Section 1.1 – Practice Problems

Write the first four terms of each of the following sequences

1. $t_n = n^2 - 2$

$t_1 = (1^2-2) = -1$ $\quad t_2 = (2^2-2) = 2$

$t_3 = (3^2-2) = 7$ $\quad t_4 = (4^2-2) = 14$

2. $t_n = \left\{\frac{n+2}{n+1}\right\}$

$t_1 = \left(\frac{1+2}{1+1}\right) = \frac{3}{2}$ $\quad t_2 = \left(\frac{2+2}{2+1}\right) = \frac{4}{3}$

$t_3 = \left(\frac{3+2}{3+1}\right) = \frac{5}{4}$ $\quad t_4 = \left(\frac{4+2}{4+1}\right) = \frac{6}{5}$

3. $t_n = \{(-1)^{n+1}n^2\}$

$t_1 = \{(-1)^{1+1}\cdot(1)^2\} = 1$ $\quad t_2 = \{(-1)^{2+1}\cdot(2)^2\} = -4$

$t_3 = \{(-1)^{3+1}\cdot(3)^2\} = 9$ $\quad t_4 = \{(-1)^{4+1}\cdot(4)^2\} = -16$

4. $t_n = \left\{\frac{3^n}{2^n+1}\right\}$

$t_1 = \left(\frac{3^1}{2^1+1}\right) = 1$ $\quad t_2 = \left(\frac{3^2}{2^2+1}\right) = \frac{9}{5}$

$t_3 = \left(\frac{3^3}{2^3+1}\right) = \frac{27}{9} = 3$ $\quad t_4 = \left(\frac{3^4}{2^4+1}\right) = \frac{81}{17}$

5. $t_n = \left\{\frac{2^n}{n^2}\right\}$

$t_1 = \left(\frac{2^1}{1^2}\right) = \frac{2}{1} = 2$ $\quad t_2 = \left(\frac{2^2}{2^2}\right) = 1$

$t_3 = \left(\frac{2^3}{3^2}\right) = \frac{8}{9}$ $\quad t_4 = \left(\frac{2^4}{4^2}\right) = \frac{16}{16} = 1$

6. $t_n = \left\{\left(\frac{2}{3}\right)^n\right\}$

$t_1 = \left(\frac{2}{3}\right)^1 = \frac{2}{3}$ $\quad t_2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

$t_3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$ $\quad t_4 = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$

Find the indicated arithmetic term

7. $a = 5, d = 3, find\ t_{12}$

$t_{12} = a + (n-1)d$

$= 5 + (12-1)3$

$= 5 + (11)3$

$= 5 + 33 = 38$

8. $a - \frac{2}{3}, d - -\frac{1}{4}, find\ t_9$

$t_9 = \frac{2}{3} + (9-1)\left(-\frac{1}{4}\right)$

$= \frac{2}{3} + 8\cdot\left(-\frac{1}{4}\right)$

$= \frac{2}{3} + \frac{8}{-4} = \frac{2}{3} - 2 = \frac{-4}{3}$

9. $a = -\frac{3}{4}, d = \frac{1}{2}, find\ t_{10}$

$t_{10} = -\frac{3}{4} + (10-1)\left(\frac{1}{2}\right)$

$= -\frac{3}{4} + 9\cdot\left(\frac{1}{2}\right)$

$= -\frac{3}{4} + \frac{9}{2} = -\frac{3}{4} + \frac{18}{4} = \frac{15}{4}$

10. $a = 2.5, d = -1.25, find\ t_{20}$

$t_{20} = 2.5 + (20-1)(-1.25)$

$= 2.5 + (19)(-1.25)$

$= 2.5 - 23.75$

$= -21.25$

11. $a = -0.75, d = 0.05, find\ t_{40}$

$t_{40} = -0.75 + (40-1)(0.05)$

$= -0.75 + 39(0.05)$

$= -0.75 + 1.95$

$= 1.20$

12. $a = -\frac{7}{4}, d = -\frac{2}{3}, find\ t_{37}$

$t_{37} = -\frac{7}{4} + (37-1)\left(-\frac{2}{3}\right)$

$= -\frac{7}{4} + 36\left(-\frac{2}{3}\right)$

$= -\frac{7}{4} - \frac{72}{3} = -\frac{7}{4} - \frac{24}{1} = -\frac{7}{4} - \frac{96}{4} = -\frac{103}{4}$

