$S_n = \frac{n}{2}(a+l)$ or $S_n = \frac{n}{2}\left(2a+(n-1)d\right)$

Section 1.1 Part 2 – Practice Problems

1. Find the sum of the arithmetic series

a) $3 + 5 + 7 + \cdots + (2n+1)$

$S_n = \frac{n}{2}(3 + (2n+1))$

$S_n = \frac{n}{2}(3 + 2n + 1)$

$S_n = \frac{n}{2}(2n+4)$

$S_n = \frac{2n^2}{2} + \frac{4n}{2} \rightarrow \boxed{S_n = n^2 + 2n}$

b) $-1 + 2 + 5 + \cdots + (3n-4)$

$S_n = \frac{n}{2}(-1 + (3n-4))$

$S_n = \frac{n}{2}(-1 + 3n - 4)$

$S_n = \frac{n}{2}(3n-5)$

$\boxed{S_n = \frac{3n^2 - 5n}{2}}$

c) $2 + 5 + 8 + \cdots + 77$

need n 1st

$t_n = a + (n-1)d$

$77 = 2 + 3(n-1)$

$75 = 3(n-1)$

$25 = n-1$

$n = 26$

$S_n = \frac{n}{2}(a+l)$

$S_n = \frac{26}{2}(2+77)$

$S_n = 13(79)$

$\boxed{S_n = 1027}$

d) $5 + 9 + 13 + \cdots + 97$

need n 1st

$t_n = a + (n-1)d$

$97 = 5 + 4(n-1)$

$92 = 4(n-1)$

$23 = n-1$

$n = 24$

$S_n = \frac{n}{2}(a+l)$

$S_{24} = \frac{24}{2}(5+97)$

$S_{24} = 12(102)$

$\boxed{S_{24} = 1224}$

e) $(-41) + (-35) + (-29) + \cdots + 541$

need n 1st

$t_n = a + (n-1)d$

$541 = -41 + 6(n-1)$

$582 = 6(n-1)$

$97 = n-1$

$n = 98$

$S_n = \frac{n}{2}(a+l)$

$S_{98} = \frac{98}{2}(-41+541)$

$S_{98} = 49(500)$

$\boxed{S_{98} = 24\,500}$

f) $2\sqrt{5} + 6\sqrt{5} + 10\sqrt{5} + \cdots + 50\sqrt{5}$

need n 1st

$t_n = a + (n-1)d$

$50\sqrt{5} = 2\sqrt{5} + 4\sqrt{5}(n-1)$

$48\sqrt{5} = 4\sqrt{5}(n-1)$

$12 = n-1$

$n = 13$

$S_{13} = \frac{13}{2}(2\sqrt{5} + 50\sqrt{5})$

$S_{13} = \frac{13}{2}(52\sqrt{5})$

$\boxed{S_{13} = 338\sqrt{5}}$

g) $39 + 33 + 27 + \cdots + (-15)$

need n

$-15 = 39 + (-6)(n-1)$

$-54 = -6(n-1)$

$9 = n-1$

$n = 10$

$S_n = \frac{10}{2}(39 + (-15))$

$S_n = 5(24)$

$\boxed{S_n = 120}$

h) $23 + 19 + 15 + \cdots + (-305)$

need n

$-305 = 23 + (-4)(n-1)$

$-328 = -4(n-1)$

$82 = n-1$

$n = 83$

$S_{83} = \frac{83}{2}(23 + (-305))$

$S_{83} = 41.5(-282)$

$\boxed{S_{83} = -11703}$

i) $\frac{1}{2} + \frac{7}{8} + \frac{5}{4} + \cdots + \frac{55}{8}$

$d = 3/8$

$\frac{55}{8} = \frac{1}{2} + \frac{3}{8}(n-1)$

$\frac{51}{8} = \frac{3}{8}(n-1)$

$17 = n-1$

$n = 18$

$S_{18} = \frac{18}{2}\left(\frac{1}{2} + \frac{55}{8}\right)$

$\boxed{S_{18} = \frac{531}{8}}$

j) $\frac{16}{3} + \frac{13}{3} + \frac{10}{3} + \cdots + \left(-\frac{65}{3}\right)$

