Section 1.1 Part 1 – Practice Problems

1. Respond to the following

a) The Domain of a sequence is the set of what find of consecutive numbers?

Natural Numbers $1, 2, 3, \ldots$

b) A sequence with a last term is what kind of sequence?

Finite (it ends)

c) A sequence with no last term is what kind of sequence

Infinite

d) What kind of sequence is this: $a_1 = 2,\ \ a_n = 2a_{n-1}$

Recursive each term given with respect to the previous

e) Write the formula for the n^{th} term of an arithmetic sequence t_n

$t_n = a + (n-1)d$ $a = t_1$

2. Write the first four terms of each of the following sequences

a) $\{n^2 - 2\}$

$n=1$ $\{1^2-2\} = -1$

$n=2$ $\{2^2-2\} = 2$

$n=3$ $\{3^2-2\} = 7$

$n=4$ $\{4^2-2\} = 14$

b) $\left\{\frac{n+2}{n+1}\right\}$

$n=1$ $\left\{\frac{1+2}{1+1}\right\} = \frac{3}{2}$

$n=2$ $\left\{\frac{2+2}{2+1}\right\} = \frac{4}{3}$

$n=3$ $\left\{\frac{3+2}{3+1}\right\} = \frac{5}{4}$

$n=4$ $\left\{\frac{4+2}{4+1}\right\} = \frac{6}{5}$

c) $\{(-1)^{n+1}n^2\}$

$n=1 \quad \{(-1)^{1+1}(1)^2\} = (-1)^2(1) = 1$

$n=2 \quad \{(-1)^{2+1}(2)^2\} = (-1)^3(4) = -4$

$n=3 \quad \{(-1)^{3+1}(3)^2\} = (-1)^4(9) = 9$

$n=4 \quad \{(-1)^{4+1}(4)^2\} = (-1)^5(16) = -16$

d) $\left\{\frac{3^n}{2^n+1}\right\}$

$n=1 \quad \left\{\frac{3^1}{2^1+1}\right\} = \frac{3}{3} = 1$

$n=2 \quad \left\{\frac{3^2}{2^2+1}\right\} = \frac{9}{5} = \frac{9}{5}$

$n=3 \quad \left\{\frac{3^3}{2^3+1}\right\} = \frac{27}{8+1} = \frac{27}{9} = 3$

$n=4 \quad \left\{\frac{3^4}{2^4+1}\right\} = \frac{81}{16+1} = \frac{81}{17} = \frac{81}{17}$