Find the number of terms in each arithmetic sequence

13. $a = 6, d = -3, t_n = -30$

$t_n = a + (n-1)d$

$t_n - a = (n-1)d$

$\frac{t_n - a}{d} = n - 1$

$n = \left(\frac{t_n - a}{d}\right) + 1$

$n = \left(\frac{-30-6}{-3}\right) + 1$

$= \left(\frac{-36}{-3}\right) + 1$

$= 12 + 1 = 13$

14. $a = -3, d = 5, t_n = 82$

$n = \left(\frac{82-(-3)}{5}\right) + 1$

$= \left(\frac{85}{5}\right) + 1$

$= 17 + 1 = 18$

15. $a = 0.6, d = 0.2, t_n = 9.2$

$n = \left(\frac{9.2-0.6}{0.2}\right) + 1$

$= 43 + 1$

$= 44$

16. $a = -0.3, d = -2.3, t_n = -39.4$

$n = \left(\frac{-39.4-(-0.3)}{-2.3}\right) + 1$

$= 17 + 1$

$= 18$

17. $-1, 4, 9, \ldots, 159$ $a = -1$ $d = 5$ $t_n = 159$

$n = \left(\frac{159-(-1)}{5}\right) + 1$

$= \left(\frac{160}{5}\right) + 1 = 33$

18. $23, 20, 17, \ldots, -100$ $a = 23$, $d = -3$, $t_n = -100$

$n = \left(\frac{-100-23}{-3}\right) + 1$

$= 41 + 1 = 42$

Find the first term in the arithmetic sequence

19. *6th term is* 10; *18th term is* 46

$18 - 6 = 12$

$46 - 10 = 36$

$12d = 36$

$d = 3$

$t_n = a + (n-1)d$

$a = t_n - (n-1)d$

$= 10 - (6-1)3$

$= 10 - 15 = -5$

20. *4th term is* 2; *18th term is* 30

$30 - 2 = 28$

$18 - 4 = 14$

$14d = 28$

$d = 2$

$a = t_n - (n-1)d$

$= 2 - (4-1)2$

$= 2 - (3)2$

$= -4$

21. *9th term is* 23; *17th term is* -1

$17 - 9 = 8$

$-1 - 23 = -24$

$8d = -24$

$d = -3$

$a = 23 - (9-1)(-3)$

$= 23 - (-24)$

$= 47$

22. *5th term is* 3; *25th term is* -57

$25 - 5 = 20$

$-57 - 3 = -60$

$20d = -60$

$d = -3$

$a = 3 - (5-1)(-3)$

$= 3 - (-12)$

$= 15$

23. *13th term is* -3; *20th term is* -17

$20 - 13 = 7$

$-17 - (-3) = -14$

$7d = -14$

$d = -2$

$a = -3 - (13-1)(-2)$

$= -3 - (-24)$

$= 21$

24. *11th term is* 37; *26th term is* 32

$26 - 11 = 15$

$32 - 37 = -5$

$15d = -5$

$d = -\frac{1}{3}$

$a = 37 - (11-1)\left(-\frac{1}{3}\right)$

$= 37 - (10)\left(-\frac{1}{3}\right)$

$= 37 + \frac{10}{3} = \frac{111}{3} + \frac{10}{3} = \frac{121}{3}$

Find the sum of the arithmetic series

25. $3 + 5 + 7 + \cdots + (2n + 1)$

$S_n = \frac{n}{2}(a+l)$ or $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_n = \frac{n}{2}(3 + 2n + 1)$

