Review 2: Algebra, Elimination, and Logic

This booklet belongs to:________________________________**Block:** _______

Section	Due Date	How Did It Go?	Corrections Made and Understood
$R2.1$			
$R2.2$			
$R2.3$			

Self-Assessment Rubric

Category	L-T Score	Learning Target Procedure	Algebraic/Arithmetic Procedure	Communication	Anecdotal Example
Extending	4	Procedural context demonstrates a detailed understanding of the learning targets	Algebraic/Arithmetic process is error free, logic is clear and easy to follow	Written output is clear, easy to follow, and shows depth of understanding	"You could teach this" or "It's an answer key"
	3.5	Procedural context demonstrates a thorough understanding of the learning targets	Algebraic/Arithmetic process contains very minor errors, logic is clear and easy to follow	Written output is clear, easy to follow, and shows depth of understanding	"Almost perfect, one or two little errors"
Proficient	3	Procedural context is clear, demonstrates sound reasoning and thought of the learning targets	Algebraic/Arithmetic process contains minor errors, logic is clear and easy to follow	Written output is clear and organized, and shows depth of understanding	"Good understanding with a few errors"
Developing	2.5	Procedural context is clear, contains errors but demonstrates sound reasoning and thought of the learning targets	Algebraic/Arithmetic process contains errors, logic is clear and easy to follow	Written output is difficult to follow, but shows an understanding of the task	"You know what to do bet not clear how to do it"
	2	Procedural context contains errors. Understanding of the learning targets is developing	Algebraic/Arithmetic process contains numerous errors, difficult to follow	Written output is difficult to follow but shows an understanding of the task	"You are on the right track but key concepts are missing"
Emerging	1	Procedural context is not clear, demonstrates minimal understanding of the learning targets	Algebraic/Arithmetic process contains numerous errors, difficult to follow	Written output is difficult to follow, but shows an understanding of the task	"You have achieved the bare minimum to meet the learning outcome"
Not Yet Meeting Outcomes	IE	Procedural context is not clear, demonstrates minimal understanding of the learning targets	Algebraic/Arithmetic process contains numerous errors, difficult to follow	Written output is difficult to follow or completely absent and lacks clarity	"Learning outcomes are not met at this time"

Learning Targets and Self-Evaluation

$L-T$	Description	Mark	
$R2-1$	• Understanding the balance of an equation, and maintaining equality • 1 and 2 Step elimination and logic (Addition and Multiplication Principle) • Grouping like terms		
$R2-2$	• Eliminating brackets (Distributive principle) • Eliminating fraction and decimals (LCM concepts of fractions) • Discovering equations from word problems (real life situations)		

Comments:

Competency Evaluation

A valuable aspect to the learning process involves self-reflection and efficacy. Research has shown that authentic self-reflection helps improve performance and effort, and can have a direct impact on the growth mindset of the individual. In order to grow and be a life-long learner we need to develop the capacity to monitor, evaluate, and know what and where we need to focus on improvement. Read the following list of Core Competency Outcomes and reflect on your behaviour, attitude, effort, and actions throughout this unit.

- Rank yourself on the left of each column: 4 (Excellent), 3 (Good), 2 (Satisfactory), 1 (Needs Improvement)

		4	3	2	1
Personal Responsibility	• I **listen** during instruction and come ready to ask questions				
	• I am **on time** for class • I am **fully prepared** for the class, with all the required supplies				
	• I am **fully prepared** for Tests				
	• I **follow** instructions keep my Workbook organized and tidy • I am **on task** during work blocks • I **complete** assignments **on time**				
Self-Regulation	• I keep track of my **Learning Targets**				
	• I take **ownership** over my goals, learning, and behaviour • I can **solve problems** myself and know when to ask for help • I can **persevere** in challenging tasks • I **am actively** engaged in lessons and discussions				
	• I only **use my phone** for school tasks				
Classroom Responsibility and Communication	• I am **focused** on the discussion and lessons • I **ask questions** during the lesson and class • I give **my best effor**t and **encourage** others to work well • I am polite and communicate questions and concerns with my peers and teacher in a timely manner • I clean up after myself and leave the classroom tidy when I leave				
Collaborative Actions	• I can **work with others** to achieve a common goal • I make **contributions** to my group • I **am kind** to others, can work collaboratively and **build relationships** with my peers • I **can identify** when others need support and provide it				
Communication Skills	• I present informative **clearly**, in an organized way • I **ask and respond** to simple direct questions • I am an **active listener**, I support and encourage the speaker • I **recognize** that there are different points of view and can disagree respectfully • I do not interrupt or speak over others				
	Overall				
Goal for next Unit – refer to the above criteria. **Please selec**t (underline/highlight) **two areas** you want to focus on					

Review Section 2.1 – One Step Equations

- When we think algebra, what comes to mind?
    - Headaches, moans & groans, anxiety…

- Don't get yourself too riled up. **Algebra** is just the **logical manipulation** of an equation.
- That's where we start. With an equation.