e) $\left\{\frac{2^n}{n^2}\right\}$

$n=1 \quad \left\{\frac{2^1}{1^2}\right\} = 2$

$n=2 \quad \left\{\frac{2^2}{2^2}\right\} = \frac{4}{4} = 1$

$n=3 \quad \left\{\frac{2^3}{3^2}\right\} = \frac{8}{9}$

$n=4 \quad \left\{\frac{2^4}{4^2}\right\} = \frac{16}{16} = 1$

f) $\left\{\left(\frac{2}{3}\right)^n\right\}$

$n=1 \quad \left(\frac{2}{3}\right)^1 = \frac{2}{3}$

$n=2 \quad \left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$

$n=3 \quad \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$

$n=4 \quad \left(\frac{2^4}{3^4}\right) = \frac{2^4}{3^4} = \frac{16}{81}$

3. Write the n^{th} term of the suggested pattern.

a) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

$n = \ 1 \ 2 \ 3 \ 4$

$\boxed{\frac{1}{n}}$

b) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

$n = \ 1 \ 2 \ 3 \ 4$

$\boxed{\frac{1}{2^{n-1}}}$

Pattern is power of 2 but n-1

2^{n-1}

c) $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \dots$

1 2 3 4

$\boxed{\frac{2^n}{3^n} \text{ or } \left(\frac{2}{3}\right)^n}$

numerator: 2^n

denominator: 3^n

d) $2, -4, 6, -8$

$\boxed{(-1)^{n+1} 2n}$

The flip flop of the negative comes from

$(-1)^{n+1}$

Then we have $2n$

4. Write the first four terms of the recursive sequence.

a) $a = 4, t_n = 2 + t_{n-1}$

$a = 4$ $\qquad a = 4$

$t_2 = 2 + t_{2-1} \rightarrow 2 + t_1 \rightarrow 2+4 = 6$

$t_3 = 2 + t_{3-1} \rightarrow 2 + t_2 \rightarrow 2+6 = 8$

$t_4 = 2 + t_{4-1} \rightarrow 2 + t_3 \rightarrow 2+8 = 10$

b) $a = 3, t_n = n - t_{n-1}$

$a = 3$ $\qquad a = 3$

$t_2 = 2 - t_{2-1} \Rightarrow 2 - t_1 \rightarrow 2-3 = -1$

$t_3 = 3 - t_{3-1} \rightarrow 3 - t_2 \rightarrow 3-(-1) = 4$

$t_4 = 4 - t_{4-1} \rightarrow 4 - t_3 \rightarrow 4-4 = 0$

c) $a = 2, a_2 = 3, a_n = a_{n-1} + a_{n-2}$

$a = 2$ $\qquad a_1 = 2$

$a_2 = 3$ $\qquad a_2 = 3$

$a_3 = a_{3-1} + a_{3-2} \rightarrow a_2 + a_1 = 2+3 = 5$

$a_4 = a_{4-1} + a_{4-2} \rightarrow a_3 + a_2 = 5+3 = 8$

d) $a_1 = -1, a_2 = 1, a_n = na_{n-1} + a_{n-2}$

$a_1 = -1$ $\qquad a_1 = -1$

$a_2 = 1$ $\qquad a_2 = 1$

$a_3 = 3(a_{3-1}) + a_{3-2}$

$= 3(a_2) + a_1$

$= 3(1) + (-1) \rightarrow 3-1 = 2$

$a_4 = 4(a_3) + a_2 \rightarrow 4(2)+1 = 9$

5. Find the sum of each sequence.

a) $\sum_{k=1}^{5} 4$

no k values to change so

$4+4+4+4+4$

$\boxed{20}$

b) $\sum_{k=1}^{4} (k^2 - 2)$

$(1^2-2)+(2^2-2)+(3^2-2)+(4^2-2)$

$-1+2+7+14$

$\boxed{22}$

c) $\sum_{k=2}^{5} (k^2 - 1)$

$(2^2-1)+(3^2-1)+(4^2-1)+(5^2-1)$

$3+8+15+24$

$\boxed{50}$

d) $\sum_{k=0}^{3} (k^3 - 1)$

$(0^3-1)+(1^3-1)+(2^3-1)+(3^3-1)$

$-1+0+7+26$

$\boxed{32}$

e) $\sum_{k=1}^{4} \frac{k^2}{2}$

$\frac{1^2}{2}+\frac{2^2}{2}+\frac{3^2}{2}+\frac{4^2}{2}$

$\frac{1+4+9+16}{2} = \frac{30}{2} = \boxed{15}$

f) $\sum_{k=6}^{8} (k+1)^2$

$(6+1)^2+(7+1)^2+(8+1)^2$

$7^2+8^2+9^2$

$49+64+81$

$\boxed{194}$

6. Express each sum using summation notation with index $k = 1$

a) $1 + 3 + 5 + 7$ but start at 1

$\underbrace{1+3}_{+2}\underbrace{+5}_{+2}\underbrace{+7}_{+2}$

