Final Exam Review Pack – Section 3

Standard Form and General Form of a Linear Equation

- If $A, B,$ and $C$ are real numbers, the equation $Ax + By = C$ is called the **Standard Form** of the equation of a line.
- It is best to write the equation with $A, B,$ and $C$ as integers, and $A \geq 0$ (Not Negative).
- Standard Form was introduced in Grade 9 and won't be used much longer
- The **General Form**: $Ax + By + C = 0$ is a more appropriate form since moving forward we will want our Linear Equations equal to 0
- So the transformation from Standard to General Form is a simple one:
  
  - Get ALL THE TERMS on the same side of the equal sign

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>General Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Ax + By = C$</td>
<td>$Ax + By + C = 0$</td>
</tr>
<tr>
<td>$A, B, C$ are Integers, $A &gt; 0$</td>
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</tbody>
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Slope-Intercept Form of a Linear Equation

- The equation $y = mx + b$ is the **Slope-Intercept Form** of the equation of a line.
- The $y$ – **intercept** of the line is $(0, b)$, and the **slope** of the line is $m$.
- The algebra of STANDARD FORM to SLOPE-INTERCEPT FORM is as follows:

$$
Ax + By = C \quad \rightarrow \quad By = -Ax + C \quad \rightarrow \quad y = -\frac{A}{B}x + \frac{C}{B}
$$

- The **slope** of $Ax + By = C$ is $-\frac{A}{B}$
- The $y$ – **intercept** of $Ax + By = C$ is $\frac{C}{B} \rightarrow (0, \frac{C}{B})$

**SLOPE-INTERCEPT FORM**

$$
y = mx + b
$$
Point-Slope Form of a Linear Equation

- The equation \( y - y_1 = m(x - x_1) \) is the POINT-SLOPE EQUATION of a line.
- The given point is \((x_1, y_1)\) and the slope of the line is \(m\).
- This formula comes from rearranging the definition of the slope, \( m = \frac{y - y_1}{x - x_1} \)

**POINT-SLOPE EQUATION**

\[
y - y_1 = m(x - x_1)
\]

**Example:**

Write the equation of a line with slope 2 that passes through \((-4, 1)\) in Slope-intercept form.

**Solution:**

\[
y - y_1 = m(x - x_1) \quad \rightarrow \quad y - 1 = 2(x - (-4))
\]

\[
y - 1 = 2(x + 4)
\]

\[
y - 1 = 2x + 8
\]

\[
y = 2x + 9
\]

**Example:**

Write the equation of a line with slope \(\frac{4}{5}\) that passes through \((3, -2)\) in Standard form. **Solution:**

\[
y - y_1 = m(x - x_1) \quad \rightarrow \quad y - (-2) = \frac{4}{5}(x - 3)
\]

\[
y + 2 = \frac{4}{5}(x - 3)
\]

\[
5(y + 2) = 4(x - 3)
\]

\[
5y + 10 = 4x - 12
\]

\[
4x - 5y = 22
\]
Rewrite the **Standard Form Equation in Slope – Intercept Form**

1. \(3x + y = -6\)
   \(y = -3x - 6\)

2. \(3x + y = -4\)
   \(y = -3x - 4\)

3. \(4x - 3y = 12\)
   \(y = \frac{4}{3}x - 4\)

4. \(-2x - 3y = 6\)
   \(y = -\frac{2}{3}x - 2\)

5. \(5x + 4y = -3\)
   \(y = -\frac{5}{4}x - \frac{3}{4}\)

6. \(6x - 3y = -4\)
   \(y = 2x + \frac{4}{3}\)

**Rewrite the Slope – Intercept Equation in Standard Form** \(Ax + By = C\) \(A > 0\) \(B, C \leq 0\) Integers

7. \(y = 2x - 7\)
   \(2x - y = 7\)

8. \(y = -3x + 5\)
   \(3x + 4y = 5\)

9. \(y = -3x\)
   \(3x + y = 0\)

10. \(y = -\frac{2}{3}x - 6\)
    \(2x + 3y = -18\)

11. \(y = -\frac{3}{4}x + 1\)
    \(3x + 4y = 4\)

12. \(y = -\frac{2}{5}x - \frac{1}{2}\)
    \(4x + 10y = -5\)

