+5z)(2x+3-4z)$
38. $(x-2y-3a)(x-2y-5a)$
39. $(5x-y+6z)(5x-y-4z)$
40. $x(x+3)(x+4)$
41. $x(7-2x)(5-2x)$
42. $(x+1)(x-1)$
43. $(2x+1)(2x-1)$
44. $(y+5)(y-5)$
45. $(5y+3)(5y-3)$
46. $(2-3z)(2+3z)$
47. $(4-5y)(4+5y)$
48. $(4x-3y)(4x+3y)$
49. $(5x^2-9y^3)(5x^2+9y^3)$
50. $4(2xy^4-1)(2xy^4+1)$
51. $5(2x-y)(2x+y)$
52. $(x+1-y)(x+1+y)$
53. $(x+4)(-x)$
54. $(x+5)^2$
55. $(x+4)^2$
56. $(x-6)^2$
57. $(y^2-3)^2$
58. $2(z-7)^2$
59. $-(3x+4y)^2$
60. $(x+3-2y)(x+3+2y)$
61. $(x^3-2y^3-a^2-3b^2)(x^3-2y^3+a^2+3b^2)$

Extra Work Space