$$\sum_{k=1}^{4} 2k-1$$

b) $1^2 + 2^2 + 3^2 + 4^2 + 5^2$

$$\sum_{k=1}^{5} k^2$$

c) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{n}{n+1}$

k

k+1

$$\sum_{k=1}^{n} \frac{k}{k+1}$$

d) $\frac{5}{1} + \frac{5^2}{2} + \frac{5^3}{3} + \cdots + \frac{5^n}{n}$ ← 5^k

k

$$\sum_{k=1}^{n} \frac{5^k}{k}$$

7. Write the first five terms of the arithmetic sequence.

a) $7, 11, 15,$ 19, 23

$\underbrace{7,11}_{4}\underbrace{,15}_{4}$

b) $15, 12, 9,$ 6, 3

$\underbrace{15,12}_{-3}\underbrace{,9}_{-3}$

c) $a = 4, d = 2$

4, 6, 8, 10, 12

d) $a = -1, d = -3$

−1, −4, −7, −10, −13

e) $a = -5, d = -\frac{3}{4}$

$-5, -\frac{23}{4}, -\frac{13}{2}, -\frac{29}{4}, -8$

$-\frac{20}{4}, -\frac{23}{4}, -\frac{26}{4}, -\frac{29}{4}, -\frac{32}{4}$

f) $a = -\frac{2}{3}, d = \frac{1 \cdot 3}{5 \cdot 3} = \frac{3}{15}$

$-\frac{2}{3}$

$-\frac{10}{15}, -\frac{7}{15}, -\frac{4}{15}, -\frac{1}{15}, \frac{2}{15}$

8. Find the indicated arithmetic term.

a) $a = 5, d = 3; find\ t_{12}$

$t_{12} = a + (n-1)d$

$t_{12} = 5 + (12-1)(3)$

$t_{12} = 5 + 11(3)$

$t_{12} = 5 + 33$

$\boxed{t_{12} = 38}$

b) $a = \frac{2}{3}, d = -\frac{1}{4}; find\ t_9$

$t_9 = \frac{2}{3} + (9-1)(-\frac{1}{4})$

$t_9 = \frac{2}{3} + 8(-\frac{1}{4})$

$t_9 = \frac{2}{3} - 2$

$t_9 = \frac{2}{3} - \frac{6}{3}$

$\boxed{t_9 = -\frac{4}{3}}$

c) $a = -\frac{3}{4}, d = \frac{1}{2}; find\ t_{10}$

$t_{10} = -\frac{3}{4} + (10-1)(\frac{1}{2})$

$= -\frac{3}{4} + 9(\frac{1}{2})$

$= -\frac{3}{4} + \frac{9}{2}$

$= -\frac{3}{4} + \frac{18}{4}$

$\boxed{t_{10} = \frac{15}{4}}$

d) $a = 2.5, d = -1.253; find\ t_{20}$

$t_{20} = 2.5 + (20-1)(-1.253)$

$t_{20} = 2.5 + 19(-1.253)$

$t_{20} = 2.5 + -23.807$

$\boxed{t_{20} = -21.307}$

e) $a = -0.75, d = 0.05; find\ t_{40}$

$t_{40} = a + (n-1)d$

$t_{40} = -0.75 + (40-1)(0.05)$

$t_{40} = -0.75 + 39(0.05)$

$t_{40} = -0.75 + 1.95$

$\boxed{t_{40} = 1.2}$

f) $a = -\frac{7}{4}, d = -\frac{2}{3}; find\ t_{37}$

$t_{37} = -\frac{7}{4} + (37-1)(-\frac{2}{3})$

$t_{37} = -\frac{7}{4} + 36(-\frac{2}{3})$

$t_{37} = -\frac{7}{4} + (-24)$

$t_{37} = -\frac{7}{4} - \frac{96}{4}$

$\boxed{t_{37} = -\frac{103}{4}}$

9. Find the number of terms in each arithmetic sequence

a) $a = 6, d = -3,\ t_n = -30$

$t_n = a + (n-1)d$

$-30 = 6 + (n-1)(-3)$

$-36 = -3(n-1)$

$12 = n-1$

$\boxed{n = 13}$

b) $a = -3, d = 5,\ t_n = 82$

$82 = -3 + (n-1)(5)$

$85 = 5(n-1)$

$17 = n-1$

$\boxed{n = 18}$

c) $a = 0.6, d = 0.2, t_n = 9.2$

$9.2 = 0.6 + (n-1)(0.2)$

$8.6 = 0.2(n-1)$

$43 = n-1$

$\boxed{n = 44}$

d) $a = -0.3, d = -2.3,\ t_n = -39.4$

$-39.4 = -0.3 + (n-1)(-2.3)$

$-39.1 = -2.3(n-1)$

$17 = n-1$

$\boxed{n = 18}$

e) $-1, 4, 9, \ldots, 159$

$t_n = a + (n-1)d$

$159 = -1 + (n-1)(5)$

$160 = 5(n-1)$

$32 = n-1$

$\boxed{n = 33}$

f) $23, 20, 17, \ldots, -100$

$-100 = 23 + (n-1)(-3)$

$-123 = -3(n-1)$

$41 = n-1$

$\boxed{n = 42}$

10. Find the first term in the arithmetic sequence

a) $6th\ term\ is\ 10; 18th\ term\ is\ 46$

$18-6 = 12$ term $\quad 12d = 36$

