**Exercise 2.3 – Practice Problems**

1. Differentiate the following

a) $f(x) = x^2 + 4x$

$f'(x) = \boxed{2x + 4}$

b) $f(x) = 3x^5 - 6x^4 + 2$

$f'(x) = \boxed{15x^4 - 24x^3}$

c) $g(x) = x^{10} + 25x^5 - 50$

$g'(x) = \boxed{10x^9 + 125x^4}$

d) $g(x) = x^2 - \frac{2}{x^2} \rightarrow x^2 - 2x^{-2}$

$g'(x) = 2x + 4x^{-3}$

$= \boxed{2x + \frac{4}{x^3}}$

e) $h(x) = \sqrt{x} - 5x^4$

$h'(x) = \frac{1}{2}x^{-\frac{1}{2}} - 20x^3$

$= \boxed{\frac{1}{2\sqrt{x}} - 20x^3}$

f) $h(x) = (x - 1)(x + 6)$

$= x^2 + 5x - 6$

$h'(x) = \boxed{2x + 5}$

g) $y = \frac{x+1}{\sqrt{x}} = \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} = \sqrt{x} + x^{-\frac{1}{2}}$

$y' = \boxed{\frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}}$

h) $y = t^5 - 6t^{-5}$

$y' = \boxed{5t^4 + 30t^{-6}}$

i) $f(t) = (1 + t)^3 = (1+t)(1+2t+t^2)$

$= 1 + 2t + t^2 + t + 2t^2 + t^3$

$= t^3 + 3t^2 + 3t + 1$

$f'(t) = \boxed{3t^2 + 6t + 3}$

j) $F(x) = \sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x} = x^{\frac{1}{2}} + x^{\frac{1}{3}} + x^{\frac{1}{4}}$

$F'(x) = \boxed{\frac{1}{2\sqrt{x}} + \frac{1}{3\sqrt[3]{x^2}} + \frac{1}{4\sqrt[4]{x^3}}}$

k) $u(t) = a + \frac{b}{t} + \frac{c}{t^2} \rightarrow a + bt^{-1} + ct^{-2}$

$u'(t) = -bt^{-2} - 2ct^{-3}$

$= \boxed{\frac{-b}{t^2} - \frac{2c}{t^3}}$

l) $v(r) = \sqrt{r}(2 + 3r) = 2\sqrt{r} + 3r^{3/2}$

$v'(r) = \frac{2}{2r^{\frac{1}{2}}} + 3 \cdot \frac{3}{2}r^{\frac{1}{2}}$

$= \boxed{\frac{1}{\sqrt{r}} + \frac{9\sqrt{r}}{2}}$

2. Find $f'(x)$ and state the Domains of $f$ and $f'$

Domain $f$ : $\{x \mid x \in \mathbb{R}\}$

Domain $f'$ : $\{x \mid x \in \mathbb{R}\}$

a) $f(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4$

$f'(x) = \boxed{1 + x + x^2 + x^3}$

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b) $f(x) = 4x - \sqrt[4]{x}$

$= 4x - x^{\frac{1}{4}}$

Domain $f$: $\{x \mid x \in \mathbb{R}, x \geq 0\}$

Domain $f'$: $\{x \mid x \in \mathbb{R}, x > 0\}$

$f'(x) = 4 - \frac{1}{4x^{3/4}}$

$= \boxed{4 - \frac{1}{4\sqrt[4]{x^3}}}$

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c) $f(x) = x + \frac{\sqrt{10}}{x^5}$

$= x + \sqrt{10}x^{-5}$

Domain $f$: $\{x \mid x \in \mathbb{R}, x \neq 0\}$

Domain $f'$: $\{x \mid x \in \mathbb{R}, x \neq 0\}$

$f'(x) = 1 - 5\sqrt{10}x^{-6}$

$= \boxed{1 - \frac{5\sqrt{10}}{x^6}}$

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d) $f(x) = \sqrt{x} + \frac{2}{\sqrt{x}}$