$d = -1$

$-\frac{65}{3} = \frac{16}{3} + (-1)(n-1)$

$-27 = -1(n-1)$

$27 = n-1$

$n = 28$

$S_{28} = \frac{28}{2}\left(\frac{16}{3} + \left(-\frac{65}{3}\right)\right)$

$S_{28} = \frac{28}{2}\left(-\frac{49}{3}\right)$

$\boxed{S_{28} = -\frac{686}{3}}$

2. Find the indicated value using the information given

a) $S_{20}, if\ a_1 = 8, a_{20} = 65$

↑ this is the last term

$S_{20} = \frac{20}{2}(8 + 65)$

$S_{20} = 10(73)$

$\boxed{S_{20} = 730}$

b) $S_{21}, if\ a_1 = 8, a_{20} = 65$

t_{21} is l

so we can use d

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{21} = \frac{21}{2}(2(8) + (21-1)(3))$

$S_{21} = 10.5(16 + 60)$

$S_{21} = 10.5(76)$

$\boxed{S_{21} = 798}$

$20 - 1 = 19$

$65 - 8 = 57$

$19d = 57$

$d = 3$

c) $S_{56}, if\ a_{56} = 13, d = -9$

$\uparrow$
ℓ

don't know a

$t_{56} = a + (n-1)d$

$13 = a + (55)(-9)$

$13 = a - 495$

$a = 508$

$S_{56} = \frac{n}{2}(a+\ell)$

$S_{56} = \frac{56}{2}(508+13)$

$\boxed{S_{56} = 14\ 588}$

d) $n\ if\ S_n = 180, a_1 = 4, t_n = 16$

$a = 4$
$\ell = 16$
$S_n = 180$
$n = ?$

$S_n = \frac{n}{2}(4+16)$

$180 = \frac{n}{2}(20)$

$180 = 10n$

$\boxed{n = 18}$

e) $d, if\ S_{40} = 680, a_1 = 11$

$n = 40$
$S_n = 680$
$a = 11$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{40} = \frac{40}{2}(2(11) + (40-1)d)$

$680 = 20(22 + d(39))$

$34 = 22 + 39d$

$12 = 39d$

$d = \frac{12}{39}$ $\boxed{d = \frac{4}{13}}$

f) $S_{62}, if\ a_1 = 10, d = 3$

$n = 62$
$a = 10$
$d = 3$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{62} = \frac{62}{2}(2(10) + (61)(3))$

$S_{62} = 31(20 + 183)$

$S_{62} = 31(203)$

$\boxed{S_{62} = 6293}$

g) $S_{19}, if\ d = 4, a_{19} = 17$

$n = 19$
$\ell = 17$
$d = 4$

$a_{19} - a_1 =$ 18 terms

$a + 18d = 17$

$a + 18(4) = 17$

$a + 72 = 17$

$a = -55$

$S_n = \frac{n}{2}(a+\ell)$

$S_{19} = \frac{19}{2}(-55+17)$

$S_{19} = 9.5(-38)$

$\boxed{S_{19} = -361}$

h) $S_{40}, if\ d = -3, a_{40} = 65$

$a_{40} - a_1 =$ 39 terms

$a + 39d = 65$

$a + 39(-3) = 65$

$a - 117 = 65$ $a = 182$

$S_{40} = \frac{40}{2}(182 + 65)$

$S_{40} = 20(247)$

$\boxed{S_{40} = 4940}$

i) $S_{40}, if\ a_5 = 42, a_{15} = -18$

$S_{40} = ?$

$15 - 5 = 10$ terms

$-18 - 42 = -60$ $\quad 10d = -60$

$d = -6$

$a_1 = 42 - 4d$

$42 - 4(-6)$

$42 + 24$

$a_1 = 66$

$S_{40} = \frac{40}{2}(2a + (n-1)d)$

$S_{40} = 20(2(66) + (39)(-6))$

$= 20(132 - 234)$

$= 20(-102)$

$\boxed{S_{40} = -2040}$

j) $S_{20}, if\ a_8 = 17, a_{15} = 38$

$15 - 8 = 7$ $\quad 7d = 21$ $\quad d = 3$

$38 - 17 = 21$

$a_1 = 17 - 7d$ $\quad a_{20} = 38 + 5d$

$a_1 = 17 - 7(3)$ $\quad = 38 + 15$

$a_1 = 17 - 21$ $\quad = 53$

$a_1 = -4$

$S_n = \frac{n}{2}(a + l)$

$S_{20} = 10(-4 + 53)$

$S_{20} = 10(49) = \boxed{490}$

3. Find the indicated sum.

a) $\sum_{n=1}^{100} n$

$n = 100$

$a_1 = 1$

$l = 100$

$S_{100} = \frac{100}{2}(1 + 100)$

$= 50(101)$

$\boxed{S_{100} = 5050}$

b) $\sum_{k=100}^{200} k$

$n = 101$

$a = 100$

$l = 200$

$S_{101} = \frac{101}{2}(100 + 200)$

$S_{101} = \frac{101(300)}{2} \rightarrow 101(150) = \boxed{15\,150}$

c) $\sum_{j=0}^{72} (3j - 4)$

$n = 73$

$a = -4$ (when $j = 0$)