- In order to be considered an equation you need a statement of inequality.

$$Either: = \quad < \quad > \quad \leq \quad \geq$$

- Whenever you have one of these in a statement it makes it an **equation**
- One side maintains equality with the other

In other words:

BALANCE

Whatever we do from this point on in an equation, we have to use logical rules in order to maintain that balance, that equality.

Addition and Subtraction

- It's called the **ADDITION PRINCIPLE (ADDING TO MAKE 0)**

Consider this,

$3 = 3$ we have BALANCE

- So if we **ADD** something to **one side** we have to **add it to both**:

$$3 + 2 = 3 + 2$$

- ❖ We use this concept to help **eliminate information** from one side of an equation
- ❖ This in turns adds it to the other side

Example: $r - 4 = 7$

On the left we have an unknown. We need to get that unknown by itself on one side of the equals sign.

How do we do that?

- Well we have -4, in order to **eliminate it**, we need it to be 0
- So what do we **add** to -4 to make it 0, we need to **add** $+\mathbf{4}$

So,

$$r - 4 + 4 = 7 + 4$$

Add $+4$ to both sides

And now, since $-4 + 4 = 0$, we get $r + 0$ on the left, which is r

So after **the elimination** we get:

$r = 11$ and we have **solved for the unknown**

- Now the previous example saw us **subtracting** from the unknown so we had to **add a positive** to both sides.
- When we **add** with the unknown, we have to **add a negative (subtract)** from both sides.

Example:

$q + 5 = 15$

$q + 5 - 5 = 15 - 5$

$q = 10$

Added a negative to both sides, in other words:

Subtracted

Example:

$r - 4 = 7$	$t + 5 = 2$
$r - 4 + 4 = 7 + 4$	$t + 5 - 5 = 2 - 5$
$r = 11$	$t = -3$

$q - 8 = 10$ $q - 8 + 8 = 10 + 8$ $q = 18$	$x + 4 = -6$ $x + 4 - 4 = -6 - 4$ $x = -10$
$a - 6 = -13$ $a - 6 + 6 = -13 + 6$ $a = -7$	$b + 8 = -2$ $b + 8 - 8 = -2 - 8$ $b = -10$

Multiplication and Division

It's called the **MULTIPLICATION PRINCIPLE (Multiplying to get 1)**

- **Multiplication and Division are inverses** of one another
- Much like **adding a negative** is the same as **subtraction**
- **Multiplying a fraction** is the same as **dividing**

Now for multiplication and division the number we want isn't 0, it's 1

- When we are **multiplying with the variable** we have to **divide** to end up with 1

Example:

$$3x = 12$$

- I don't want $3x$, I want $1x$, so I'll have to **divide by $\mathbf{3}$ (or multiply by $\mathbf{\frac{1}{3}}$**)

$$\frac{3x}{3} = 1x$$

But don't forget the whole **balance thing**. We need to **divide both sides**

$$\frac{3x}{3} = \frac{12}{3}, \qquad 1x = 4 \text{ or } x = 4$$

- So when we **multiply** with the **variable**, we do the **inverse**, **division**
- Then, if we **divide** with the **variable**, we do the **inverse**, **multiplication**

Consider this,

$$\frac{1}{2} * 2 = 1$$

- If you **multiply a fraction** by its **denominator** the cancel one another out, because the top and bottom divide to give you 1

$$\frac{1}{2} * 2 = \frac{1 * 2}{2} = \frac{2}{2} = 1$$

So,

$$\frac{t}{5} = 10$$

- Since we are **dividing** with the **variable**, we have to **multiply**

$$\frac{t}{5} = 10, \qquad 5 * \frac{t}{5} = 10 * 5, \qquad \frac{5t}{5} = 50, \qquad t = 50$$

Multiply both sides by 5 Divide the left out

Examples:

$5x = 10$	$-3r = 27$
$\frac{5x}{5} = \frac{10}{5}$	$\frac{-3r}{-3} = \frac{27}{-3}$
$x = 2$	$r = -9$

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$4a = 3$ $\frac{4a}{4} = \frac{3}{4}$ $a = \frac{3}{4}$	$8n = 2$ $\frac{8n}{8} = \frac{2}{8}$ $n = \frac{1}{4}$
$\frac{q}{5} = 2$ $5 * \frac{q}{5} = 2 * 5$ $q = 10$	$\frac{d}{-4} = -8$ $-4 * \frac{d}{-4} = -8 * -4$ $d = 32$
$\frac{b}{7} = 2$ $7 * \frac{b}{7} = 2 * 7$ $b = 14$	$\frac{v}{4} = -12$ $4 * \frac{v}{4} = -12 * 4$ $v = -48$

- These are all $\mathbf{1 - \mathit{Step}}$ equations
- They take 1 step to get your answer
- Addition, Subtraction, Multiplication, and Division

Next we will see examples that require **2 or more Steps**

Two Steps

What if you have a fraction multiplying with a variable?

- Well we just need $\mathbf{2\ \mathit{Steps}}$

Example:

$$\frac{2}{3}x = 6$$

- **First Multiply** by the **Denominator** on **both** sides

$$3 * \frac{2}{3}x = 6 * 3$$

- The $3's$ cancel on the left

$$2x = 18$$

- **Then Divide** by the **Numerator** on **both** sides

$$\frac{2x}{2} = \frac{18}{2}$$

- The $2's$ cancel out on the left

$$x = 9$$

Example:

$$\frac{4}{5}x = 4 \quad \rightarrow \quad 5 * \frac{4}{5}x = 4 * 5 \quad \rightarrow \quad 4x = 20 \quad \rightarrow \quad \frac{4x}{4} = \frac{20}{4} \quad \rightarrow \quad x = 5$$

- These can be done in 1 step by **multiplying by the reciprocal**.
- It only works in when you have **1 fraction**, a **variable** and the **answer**

$$\frac{2}{3}x = 8 \quad \rightarrow \quad \frac{3}{2} * \frac{2}{3}x = 8 * \frac{3}{2} \quad \rightarrow \quad x = \frac{24}{2} \quad \rightarrow \quad x = 12$$

Those are all TWO STEP fraction examples, next are TWO STEP, multi operation examples.

- When you isolate the variable term first, it makes life much easier.
- Then solve for the variable

Example: What is the unknown value. $3x - 4 = 8$

Solution:

$$3x - 4 = 8$$

$$3x - 4 + 4 = 8 + 4$$ Add 4 to both sides

$$3x = 12$$

$$\frac{3x}{3} = \frac{12}{3} \quad \rightarrow \quad \boldsymbol{x = 4}$$ Divide both sides by 3

Example: Solve for the unknown variable. $-2t - 4 = -19$

Solution:

$$-2t - 4 = -19$$

$$-2t - 4 + 4 = -19 + 4$$ Add 4 to both sides

$$-2t = -15$$

$$\frac{-2t}{2} = \frac{-15}{-2} \quad \rightarrow \quad \boldsymbol{t = \frac{15}{2}}$$ Divide both sides by -2

Review Section 2.1 – Practice Questions

Addition and Subtraction Principle. ISOLATE THE VARIABLE, show steps.

1. $w + 4 = 7$	2. $x + 16 = -4$	3. $t - 12 = -4$
4. $t + 9 = -3$	5. $k - 6 = -8$	6. $w + (-3) = -8$
7. $z - (-2) = 5$	8. $8 - j = 7$	9. $r - (-5) = 12$
10. $12 - 2l = -4$	11. $15 - 3r = 12$	12. $4x + 23 = -7$
13. $-5j + 7 = -4$	14. $23 + 4f = -17$	15. $-6j + (-5) = -11$

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Multiplication and Division Principle. ISOLATE THE VARIABLE, show steps.