$46-10 = 36$ diff $\quad d = 3$

use either tn n combo

$10 = a+(6-1)(3)$

$10 = a+(5)(3)$

$10 = a+15$

$\boxed{a = -5}$

b) $4th\ term\ is\ 2; 18th\ term\ is\ 30$

$18-4 = 14$ terms $\quad 14d = 28$

$30-2 = 28 \quad d = 2$

$2 = a+(4-1)(2)$

$2 = a+3(2)$

$2 = a+6$

$\boxed{a = -4}$

c) $9th\ term\ is\ 23; 17th\ term\ is -1$

$17-9 = 8 \quad 8d = -24$

$-1-23 = -24 \quad d = -3$

$23 = a+(9-1)(-3)$

$23 = a+8(-3)$

$23 = a-24$

$\boxed{a = 47}$

d) $5th\ term\ is\ 3; 25th\ term\ is -57$

$25-5 = 20 \quad 20d = -60$

$-57-3 = -60 \quad d = -3$

$3 = a+(5-1)(-3)$

$3 = a+4(-3)$

$3 = a-12$

$\boxed{a = 15}$

e) $13th\ term\ is -3; 20th\ term\ is -17$

$20-13=7$

$-17-(-3)=-14$

$7d=-14$

$d=-2$

$-3=a+(13-1)(-2)$

$-3=a-24$

$\boxed{a=21}$

f) $11th\ term\ is\ 37; 26th\ term\ is 32$

$26-11=15$

$32-37=-5$

$15d=-5$

$d=-\frac{5}{15} \rightarrow d=-\frac{1}{3}$

$37=a+(11-1)\left(-\frac{1}{3}\right)$

$37=a+10\left(-\frac{1}{3}\right)$

$37=a-\frac{10}{3}$

$37+\frac{10}{3}=a$

$37+3\frac{1}{3}=a$

$\boxed{a=40\frac{1}{3}}$

11. Find x so that the values given are consecutive terms of an arithmetic sequence

a) $x+3, 2x+1, and\ 5x+2$

$(2x+1)-(x+3)=d$ $(5x+2)-(2x+1)=d$

$d=d$ so

$(2x+1)-(x+3)=(5x+2)-(2x+1)$

$2x+1-x-3=5x+2-2x-1$

$x-2=3x+1$

$-2=2x+1$

$-3=2x$

$\boxed{x=\frac{-3}{2}}$

b) $2x, 3x+2, and\ 5x+3$

$(3x+2)-2x=d$ $(5x+3)-(3x+2)=d$

$3x+2-2x=5x+3-3x-2$

$x+2=2x+1$

$2=x+1$

$\boxed{1=x}$

c) $x - 1, \frac{1}{2}x + 4, and\ 1 - 2x$

$\left(\frac{1}{2}x+4\right) - (x-1) = d \quad (1-2x) - \left(\frac{1}{2}x+4\right) = d$

$\frac{1}{2}x + 4 - x + 1 = 1 - 2x - \frac{1}{2}x - 4$

$-\frac{1}{2}x + 5 = -2\frac{1}{2}x - 3$

$5 = -2x - 3$

$8 = -2x$

$\boxed{x = -4}$

d) $2x - 1, x + 1, and\ 3x + 9$

$(x+1) - (2x-1) = d \quad (3x+9) - (x+1) = d$

$x + 1 - 2x + 1 = 3x + 9 - x - 1$

$-x + 2 = 2x + 8$

$-6 = 3x$

$\boxed{x = -2}$

e) $x + 4, x^2 + 5, and\ x + 30$

$(x^2+5) - (x+4) = d \quad (x+30) - (x^2+5) = d$

$x^2 + 5 - x - 4 = x + 30 - x^2 - 5$

$2x^2 - 2x + 1 = 25$

$2x^2 - 2x - 24 = 0$

$2(x^2 - x - 12) = 0$

$2(x-4)(x+3) = 0$

$\boxed{x = 4 \text{ or } x = -3}$

f) $8x + 7, 2x + 5, and\ 2x^2 + x$

$(2x+5) - (8x+7) = d \quad (2x^2+x) - (2x+5) = d$

$2x + 5 - 8x - 7 = 2x^2 + x - 2x - 5$

$-6x - 2 = 2x^2 - x - 5$

$0 = 2x^2 + 5x - 3$

$0 = (2x-1)(x+3)$

$\boxed{x = \frac{1}{2} \text{ or } x = -3}$

12. If t_n is a term of an arithmetic sequence, what is $t_n - t_{n-1}$ equal to?

Let n = at least 2

$t_2 - t_{2-1} \rightarrow t_2 - t_1 = d$

$t_n - t_{n-1}$ is the difference

13. List the first seven numbers of the Fibonacci Sequence: $a_1 = 1, a_2 = 1, a_n = a_{n-1} + a_{n-2}, n > 2$

$a_1 = 1$

$a_2 = 1$

$a_3 = a_{3-1} + a_{3-2}$
$= a_2 + a_1$
$= 2$

$a_4 = a_3 + a_2$
$= 3$

$a_5 = a_4 + a_3$
$= 5$

$a_6 = a_5 + a_4$
$= 8$

$a_7 = a_6 + a_5$
$= 13$

$\boxed{1, 1, 2, 3, 5, 8, 13}$

14. The starting salary of an employee is \$23 750. If each year a \$1250 raise is given, in how many years will the employee's salary be \$50 000?