$l = 212$ (when $j = 72$)

$S_{73} = \frac{73}{2}(-4 + 212) = \frac{73}{2}(208)$

$= \boxed{7592}$

d) $\sum_{x=7}^{24} (2x + 5)$

$n = 18$

$a = 19$ (when $x = 7$)

$l = 53$ (when $x = 24$)

$S_{18} = \frac{18}{2}(19 + 53)$

$= 9(72) = \boxed{648}$

e) $\sum_{y=11}^{48}\left(\frac{y+4}{2}\right)$

$n = 38$

$a = \frac{15}{2}$

$l = 26$

$S_{38} = \frac{38}{2}\left(\frac{15}{2} + 26\right)$

$S_{38} = 19\left(\frac{15}{2} + \frac{52}{2}\right)$

$S_{38} = 19\left(\frac{67}{2}\right)$

$\boxed{S_{38} = 636\frac{1}{2}}$

f) $\sum_{z=51}^{100}(200 - z) - \sum_{z=1}^{50}(200 - z)$ → $6225 - 8725$

$\boxed{-2500}$

$n = 50$
$a = 149$
$l = 100$

$n = 50$
$a = 199$
$l = 150$

$S_{50} = 25(149 + 100)$
$= 6225$

$S_{50} = 25(199 + 150)$
$= 8725$

4. Insert k arithmetic means between the given pair of numbers.

a) $5, 10, k = 2$

$5, \_, \_, 10$ (d d d)

$5 + 3d = 10$

$3d = 5$

$d = \frac{5}{3}$

$5 + \frac{5}{3}$

$\frac{15}{3} + \frac{5}{3} = \frac{20}{3}$

$5, \frac{26}{3}, \frac{25}{3}, \frac{30}{3}$

↓

$\boxed{5, \frac{20}{3}, \frac{25}{3}, 10}$

b) $3, 6, k = 3$

$3, \_, \_, \_, 6$

$3 + 4d = 6$

$4d = 3$

$d = 3/4$

$3 + 3/4$

$\frac{12}{4} + 3/4 = \frac{15}{4}$

$3, \frac{15}{4}, \frac{18}{4}, \frac{21}{4}, \frac{24}{4}$

↓

$\boxed{3, \frac{15}{4}, \frac{9}{2}, \frac{21}{4}, 6}$

c) $a, b, k = 2$

$a, \overset{d}{\frown}\_, \overset{d}{\frown}\_, \overset{d}{\frown} b$

$a + 3d = b \rightarrow 3d = b - a$

$d = \frac{b-a}{3}$

$a + \frac{b-a}{3} \rightarrow \frac{3a}{3} + \frac{b-a}{3}$

$= \frac{2a+b}{3}$

$\frac{2a+b}{3} + \frac{b-a}{3} = \frac{a+2b}{3}$

$\boxed{a, \frac{2a+b}{3}, \frac{a+2b}{3}, b}$

d) $a, b, k = 3$

$a, \overset{d}{\frown}\_, \overset{d}{\frown}\_, \overset{d}{\frown}\_, \overset{d}{\frown} b$

