Exercise 1 and 2 Review – Practice Problems

Exercise 1

1. Find the Domain of the following functions

a) $f(x) = 1 - 18x$

No restrictions

$x \in \mathbb{R}$ $D = \{x \mid x \in \mathbb{R}\}$

b) $g(x) = x^4 - x^2 + 15x$

No restrictions

$x \in \mathbb{R}$ $D = \{x \mid x \in \mathbb{R}\}$

c) $h(x) = \sqrt{x - 5}$

$x - 5 \geq 0$

$x \geq 5$

$D = \{x \mid x \in \mathbb{R}, x \geq 5\}$

d) $F(x) = \sqrt[4]{-x}$ ← needs to be positive

so $-x \geq 0$

$x \leq 0$

$D = \{x \mid x \in \mathbb{R}, x \leq 0\}$

e) $G(x) = \sqrt{1 - x^2}$

$1 - x^2 \geq 0$

$1 \geq x^2$

$D = \{x \mid x \in \mathbb{R}, -1 \leq x \leq 1\}$

x ✓ x

-1 1

f) $H(x) = \sqrt{x^2 - 2}$

$x^2 - 2 \geq 0$

$x^2 \geq 2$

$|x| \geq \sqrt{2}$

$D = \{x \mid x \in \mathbb{R}, |x| \geq \sqrt{2}\}$

✓ x ✓

$-\sqrt{2}$ $\sqrt{2}$

g) $y = \dfrac{3 + x}{3 - x}$ ← can't be 0

$x \neq 3$

$D = \{x \mid x \in \mathbb{R}, x \neq 3\}$

h) $y = \dfrac{x^2}{x^2 + 4x - 5}$ $(x+5)(x-1)$

$D = \{x \mid x \in \mathbb{R}, x \neq -5, x \neq 1\}$

i) $y = \dfrac{1}{\sqrt{t^2 + 5}}$ ← $t^2 + 5 > 0$

can't be zero either

$t^2 > -5$

$t > \sqrt{-5}$ ← not a number.

