

Review and Preview Practice Problems

Exercise 1

1. Factor the following

a) $x^2 - x - 2$

$$(x-2)(x+1)$$

b) $x^2 - 9x + 14$

$$(x-7)(x-2)$$

c) $x^2 + 7x + 12$

$$(x+3)(x+4)$$

d) $2x^2 - x - 1$

$$(2x^2 - 2x)(x-1) \rightarrow (2x+1)(x-1)$$

$$2x(x-1) + 1(x-1)$$

e) $5x^2 + 13x + 6$

$$(5x^2 + 10x)(x+2) \rightarrow (5x+3)(x+2)$$

$$5x(x+2) + 3(x+2)$$

f) $6y^2 - 11y + 3$

$$(6y^2 - 9y)(x-2y+3) \rightarrow (3y-1)(2y-3)$$

$$3y(2y-3) - 1(2y-3)$$

g) $t^3 + 2t^2 - 3t$

$$t(t^2 + 2t - 3)$$

$$t(t+3)(t-1)$$

h) $3x^4 + 7x^3 + 2x^2$

$$x^2(3x^2 + 7x + 2)$$

$$x^2[(3x^2 + 6x)(x+2)] \rightarrow x(3x+1)(x+2)$$

$$x^2[3x(x+2) + 1(x+2)]$$

2. Factor the following

a) $4x^2 - 25$

$$\text{Diff of Squares}$$

$$(2x+5)(2x-5)$$

b) $x^3 - 1$

$$(x-1)(x^2 + x + 1)$$

c) $t^3 + 64$

$$(t+4)(t^2 - 4t + 16)$$

d) $y^3 - 9y$

$$y(y^2 - 9) \rightarrow y(y+3)(y-3)$$

e) $8c^3 - 27d^3$

$$a=2c \quad b=3d$$

$$(2c-3d)(4c^2 + 6cd + 9d^2)$$

f) $x^6 + 8$

$$a=x^2 \quad b=2$$

$$(x^2+2)(x^4 - 2x^2 + 4)$$

g) $x^4 - 16$

$$(x^2-4)(x^2+4)$$

$$(x+2)(x-2)(x^2+4)$$

h) $r^8 - 1$

$$(r^4-1)(r^4+1)$$

$$(r^2+1)(r^2-1)(r^4+1)$$

$$3 (r^2+1)(r^4+1)(r+1)(r-1)$$

3. Factor the following

a) $(x^3 - x^2) - 16x + 16$

$$x^2(x-1) - 16(x-1)$$

$$(x^2 - 16)(x-1)$$

$$(x-4)(x+4)(x-1)$$

b) $x^3 - 7x + 6$ $P(1) = 0$

$$(x-1)(x^2+x-6)$$

$$(x-1)(x+3)(x-2)$$

$$\begin{array}{r} x^2+x-6 \\ x-1 \overline{) x^3+0x^2-7x+6} \\ \underline{-x^3-x^2} \\ x^2-7x+6 \\ \underline{-x^2-x} \\ -6x+6 \\ \underline{-6x+6} \\ 0 \end{array}$$

c) $x^3 + 5x^2 - 2x - 24$ $P(2) = 0$

$$\begin{array}{r} x^2+7x+12 \\ x-2 \overline{) x^3+5x^2-2x-24} \\ \underline{x^3-2x^2} \\ 7x^2-2x \\ \underline{7x^2+14x} \\ 12x-24 \\ \underline{12x-24} \\ 0 \end{array}$$

$(x-2)(x^2+7x+12)$
 \downarrow
 $(x-2)(x+3)(x+4)$

d) $x^3 + 2x^2 - 11x - 12$ $P(3) = 0$

$$(x-3)(x^2+5x+4)$$

$$(x-3)(x+1)(x+4)$$

$$\begin{array}{r} x^2+5x+4 \\ x-3 \overline{) x^3+2x^2-11x-12} \\ \underline{x^3-3x^2} \\ 5x^2-11x \\ \underline{5x^2+15x} \\ 4x-12 \\ \underline{4x-12} \\ 0 \end{array}$$

e) $4x^3 + 12x^2 + 5x - 6$ $P(-2) = 0$

$$\begin{array}{r} 4x^2+4x-3 \\ x+2 \overline{) 4x^3+12x^2+5x-6} \\ \underline{4x^3+8x^2} \\ 4x^2+5x \\ \underline{4x^2+8x} \\ -3x-6 \\ \underline{-3x-6} \\ 0 \end{array}$$

