2.8 Higher Derivatives

Since the derivative of a function f is itself a function f', we can therefore take the derivative of this function as well (f')'. The result is a new function called the **second derivative** of f and is denoted f''("f double-prime").

If y = f(x) and using Leibniz notation, then

$$f'' = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

Which is abbreviated as

$$f''(x) = \frac{d^2y}{dx^2}$$

Using the D-notation, the symbol D^2 indicates the differentiation operation is to be performed twice. In this way, we have the following notations for the second derivative:

$$y'' = f''(x) = \frac{d^2y}{dx^2} = D^2f(x) = D_x^2f(x)$$

 $\frac{\mathbf{Ex. 1}}{\mathbf{Find}} \frac{d^2y}{dx^2} \text{ if } y = x^6.$

Find the second derivative of $f(x) = 5x^2 + \sqrt{x}$.

$$f(x) = 10x + \frac{1}{2}x^{-\frac{1}{2}}$$
 $f''(x) = 10 - \frac{1}{4}x^{-\frac{3}{2}} = 10 - \frac{1}{4x^{\frac{3}{2}}}$

$$\frac{Ex.3}{Find f''(1) if f(x)} = (2-x^2)^{10}.$$
 Product Rule and Chain Rule
$$f'(x) = 10(2-x^2)^{9} \cdot (-2x)$$

$$f'(x) = -20x \cdot 9(2-x^2)^{8}(-2x) + -20(2-x^2)^{9}$$

$$= -20x(2-x^2)^{9}$$

$$= 40x^2 \cdot 9(2-x^2)^{8} + -20(2-x^2)^{9}$$

$$= 40x^2 \cdot 9(2-x^2)^{8} + -20(2-x^2)^{9}$$

$$= 40x^2 \cdot 9(2-x^2)^{8} + -20(2-x^2)^{9} + -20(2-x^2)^{9}$$

What is the meaning of the second derivative? Since the first derivative is the slope of the tangent line and/or the rate of change of a function, the second derivative is a rate of change of the tangent line slope. This concept will be explored later.

Calculus 12

Higher derivatives can also be computed. The third derivative is the derivative of the second derivative: f''' = (f'')'. Other notations are as follows:

$$y''' = f'''(x) = \frac{d^3x}{dx^3} = D^3f(x) = D_x^3f(x)$$

For derivatives beyond the third derivative the prime notation becomes cumbersome and instead, the fourth derivative for instance is denoted $f^{(4)}$ instead of f''''. In general, the **nth derivative** of f is denoted by $f^{(n)}$ and is obtained by differentiating n times. Written in general terms

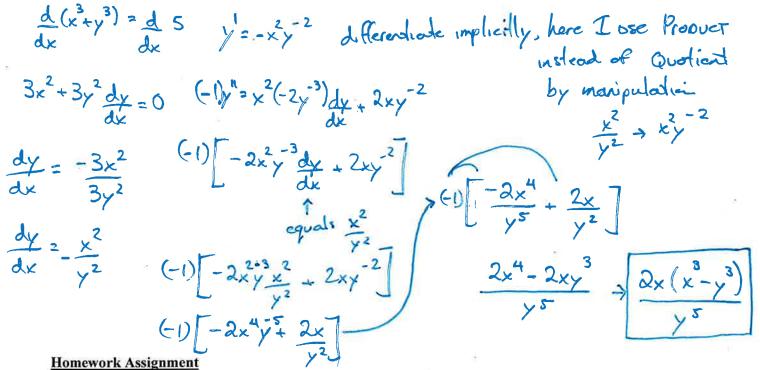
$$y^{(n)} = f^{(n)}(x) = \frac{d^n y}{dx^n} = D^n f(x) = D_x^n f(x)$$

<u>Ex. 4</u>

Find the first five derivatives of $y = x^4 + 2x^3 - 5x^2 + 3x - 6$.

$$y'' = 4x^3 + 6x^2 - 10x + 3$$
 $y''' = 12x^2 + 12x - 10$
 $y'''' = 24x + 12$
 $y''' = 0$

Ex. 5
If $x^3 + y^3 = 5$, use implicit differentiation to find y''.



Exercise 2.8: #1 odd, 2bd, 3-6, 7ac

Review Assignments (Does not need to be included as part of your HW Assignment)

Exercise 2.9: #1 - 3, 4odd, 5 - 14, 15ac, 16, 17