$= \sqrt{x} + 2x^{-\frac{1}{2}}$

Domain $f$: $\{x \mid x \in \mathbb{R}, x > 0\}$

$f'$: $\{x \mid x \in \mathbb{R}, x > 0\}$

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{2}{2x^{3/2}}$

$= \boxed{\frac{1}{2\sqrt{x}} - \frac{1}{\sqrt{x}^3}}$

3. Find the equation of the tangent line to the curve at the given point.

a) $y = x^3 - x^2 + x - 1$; $(1, 0)$

$y' = 3x^2 - 2x + 1$

at $x = 1$

$m = 3(1)^2 - 2(1) + 1$

$= 2$

$y = 2x + b$

$0 = 2(1) + b$

$b = -2$

$\boxed{y = 2x - 2}$

b) $y = 7\sqrt{x} - 3x$; $(1, 4)$

$y' = \frac{7}{2}x^{-\frac{1}{2}} - 3$

$= \frac{7}{2\sqrt{x}} - 3$ at $x = 1$

$\frac{7}{2} - 3$

$= \frac{7}{2} - \frac{6}{2}$

$= \frac{1}{2}$

$y = \frac{1}{2}x + b$

$4 = \frac{1}{2}(1) + b$

$4 = \frac{1}{2} + b$

$\frac{7}{2} = b$

$\boxed{y = \frac{1}{2}x + \frac{7}{2}}$

c) $y = x + \frac{6}{x}$; $(2, 5)$

$y = x + 6x^{-1}$

$y' = 1 - 6x^{-2}$

$= 1 - \frac{6}{x^2}$ at $x = 2$

$y' = 1 - \frac{6}{2^2}$

$1 - 6/4$

$1 - 3/2$

$y' = -\frac{1}{2}$

$y = -\frac{1}{2}x + b$

$5 = -\frac{1}{2}(2) + b$

$5 = -1 + b$

$6 = b$

$\boxed{y = -\frac{1}{2}x + 6}$

d) $(x^2 + 1)^2$; $(-1, 4)$

$(x^2 + 1)(x^2 + 1)$

$x^4 + 2x^2 + 1$

$y' = 4x^3 + 4x$

at $x = -1$

$y' = 4(-1)^3 + 4(-1)$

$= -4 - 4$

$= -8$

$y = -8x + b$

$4 = -8(-1) + b$

$4 = 8 + b$

$-4 = b$

$\boxed{y = -8x - 4}$

4. If a ball is thrown upward with a velocity of 40m/s, its height in metres after $t$ seconds is:

$$h = 40t - 5t^2$$

Find the velocity of the ball after 2s, 4s, and 5s

$v(t) = h'(t)$

$h'(t) = 40 - 10t$

at $t = 2$: $40 - 10(2)$ $\boxed{20\text{ m/s}}$

at $t = 4$: $40 - 10(4)$ $\boxed{0\text{ m/s}}$

at $t = 5$: $40 - 10(5)$ $\boxed{-10\text{ m/s}}$

5. The displacement in metres of a particle moving in a straight line is given by: $s = 8t^2 - 5t + 6$, where $t$ is measured in seconds. Find the velocity of the particle after 1s, 2s, and 5s.

$v(t) = s'(t)$

$s'(t) = 16t - 5$

at 1s $\boxed{11\text{ m/s}}$

2s $\boxed{27\text{ m/s}}$

5s $\boxed{75\text{ m/s}}$

6. At what point on the curve $y = x^4 - 25x + 2$ is the tangent line parallel to the line $7x - y = 2$?

same slope

$y = 7x - 2$

$m = 7$

$y' = 4x^3 - 25$

$4x^3 - 25 = 7$

$4x^3 = 32$

$x^3 = 8$  $x = 2$

when $x = 2$

$y = 2^4 - 25(2) + 2$

$= 16 - 50 + 2$

$= -32$

$\boxed{(2, -32)}$

7. At what point does the curve $y = x^3 + 3x^2 - 24x + 1$ have a horizontal tangent?

occurs at $m = 0$

$y' = 3x^2 + 6x - 24$

$3x^2 + 6x - 24 = 0$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4$  $x = 2$