16. $3x = 12$	17. $2x = 24$	18. $4t = -13$
19. $-3t = -6$	20. $-4r = 12$	21. $-12m = 156$
22. $3t = 17$	23. $-x = 4$	24. $7h = 2$
25. $\frac{z}{7} = 9$	26. $\frac{k}{6} = -2$	27. $\frac{t}{8} = 4$
28. $\frac{r}{3} = -3$	29. $\frac{j}{-4} = -6$	30. $\frac{r}{6} = 35$
31. $\frac{t}{-2} = 5$	32. $\frac{a}{7} = 0$	33. $\frac{-w}{7} = -4$
34. $\frac{2}{3}x = 8$	35. $\frac{-2}{5}x = 4$	36. $\frac{2}{r} = 4$
37. $\frac{3}{-7}x = 5$	38. $-\frac{6}{7}x = 2$	39. $\frac{2}{5}t = 13$

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Review Section 2.2 – Eliminating Brackets and Fractions

Eliminating Brackets

- In math we have a term called **Distributivity**

Example: $a(b + c) = ab + ac$

- $a\ times\ (b + c) = a\ times\ b\ plus\ a\ times\ c$

- $the\ a\ multiplies\ with\ both\ terms\ inside\ the\ brackets$

- This is **DISTRIBUTIVITY**

- I use the term **WATERBOMB**

$$a(b + c) = ab + ac$$

Example:

$2(r + 6) = 2$

$2(r + 6) = 2$ Waterbomb

$2r + 12 = 2$

$2r + 12 - 12 = 2 - 12$ Subtract 12 from both sides

$2r = -10$

$\frac{2r}{2} = \frac{-10}{2}$ Divide both sides by 2

$$r = -5$$

- Whenever there are **Brackets**, you **multiply in to them**

- **DISTRIBUTE**, **WATERBOMB**, whichever term you prefer

Example: $4(s+4) = 28$

$4s + 16 = 28$
- Multiply in the 4 to both terms in the brackets

$4s + 16 - 16 = 28 - 16$
- Subtract 16 from both sides of the equation

$4s = 12$

$\frac{4s}{4} = \frac{12}{4}$
- Dive both sides by 4 to isolate the variable

$s = 3$

➢ Even if is just a negative symbol $-$, this means -1

Example: $-(r-5) = 10$

$-r + 5 = 10$
- Multiply the -1 into the terms in the brackets

$-r + 5 - 5 = 10 - 5$
- Subtract 5 from both sides of the equation

$-r = 5$
- We don't want $-r$, we want r

$(-1)(-r) = 5(-1)$
- Multiply both sides by -1

$r = -5$

- Every step is logical and maintains the balance
- Next we will look at dealing with Multiple Fractions

Eliminating Multiple Fractions

- ➢ How do you get rid of 1 denominator?

$\frac{t}{2} = 4$

- We **multiply** that **fraction** by the **denominator** or any **MULTIPLE** of it

❖ You see we **multiply by a multiple** because it gives us a **WHOLE NUMBER RESULT**, **ELIMINATING** the **FRACTION**

Watch:

Multiple of 2

$4 * \frac{1}{2} = 2$

Whole Number Result

Multiple of 2

$8 * \frac{1}{2} = 4$

Whole Number Result

- So what if we have multiple denominators?

$$\frac{1}{2}x + \frac{2}{3} = \frac{1}{4}$$

- To **cancel out** the $2, 3, and\ 4$ respectively I would need to multiply everything by $2\ and\ 3\ and\ 4$.

- But to get rid of **all of them at once**, I need to multiply them all by their **Lowest Common Multiple!**

- Using the **LCM** will give me a **WHOLE NUMBER** result for **every fraction**

Example:

$$\frac{1}{2}x - \frac{2}{3} = \frac{1}{4}$$

- 1st what is the LCM of $2, 3, and\ 4$? $It's\ 12!$
- Then **multiply every term** by it

$$\mathbf{12} * \frac{1}{2}x - \frac{2}{3} * \mathbf{12} = \frac{1}{4} * \mathbf{12}$$

- Multiply every fraction by the LCM

$$\frac{12}{2}x - \frac{24}{3} = \frac{12}{4}$$

- Remember the multiplication is with the numerator only

$$6x - 8 = 3$$

- Simplify the fractions

$$6x - 8 + 8 = 3 + 8$$

- Add 8 to both sides of the equation

$$6x = 11$$

$$\frac{6x}{6} = \frac{11}{6}$$

- Divide both sides by 6

$$x = \frac{11}{6}$$

- You **MUST** multiply every term to **KEEP THAT BALANCE!!!**

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Review Section 2.2 – Practice Questions

- Eliminate the **Brackets (WATERBOMB)** – Distributive Property
- Then solve for the unknown – these are MULTI-STEP Equations

1. $2(x+4)=8$	2. $-3(s-7)=-5$
3. $4(t+2)=2(t-3)$	4. $-5(6-z)=3(z+4)$
5. $-2(4t+54)=3(-t+5)$	6. $3(3q-4)=2(4q+5)$
7. $3(4r-3)=5(-2r+6)+2$	8. $8(3t-12)=12t$

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Eliminate the fractions, using LCM, then solve for the unknown.