**Rewrite the Point – Slope Equation in Slope – Intercept Form**

13. \(y + 2 = -3(x + 1)\)
    \(y = -3x - 3 - 2\)
    \(y = -3x - 5\)

14. \(y + 4 = -2(x + 3)\)
    \(y + 4 = -2x - 6\)
    \(y = -2x - 10\)
15. \( y - 1 = -\frac{1}{3} (x + 2) \)
   \[ y - 1 = -\frac{1}{3}x - \frac{2}{3} \]
   \[ y = -\frac{1}{3}x + \frac{1}{3} \]

16. \( y - 4 = -\frac{2}{5} (x - 3) \)
   \[ y - 4 = -\frac{2}{5}x + \frac{6}{5} \]
   \[ y = -\frac{2}{5}x + \frac{26}{5} \]

Rewrite the Point – Slope Equation in Standard Form

17. \( y + 2 = -3(x + 1) \)
   \[ y + 2 = -3x - 3 \]
   \[ -y + 3 \]
   \[ 5 = -3x - y \]
   \[ 3x + y = -5 \]

18. \( y + 4 = 2(x + 5) \)
   \[ y + 4 = 2x + 10 \]
   \[ 2x - y = -6 \]

19. \( y + 1 = -\frac{1}{3} (x + 2) \)
   \[ y + 1 = -\frac{1}{3}x - \frac{2}{3} \]
   \[ 3y + 3 = -x - 2 \]
   \[ x + 3y = -5 \]

20. \( y - 4 = \frac{2}{5} (x + 3) \)
   \[ y - 4 = \frac{2}{5}x + \frac{6}{5} \]
   \[ 5y - 20 = 2x + 6 \]
   \[ 2x - 5y = -26 \]

Write the equation of each line in Slope – Intercept form

21. \((0, -3); m = 2\)
   \[ y = 2x - 3 \]

22. \((0, 5); m = \frac{1}{2}\)
   \[ y = \frac{1}{2}x + 5 \]

23. \((0, 1); m = 0\)
   \[ y = 1 \]

24. \((0, 7); m = -\frac{2}{3}\)
   \[ y = -\frac{2}{3}x + 7 \]
Graph the linear Equation

25. $-4x - 3y = 12$

- $x = 0$
- $y = -4$
- $x = -3$
- $y = 0$
- $x = 3$
- $y = -8$

26. $y = -\frac{2}{3}x - 4$

27. $y + 3 = -\frac{1}{2}(x - 4)$

- Point $A(4, -3)$
- Slope $-\frac{1}{2}$

28. $3x - 2y = 10$

- $x = 0$
- $y = -5$
- Find slope
- $3x - 2y = 10$
- $-2y = -3x + 10$
- $y = \frac{3}{2}y - 5$
29. \( y - 2 = \frac{2}{3}(x - 7) \)

30. \( 5x + 2y = 0 \)

\[ x = 0 \quad y > 0 \]

\[ 2y = -5x \]
\[ y = -\frac{5}{2}x \]

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For each pair of equations, determine whether they are parallel, perpendicular, or neither

31. \( 2x - 5y = 7 \) and \( 4x - 10y = 2 \)

\[ -5y = -2x + 7 \]
\[ y = \frac{2}{5}x - \frac{7}{5} \]

\[ -10y = -4x + 2 \]

Parallel

32. \( 4x + 3y = 7 \) and \( 8x + 6y = 0 \)

\[ 3y = -4x + 7 \]
\[ y = \frac{4}{3}x + \frac{7}{3} \]

\[ 6y = -8x \]
\[ y = \frac{-4}{3}x \]

Parallel

33. \( 4x - 3y = -6 \) and \( 4x + 6y = 3 \)

\[ -3y = -4x - 6 \]
\[ y = \frac{4}{3}x + 2 \]

NEITHER

34. \( 3x + 5y = 4 \) and \( 5x + 3y = 4 \)

\[ 5y = -3x + 4 \]
\[ y = \frac{-3}{5}x + \frac{4}{5} \]

\[ 3y = -5x + 4 \]
\[ y = \frac{-5}{3}x + \frac{4}{3} \]

NEITHER
Write the equation of a line passing through the given set of points in Slope – Intercept form