if $x = -4$

$y = (-4)^3 + 3(-4)^2 - 24(-4) + 1$

$= -64 + 48 + 96 + 1$

31 $= 81$

$\boxed{(-4, 81)}$

$x = 2$

$y = 2^3 + 3(2)^2 - 48 + 1$

$= 8 + 12 - 48 + 1$

$= -27$

$\boxed{(2, -27)}$

8. Show that the curve $y = 10x^3 + 4x + 2$ has no ~~traghent~~ tangent lines with a slope of 3.

$y' = 30x^2 + 4$

$30x^2 + 4 = 3$

$30x^2 = -1$

$x^2 = -\frac{1}{30}$ → $x = \sqrt{\frac{-1}{30}}$

NOT POSSIBLE

9. Find the equation of both lines that pass through the origin and are tangent to the parabola $y = 1 + x^2$

$y' = 2x$ → $2a$ at point a

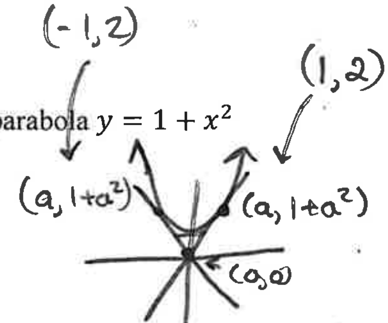
$\frac{1+a^2-0}{a-0} = \frac{1+a^2}{a}$

$\boxed{\begin{matrix} y = 2x \\ y = -2x \end{matrix}}$

$2a = \frac{1+a^2}{a}$

$2a^2 = 1 + a^2$ → $a^2 = 1$

$a = \pm 1$



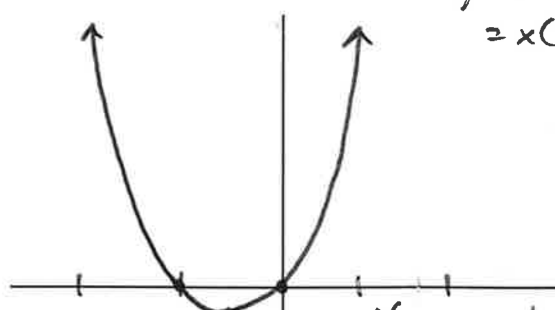

Same slope but negative

10. Find the equations of the tangent lines to the parabola $y = x^2 + x$ that pass through the point $(2, -3)$. Sketch the curve and the tangents.

$y' = 2x + 1$ → $2a+1$

Let $(a, a^2+a)$ be on the parabola

$y = x^2 + x$
$= x(x+1)$



then $m = \frac{a^2 + a - (-3)}{a-2}$

$2a+1 = \frac{a^2+a+3}{a-2}$

$(2a+1)(a-2) = a^2 + a + 3$

$2a^2 - 3a - 2 = a^2 + a + 3$

$a^2 - 4a + 5 = 0$ $(a-5)(a+1) = 0$ $\boxed{a = 5 \quad a = -1}$

if $a = 5$
$y = 30$
$m = 11$

$y = 11x + b$
$30 = 11(5) + b$
$30 = 55 + b$ $b = -25$
$\boxed{y = 11x - 25}$

if $a = -1$
$y = 0$
$m = -1$

$y = -1x + b$
$0 = -1(-1) + b$
$b = -1$

$\boxed{y = -1x - 1}$

11. Find the $x$-coordinates of the points on the hyperbola $xy = 1$ where the tangents from the point $(1, -1)$ intersect the curve.

Let $(a, \frac{1}{a})$ be on the curve.