NO Restrictions $D = \{t \mid t \in \mathbb{R}\}$

j) $y = \dfrac{t}{\sqrt{t^2 - 5t + 6}}$ $(t-2)(t-3) > 0$

✓ x ✓

2 3

$D = \{t \mid t \in \mathbb{R}, t < 2, t > 3\}$

k) $f(x) = \sqrt{x} + \sqrt{4 - x}$

$x \geq 0$ $4 - x \geq 0$

$4 \geq x$

combine these

$D = \{x \mid x \in \mathbb{R}, 0 \leq x \leq 4\}$

l) $f(x) = \sqrt{2 - \sqrt{4 - x}}$

$4 - x \geq 0$

$x \leq 4$

$\sqrt{4-x} \leq 2$

$4 - x \leq 4$

$x \geq 0$

$D = \{x \mid x \in \mathbb{R}, 0 \leq x \leq 4\}$

Exercise 2

1. Find: $f \circ g, g \circ f, f \circ f$, and $g \circ g$

a) $f(x) = 2x - 1; g(x) = 4 - 3x$

$f \circ g(x)$
$2(4-3x)-1$
$8-6x-1$
$\boxed{-6x+7}$

$g \circ f(x)$
$4-3(2x-1)$
$4-6x+3$
$\boxed{-6x+7}$

$f \circ f(x)$
$2(2x-1)-1$
$4x-2-1$
$\boxed{4x-3}$

$g \circ g(x)$
$4-3(4-3x)$
$4-12+9x$
$\boxed{9x-8}$

b) $f(x) = x^2; g(x) = x + 1$

$f \circ g(x)$
$(x+1)^2$
$\boxed{x^2+2x+1}$

$g \circ f(x)$
$\boxed{x^2+1}$

$f \circ f(x)$
$(x^2)^2$
$\boxed{x^4}$

$g \circ g(x)$
$x+1+1$
$\boxed{x+2}$

c) $f(x) = 1 - x^2; g(x) = 5$

$f \circ g(x)$
$1-5^2$
$\boxed{-24}$

$g \circ f(x)$
$\boxed{5}$

$f \circ f(x)$
$1-(1-x^2)^2$
$1-[1-2x^2+x^4]$
$\boxed{-x^4+2x^2}$

$g \circ g(x)$
$\boxed{5}$

d) $f(x) = \sqrt{x}; g(x) = x^2 - 4$

$f \circ g(x)$
$\boxed{\sqrt{x^2-4}}$

$g \circ f(x)$
$\sqrt{x}^2-4$
$\boxed{x-4}$

$f \circ f(x)$
$\sqrt{\sqrt{x}}$
$\boxed{\sqrt[4]{x}}$

$g \circ g(x)$
$(x^2-4)^2-4$
x^4-8x^2+16-4
$\boxed{x^4-8x^2+12}$

e) $f(x) = 3x - 5; g(x) = \frac{1}{x}$

$f \circ g(x)$: $3\left(\frac{1}{x}\right) - 5$, $\boxed{\frac{3}{x} - 5}$

$g \circ f(x)$: $\boxed{\frac{1}{3x+5}}$

$f \circ f(x)$: $3(3x-5)-5$, $9x-15-5$, $\boxed{9x-20}$

$g \circ g(x)$: $\frac{1}{\frac{1}{x}}$, $\boxed{x}$

f) $f(x) = \frac{1}{1-x}; g(x) = \frac{x-2}{x+2}$

$f \circ g(x)$: $\frac{1}{1-\frac{(x-2)}{(x+2)}}$, $\frac{1}{\frac{x+2-(x-2)}{x+2}}$, $\frac{1}{\frac{4}{x+2}}$, $\boxed{\frac{x+2}{4}}$

$g \circ f(x)$: $\frac{\frac{1}{1-x} - 2}{\frac{1}{1-x} + 2}$, $\frac{\frac{1-2(1-x)}{1-x}}{\frac{1+2(1-x)}{1-x}}$, $\frac{\frac{2x-1}{1-x}}{\frac{-2x+3}{1-x}}$, $\boxed{\frac{2x-1}{3-2x}}$

$f \circ f(x)$: $\frac{1}{1-\frac{1}{1-x}}$, $\frac{1}{\frac{1-x-1}{1-x}}$, $\frac{1}{\frac{-x}{1-x}}$, $\frac{1-x}{-x}$, $\boxed{\frac{x-1}{x}}$

$g \circ g(x)$: $\frac{\frac{x-2}{x+2} - 2}{\frac{x-2}{x+2} + 2}$, $\frac{\frac{x-2-2(x+2)}{x+2}}{\frac{x-2+2(x+2)}{x+2}}$, $\frac{\frac{-x-6}{x+2}}{\frac{3x+2}{x+2}}$, $\boxed{\frac{-x-6}{3x+2}}$

g) $f(x) = \sqrt{x}; g(x) = \sqrt{1+x}$

$f \circ g(x)$: $\sqrt{\sqrt{1+x}}$, $\boxed{(1+x)^{\frac{1}{4}}}$

$g \circ f(x)$: $\boxed{\sqrt{1+\sqrt{x}}}$

$f \circ f(x)$: $\sqrt{\sqrt{x}}$, $\boxed{\sqrt[4]{x}}$

$g \circ g(x)$: $\boxed{\sqrt{1+\sqrt{1+x}}}$

2. Find functions f and g such that $h(x) = f(g(x))$

a) $h(x) = (2x+1)^9$

$g(x) = (2x+1)$

$f(x) = x^9$

b) $h(x) = 1 + 2x^2 + 3x^4$

$g(x) = x^2$

$f(x) = 1 + 2x + 3x^2$

c) $h(x) = \frac{1}{x^2 - 7}$

$g(x) = x^2 - 7$

$f(x) = \frac{1}{x}$

d) $h(x) = \sqrt{6+x}$

$g(x) = 6 + x$

$f(x) = \sqrt{x}$