$= \frac{n}{2}(4 + 2n)$

$= \frac{4n}{2} + \frac{2n^2}{2} = 2n + n^2$

26. $-1 + 2 + 5 + \cdots + (3n - 4)$

$S_n = \frac{n}{2}(-1 + 3n - 4)$

$= \frac{n}{2}(-5 + 3n)$

$= -\frac{5n}{2} + \frac{3n^2}{2}$

27. $2 + 5 + 8 + \cdots + 77$

$S_n = \frac{n}{2}(a+l) = \frac{26}{2}(2 + 77) = 13(79)$

$= 1027$

$n = \left(\frac{l - a}{d}\right) + 1 = \left(\frac{77 - 2}{3}\right) + 1 = 26$

28. $5 + 9 + 13 + \cdots + 97$

$n = \left(\frac{97 - 5}{4}\right) + 1 = 24$

$S_n = \frac{24}{2}(5 + 97) = 12(102) = 1224$

29. $(-41) + (-35) + (-29) + \cdots + 541$

$n = \left(\frac{541 - (-41)}{6}\right) + 1$

$= 98$

$S_n = \frac{98}{2}(-41 + 541)$

$= 49(500)$

$= 24,500$

30. $2\sqrt{5} + 6\sqrt{5} + 10\sqrt{5} + \cdots + 50\sqrt{5}$

$t_n = a + (n-1)d$

$50\sqrt{5} = 2\sqrt{5} + (n-1)4\sqrt{5}$

$48\sqrt{5} = (n-1)4\sqrt{5}$

$\frac{48\sqrt{5}}{4\sqrt{5}} = n - 1$

$12 = n - 1$

$13 = n$

$S_n = \frac{n}{2}(a + l)$

$S_{13} = \frac{13}{2}(2\sqrt{5} + 50\sqrt{5})$

$S_{13} = \frac{13}{2}(52\sqrt{5})$

$S_{13} = 338\sqrt{5}$

31. $39 + 33 + 27 + \cdots + (-15)$

$n = \left(\frac{-15 - 39}{-6}\right) + 1$

$= 9 + 1$

$= 10$

$S_n = \frac{10}{2}(39 + (-15))$

$= 5(24)$

$= 120$

32. $23 + 19 + 15 + \cdots + (-305)$

$n = \left(\frac{-305 - 23}{-4}\right) + 1$

$= 83$

$S_n = \frac{83}{2}(23 + (-305))$

$= \frac{83}{2}(-282)$

$= -11,703$

t and a are synonymous

Find the indicated value using the information given

33. $S_{20}, if\ a_1 = 8, a_{20} = 65$

$S_{20} = \frac{20}{2}(8+65)$

$= 10(73)$

$= 730$

34. $S_{21}, if\ a_1 = 8, a_{20} = 65$

$a_{20} - a_1 = 19$ terms

$65 - 8 = 57$

$19d = 57$

$d = 3$

so $t_{21} = 65 + 3$

$= 68$

$S_{21} = \frac{21}{2}(8+65)$

$= \frac{1533}{2} = 766.5$

$S_{21} = \frac{21}{2}(8+68) = 798$

35. $S_{56}, if\ a_{56} = 13, d = -9$

a_{56}

$a + \_\_ + \ldots + 22 + 13$

$t_n = a + (n-1)d$

$13 = a + (56-1)(-9)$

$508 = a$

$S_n = \frac{n}{2}(a+l)$

$S_{56} = \frac{56}{2}(508+13)$

$S_{56} = 14588$

36. $n\ if\ S_n = 180, a_1 = 4, a_n = 16$

$n = 2 \cdot \left(\frac{S_n}{a+l}\right)$

$= 2 \cdot \left(\frac{180}{4+16}\right)$

$= 2 \cdot \left(\frac{180}{20}\right)$

$= 18$

37. $d, if\ S_{40} = 680, a_1 = 11$

$11 + \ldots + \frac{a_{40}}{\_\_}$ $\quad n = 40$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$680 = \frac{40}{2}[2(11) + (40-1)d]$

$12 = 39d$

$d = \frac{12}{39} \Rightarrow d = \frac{4}{13}$

38. $S_{62}, if\ a_1 = 10, d = 3$

$S_{62} = \frac{62}{2}(2(10) + (62-1)3)$

$= 31(20 + 183)$

$= 6293$

39. $S_{19}, if\ d = 4, a_{19} = 17$

$t_n = a + (n-1)d$

$17 = a + (19-1)4$

$a = -55$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{19} = \frac{19}{2}[2(-55) + (19-1)4]$

$S_{19} = -361$

40. $S_{40}, if\ d = -3, a_{40} = 65$

a_{40}

$\_\_ + \ldots + 65$

$t_n = a + (n-1)d$

$65 = a + (40-1)(-3)$

$a = 182$

$S_n = \frac{n}{2}(a+l)$

$= \frac{40}{2}(182+65)$

$= 4940$