$xy = 1 \rightarrow y = \frac{1}{x}$

$y' = -\frac{1}{x^2}$

$= -\frac{1}{a^2}$

$\frac{\frac{1}{a} - (-1)}{a-1} = m \rightarrow \frac{\frac{1}{a}+1}{a-1} = -\frac{1}{a^2} \rightarrow a^2\left(\frac{1}{a}+1\right) = -1(a-1)$

$\rightarrow a + a^2 = -a + 1$

$a^2 + 2a - 1 = 0$

Quad Equation or Desmos

$\boxed{x = -1 \pm \sqrt{2}}$

12. Let

$$f(x) = \begin{cases} 2x+3 & \text{if} \quad x < -1 \\ x^2 & \text{if} \quad -1 \le x \le 1 \\ 3-2x & \text{if} \quad x > 1 \end{cases}$$

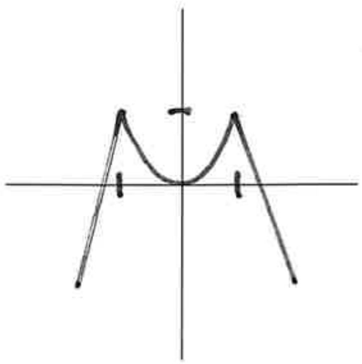
a) Where is $f$ differentiable

consider the graph    NOT Differentiable at $x = -1$ or $x = 1$

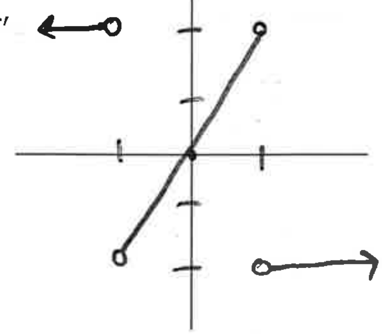
* sharp edges *

b) Find an expression for $f'$ and sketch the graphs of $f$ and $f'$

$f$



$f'$



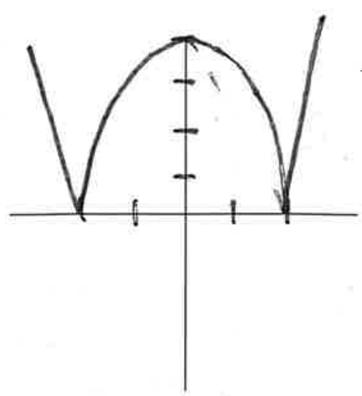
$$f'(x) \begin{cases} 2 & \text{if } x < -1 \\ 2x & \text{if } -1 \le x \le 1 \\ -2 & \text{if } x > 1 \end{cases}$$

13.

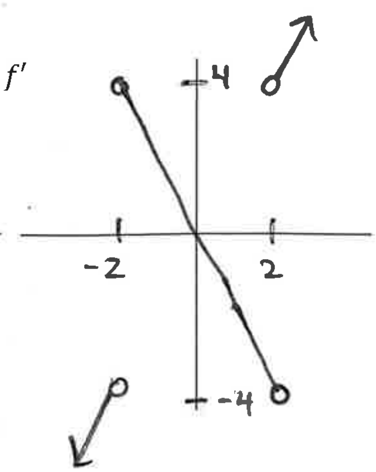
a) Sketch the graph of $f(x) = |x^2 - 4|$.
b) For what values of $x$ is $f$ not differentiable. → not differentiable at $x = 2$ $x = -2$
c) Find a formula for $f'$ and sketch its graph.

$$f(x) \begin{cases} x^2-4 & \text{if } x < -2 \\ -(x^2-4) & \text{if } -2 \le x \le 2 \\ x^2-4 & \text{if } x > 2 \end{cases}$$

$f$



$f'$




$f'(x)$ $2x$ if $x < -2$

$-2x$ if $-2 \le x \le 2$

$2x$ if $x > 2$