$a + 4d = b \rightarrow 4d = b - a \quad d = \frac{b-a}{4}$

$a + \frac{b-a}{4} \rightarrow \frac{4a+b-a}{4} = \frac{3a+b}{4}$

$\frac{3a+b}{4} + \frac{b-a}{4} \rightarrow \frac{2a+2b}{4} = \frac{a+b}{2}$

$\frac{2a+2b}{4} + \frac{b-a}{4} \rightarrow \frac{a+3b}{4}$

$\boxed{a, \frac{3a+b}{4}, \frac{a+b}{2}, \frac{a+3b}{4}, b}$

5. Solve for b: $\sum_{x=2}^{b}(23 - 2x) = 91$

$b - 2 = n - 1$

$b - 1 = n$

when $x = 2 \quad a_1 = 19$

$x = b \quad \ell = 23 - 2b$

$S_n = \frac{n}{2}(a + \ell)$

$91 = \frac{b-1}{2}(19 + 23 - 2b)$

$91 = \frac{b-1}{2}(42 - 2b) \rightarrow 91 = (b-1)(21-b)$

$\leftarrow$ divides

$91 = 21b - b^2 - 21 + b$

$b^2 - 22b + 112 = 0$

$(b-8)(b-14) = 0$

$\boxed{b = 8 \quad b = 14}$

6. Find the sum: $\sum_{x=a}^{b} 5$

$a = 5$

$b = 5$

$b - a + 1 = n$

$S_{b-a+1} = \frac{b-a+1}{2}(5+5)$

$S_{b-a+1} = \frac{b-a+1}{2}(10)$

$S_{b-a+1} = 5(b-a+1)$

$= 5b - 5a + 5$

$= \boxed{-5a + 5b + 5}$

7. What is the last element in the 20^{th} row?

1
2 3
4 5 6
7 8 9 10

Last term: 1, 3, 6, 10

1, 1+2, 1+2+3, 1+2+3+4

So 20^{th} row: $a = 1$ $n = 20$
$l = 20$

$S_{20} = \frac{20}{2}(1+20)$

$10(21)$

$\boxed{S_{20} = 210}$

8. How many terms of the arithmetic series $1491 + 1484 + 1477 + \cdots$ are needed to give the sum of zero?

$d = -7$ $n = ?$

$S_n = 0$ $a = 1491$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$2 \cdot 0 = 2\frac{n}{2}(2(1491) + (n-1)(-7))$

$0 = n(2982 - 7(n-1))$

$0 = 2982n - 7n^2 + 7n$

$0 = -7n^2 + 2989n$

$0 = -7n(n-427)$

$\boxed{n = 0 \quad n = 427}$

9. An auditorium has eight seats in the first row. Each subsequent row has four more seats than the previous row. How many seats are there in the 50^{th} row of the auditorium?

8, 12, 16, ...

$a = 8$
$d = 4$
$n = 50$

$t_{50} = a + (n-1)d$

$t_{50} = 8 + 49(4)$

$t_{50} = 8 + 196$

$\boxed{t_{50} = 204}$

10. If \$1000 is deposited into the bank the day a child is born, and \$100 more than the previous deposit is made each year until the child's $18th$ birthday, how much will be in the account, excluding interest?

$18 - 0 + 1 = 19$

$n = 19$
$a = 1000$
$d = 100$

$S_{19} = \frac{19}{2}(2(1000) + (19-1)(100))$

$S_{19} = \frac{19}{2}(2000 + 1800)$

$S_{19} = \frac{19}{2}(3800)$

$S_{19} = 19(1900)$

$\boxed{S_{19} = \$36\,100}$

$\text{If } d=6 \rightarrow a=-5$ $\boxed{-5, 1, 7}$

$\text{If } d=-6 \rightarrow a=7$ $\boxed{7, 1, -5}$

11. Find the sum of all multiples of 6 between 50 *and* 500.

1st multiple of 6 after 50 is 54

last multiple of 6 before 500 is 498

$t_n = 54 + (n-1)(6)$ ← common difference gives multiple of 6

$498 = 54 + 6(n-1)$

$444 = 6(n-1)$

$74 = n-1$

$n = 75$

$S_{75} = \frac{75}{2}(54+498)$

$S_{75} = \frac{75}{2}(552)$

$\boxed{S_{75} = 20700}$

12. The sum of three consecutive terms of an arithmetic sequence is 3. The sum of their squares is 75. Find the three numbers.

Term 1: a

Term 2: $a+d$

Term 3: $a+2d$

$a+(a+d)+(a+2d)=3$

$3a+3d=3$

$a+d=1$

$\boxed{a=1-d}$

$a^2+(a+d)^2+(a+2d)^2=75$

$(1-d)^2+(1-d+d)^2+(1-d+2d)^2=75$

$1-2d+d^2+1^2+(d+1)^2=75$

$1-2d+d^2+1+d^2+2d+1=75$

$2d^2+3=75$

$2d^2-72=0$

$d^2-36=0$

$d^2=36$

$\boxed{d=\pm 6}$

13. If 20 people in a class shake hands with each other exactly once, how many handshakes will take place?

1st person shakes 19 hands

2nd " " 18 hands

"

"

"

$19+18+17+\ldots+1$

$S_{19} = \frac{19}{2}(19+1)$

$S_{19} = \frac{19}{2}(20) = 19(10) = \boxed{190}$

14. If the sum of the terms of an arithmetic series is 234, and the middle term is 26, find the number of terms in the series.

$S_n = 234$

$a+xd=26 \rightarrow a=26-xd$

$26+xd=l$ $\quad l=26+xd$

$234 = \frac{n}{2}(26-xd+26+xd)$

$234 = \frac{n}{2}(52)$

$234 = 26n$

$\boxed{n=9}$

See Website for a Detailed Answer Key