9. $\frac{t}{6} + \frac{1}{3} = \frac{1}{2}$

10. $\frac{7}{8}x - \frac{1}{16} = \frac{1}{4} + x$

11. $\frac{2}{3}x - \frac{1}{4}x = \frac{1}{2}x + 1$

12. $\frac{7}{2}q - 3q = +\frac{3}{2} + \frac{5}{2}q$

13. $1 + \frac{y}{5} = \frac{2}{3}y + \frac{12}{5}$

14. $\frac{4}{5}x - \frac{1}{2}x = \frac{3}{10}x + 4$

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Review Section 2.3 – Word Problems

- The hard part is **converting the words** to **variables and numbers**
- Start by **identifying your unknowns** and any other unknown with respect to your first one (you'll see in an example)

Example:

A number multiplied by 2 and then adding 7 results in 23, what is the number?

- How do I write this?
- Follow along

Let n be our number	Multiplied by two means	Adding 7	Result is 23
n	$2n$	$2n+7$	$=23$

So,

$$2n+7=23$$

$$2n+7-7=23-7$$

$$2n=16$$

$$\frac{2n}{2}=\frac{16}{2}$$

$$n=8$$

Example:

The **sum** of 3 consecutive numbers is 48. What are the numbers?

- Consecutive means one right after another so, if my first number is n:

I have: n $\quad n+1$ $\quad n+2$

Sum means addition:

$$n+n+1+n+2=48 \quad \rightarrow \quad 3n+3=48$$

$$3n=45$$

$$\frac{3n}{3}=\frac{45}{3} \quad \rightarrow \quad n=15$$

The Numbers are:

$15, 16, and\ 17$

<u>**Review Section 2.3 – Practice Questions**</u>

Word Problems: Situations that you may come across that you don't even realize involve math while you figure them out!

1. You open a book and the sum (addition) of the two pages is 111. What are the two pages?

2. In Victoria, a taxi charges $3.00 and then $0.60 for every kilometer you travel. How far can you go for $19.20?

3. The second angle of a triangle is four times as large as the first angle. The third angle is 30 degrees bigger than the first. What are the measurements of the three angles? (Angles in a triangle add to 180 degree)

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4. If I have $2.05 and there are 3 dimes in the pile, how many quarters do I have?

5. If you double a number and add 36, you get five times the original number. What is the original number?

6. Three consecutive numbers such that first number plus twice the second, plus five less than the third is 27. What are the three numbers?

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Extra Work Space

Answer Key

Section R-2.1

1. $w = 3$
2. $x = -20$
3. $t = 8$
4. $t = -12$
5. $k = -2$
6. $w = -5$
7. $z = 3$
8. $j = 1$
9. $r = 7$
10. $l = 8$
11. $r = 1$
12. $x = -\frac{15}{2}$
13. $j = \frac{11}{5}$
14. $f = -10$
15. $j = 1$
16. $x = 4$
17. $x = 12$
18. $t = -\frac{13}{4}$
19. $t = 2$
20. $r = -3$
21. $m = -13$
22. $t = \frac{17}{3}$
23. $x = -4$
24. $h = \frac{2}{7}$
25. $z = 63$
26. $k = -12$
27. $t = 32$
28. $r = -9$
29. $j = 24$
30. $r = 210$
31. $t = -10$
32. $a = 0$
33. $w = 28$
34. $x = 12$
35. $x = -10$
36. $r = \frac{1}{2}$
37. $x = -\frac{35}{3}$
38. $x = -\frac{7}{3}$
39. $t = \frac{65}{2}$

Section R-2.2

1. $x = 0$
2. $s = \frac{26}{3}$
3. $t = -7$
4. $z = 21$
5. $t = -\frac{123}{5}$
6. $q = 22$
7. $r = \frac{41}{22}$
8. $t = 8$
9. $t = 1$
10. $x = -\frac{5}{2}$
11. $x = -12$
12. $q = -\frac{3}{4}$
13. $y = -3$
14. $No\ Solution$

Section R-2.3

1. $55\ and\ 56$
2. $27km$
3. $25°, 100°, 55°$
4. $7\ quarters$
5. $n = 12$
6. $7, 8, 9$