$t_n = 50\,000 \quad t_1 = 23\,750 \quad d = 1250 \quad n = ?$

$50\,000 = 23\,750 + (n-1)(1250)$

$26250 = 1250(n-1)$

$21 = n - 1$

$\boxed{n = 22}$

$\boxed{22 \text{ years}}$

15. An auditorium has 8 seats in the first row. Each subsequent row has 4 more seats than the previous row. What row has 140 seats?

$a = 8$

$d = 4$

$t_n = 140$

$n = ?$

$140 = 8 + (n-1)4$

$132 = 4(n-1)$

$33 = n - 1$

$n = 34$

$\boxed{34^{th} \text{ row}}$

16. A well drilling company charges \$8.00 for the first meter, then \$8.75 for the second meter and so on in an arithmetic sequence. At this rate, what would be the cost to drill the meter of a well 120 *meters* deep?

$a = 8$

$d = 0.75$

$n = 120$

0.75 more per meter at each additional meter in depth

$t_{120} = 8 + (120-1)(0.75)$

$= 8 + 89.25$

$t_{120} = 97.25$

$\boxed{\$97.25}$

17. Assume that leading up to your death you needed 15 *minutes* more sleep each night. When you hit an additional 24 *hours* sleep you die. If you needed 8 *hours* sleep on September 1st, what day do you die?

$a = 8$

$d = 15 \text{ mins} = \frac{1}{4} \text{hr}$

$t_n = 24$

$24 = 8 + (n-1)(\frac{1}{4})$

$16 = \frac{1}{4}(n-1)$

$64 = n-1$

$65 = n$

Sept 30 day
Oct 31 day
61

$\boxed{\text{Nov 4th}}$

18. The first three terms of an arithmetic sequence are: $x-3, \frac{x^2}{25}+9,$ *and* $3x-11$. Determine the fourth term.

need x then d

$\left(\frac{x^2}{25}+9\right) - (x-3) = (3x-11) - \left(\frac{x^2}{25}+9\right)$

$\frac{x^2}{25}+9-x+3 = 3x-11-\frac{x^2}{25}-9$

$\frac{x^2}{25} - x + 12 = 3x - 20 - \frac{x^2}{25} \rightarrow \frac{2x^2}{25} - 4x + 32 = 0$

$\Rightarrow 2x^2 - 100x + 800 = 0$

$x^2 - 50x + 400 = 0$

$(x-10)(x-40) = 0$

$x = 10$
$x = 40$

if $x=10$		$x = 40$	
7		37	
13	$t_4 = 25$	73	$t_4 = 145$
19		109	

19. The first, third, and fifth terms of an arithmetic sequence are: $2x-1, x^2-3,$ *and* $11-x^2$ respectively. Determine the second term.

$(x^2-3)-(2x-1) = 2d$

$(11-x^2)-(x^2-3) = 2d$

$(x^2-3)-(2x-1) = (11-x^2)-(x^2-3)$

$x^2-3-2x+1 = 11-x^2-x^2+3$

$x^2-2x-2 = -2x^2+14$

$3x^2-2x-16 = 0$

$(3x-8)(x+2)$

intuitive factoring use grouping or AC method

$x = -2$

$x = 8/3$

$\boxed{\text{if } x = -2}$

$(-2)^2-3-[2(-2)-1] = 2d$

$4-3-(-5) = 2d \quad 2d = 6 \quad \boxed{d=3}$

$\boxed{t_2 = -5+3 = -2}$

if $x = 8/3$ $\quad (\frac{8}{3})^2-3-[2(\frac{8}{3})-1] = 2d$

$\frac{64}{9} - 3 - \frac{13}{3} = 2d$

$2d = -\frac{2}{9} \quad \boxed{d = \frac{-1}{9}}$

$\boxed{t_2 = 38/9}$

$t_2 = (\frac{16}{3}-1)+(\frac{-1}{9})$