35. \((-3, 5)\) and \((2, -4)\)  
   \[ \text{Slope: } -\frac{4-5}{2-(-3)} = -\frac{9}{5} \]
   \[ y - 5 = -\frac{9}{5}(x + 3) \]
   \[ y = -\frac{9}{5}x - \frac{27}{5} \]
   \[ y = -\frac{9}{5}x - \frac{27}{5} \]

36. \((5, 2)\) and \((-3, 1)\)  
   \[ \text{Slope: } -\frac{2-1}{5-(-3)} = -\frac{1}{8} \]
   \[ y - 1 = -\frac{1}{8}(x + 3) \]
   \[ y = -\frac{1}{8}x + \frac{1}{8} \]

37. \((4, 1)\) and \((-2, 3)\)  
   \[ \text{Slope: } \frac{3-1}{-2-4} = \frac{2}{-6} = -\frac{1}{3} \]
   \[ y - 1 = -\frac{1}{3}(x - 1) \]
   \[ y = -\frac{1}{3}x + \frac{1}{3} \]

38. \((-1, 2)\) and \((6, -4)\)  
   \[ \text{Slope: } -\frac{4-2}{6-(-1)} = -\frac{6}{7} \]
   \[ y - 2 = -\frac{6}{7}(x + 1) \]
   \[ y = -\frac{6}{7}x - \frac{6}{7} + 2 \]
   \[ y = -\frac{6}{7}x + \frac{8}{7} \]

Find the equation of the line in General Form, that passes through the given point and is:

Perpendicular to the given line.

39. \(P(0, 0); y = -2x - 5\)  
   \[ \text{Perp Slope: } -\frac{1}{2} \]
   \[ y - 0 = -\frac{1}{2}(x - 0) \]
   \[ y = -\frac{1}{2}x \]

40. \(P(0, 0); 2x = 2y - 5\)  
   \[ 2y = 2x + 5 \]
   \[ y = x + \frac{5}{2} \]
   \[ \text{Perp Slope: } 1 \]

Parallel to the given line.

41. \(P(-1, 3); 3x - y = 6\)  
   \[ y = -3x + 6 \]
   \[ y - 3 = -3(x + 1) \]
   \[ y = -\frac{3}{3}x - \frac{1}{3} \]
   \[ y = -\frac{1}{3}x + \frac{8}{3} \]

42. \(P(2, 0); 2x + 5y = 3\)  
   \[ 5y = -2x + 3 \]
   \[ y = -\frac{2}{5}x + \frac{3}{5} \]
   \[ y - 0 = -\frac{2}{5}(x - 2) \]
   \[ y = -\frac{2}{5}x + \frac{14}{5} \]
   \[ \text{Parallel Slope: } -\frac{2}{5} \]
43. An insurance company purchased computers for its office. The value of the computers after three years was $80,000, and $42,000 after five years. Determine the purchase price of the computers.

\[
\begin{align*}
(3, 80,000) & \quad (5, 42,000) \\
\text{Slope: } & = \frac{42,000 - 80,000}{5 - 3} = \frac{-38,000}{2} = -19,000 \\
& \quad y - 80,000 = -19,000(x - 3) \\
& \quad y = -19,000x + 57,000 + 80,000 \\
& \quad y = -19,000x + 137,000 \\
\end{align*}
\]

44. In her first year of practice, a psychologist has 210 patients. By the third year, the number of patients grew to 296. If this trend continues, how many patients will she have in the sixth year?

\[
\begin{align*}
(1, 210) & \quad (3, 296) \\
\text{Slope: } & = \frac{296 - 210}{3 - 1} = \frac{86}{2} = 43 \text{ patients/yr} \\
& \quad y - 210 = 43(x - 1) \\
& \quad y = 43x - 43 + 210 \\
& \quad y = 43x + 167 \\
& \quad y = 425 \\
\end{align*}
\]

45. A printer costs $1020 new and is expected to be worth $200 after six years. What will it be worth after three years?

\[
\begin{align*}
(0, 1020) & \quad (6, 200) \\
\text{Slope: } & = \frac{200 - 1020}{6 - 0} = \frac{-820}{6} = -\frac{410}{3} \\
& \quad y - 1020 = -\frac{410}{3}(x - 0) \\
& \quad y = -\frac{410}{3}x + 1020 \\
\text{when } x = 3 \\
& \quad y = -\frac{410}{3}(3) + 1020 = -410 + 1020 = 610 \\
\end{align*}
\]