$(x+2)(4x^2+4x-3)$
 $(x+2)[4x^2+6x-2x-3]$
 $(x+2)[2x(2x+3)-1(2x+3)]$
 $(x+2)(2x+3)(2x-1)$

f) $x^4 - 3x^3 - 7x^2 + 27x - 18$ $P(1) = 0$

$$\begin{array}{r} x^3-2x^2-9x+18 \\ x-1 \overline{) x^4-3x^3-7x^2+27x-18} \\ \underline{x^4-x^3} \\ -2x^3-7x^2 \\ \underline{-2x^3+2x^2} \\ -9x^2+27x \\ \underline{-9x^2+9x} \\ -9x+18 \\ \underline{-9x+18} \\ 0 \end{array}$$

$(x-1)(x^3-2x^2-9x+18)$
 $P(2) = 0$
 $x-2 \overline{) x^3-2x^2-9x+18}$
 $\underline{x^3-2x^2} $
 $-9x+18$
 $\underline{-9x+18}$
 0

$(x-1)(x^2-9)(x-2)$
 $(x-1)(x+3)(x-3)(x-2)$

4. Factor the following

a) $x^{\frac{5}{2}} - x^{\frac{1}{2}}$

$$x^{\frac{1}{2}}(x^2 - 1)$$

$$x^{\frac{1}{2}}(x-1)(x+1)$$

b) $x + 5 + 6x^{-1}$

$$x^{-1}(x^2 + 5x + 6)$$

$$x^{-1}(x+2)(x+3)$$

c) $x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 8x^{-\frac{1}{2}}$

$$x^{-\frac{1}{2}}(x^2 + 2x - 8)$$

$$x^{-\frac{1}{2}}(x+4)(x-2)$$

$$x^{-\frac{1}{2}}(x+4)(x-2)$$

d) $2x^{\frac{7}{2}} - 2x^{\frac{1}{2}}$

$$2x^{\frac{1}{2}}(x^3 - 1)$$

$$2x^{\frac{1}{2}}(x-1)(x^2+x+1)$$

$$2x^{\frac{1}{2}}(x-1)(x^2+x+1)$$

e) $1 + 2x^{-1} + x^{-2}$

$$x^{-2}(x^2 + 2x + 1)$$

$$x^{-2}(x+1)(x+1)$$

$$x^{-2}(x+1)^2$$

f) $(x^2 + 1)^{\frac{1}{2}} + 3(x^2 + 1)^{-\frac{1}{2}}$

let $(x^2 + 1) = a$

$$a^{\frac{1}{2}} + 3a^{-\frac{1}{2}}$$

$$a^{-\frac{1}{2}}(a^{\frac{3}{2}} + 3)$$

$$a^{-\frac{1}{2}}(a+3)$$

$$a^{-\frac{1}{2}}(x^2 + 1 + 3)$$

$$(x^2 + 1)^{-\frac{1}{2}}(x^2 + 4)$$

Exercise 2

1. Rationalize the Numerator

a) $\frac{\sqrt{x}-3}{x-9} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3}$

$\frac{x-9}{(x-9)(\sqrt{x}+3)} \rightarrow \frac{\cancel{(x-9)}}{\cancel{(x-9)}(\sqrt{x}+3)}$

$$\boxed{\frac{1}{\sqrt{x}+3}}$$

b) $\frac{\frac{1}{\sqrt{x}}-1}{x-1} \rightarrow \frac{\frac{1-\sqrt{x}}{\sqrt{x}}}{x-1} \rightarrow \frac{1-\sqrt{x}}{\sqrt{x}(x-1)}$

$\frac{1-\sqrt{x}}{\sqrt{x}(x-1)} \cdot \frac{1+\sqrt{x}}{1+\sqrt{x}} \rightarrow \frac{1-x}{\sqrt{x}(x-1)(1+\sqrt{x})}$ ← factor out -1

$\frac{-1\cancel{(x-1)}}{\sqrt{x}\cancel{(x-1)}(1+\sqrt{x})} \rightarrow \boxed{\frac{-1}{\sqrt{x}(1+\sqrt{x})}}$

c) $\frac{x\sqrt{x}-8}{x-4} \cdot \frac{x\sqrt{x}+8}{x\sqrt{x}+8}$

$\frac{x^3-64}{(x-4)(x\sqrt{x}+8)}$ Diff of cubes

$\frac{\cancel{(x-4)}(x^2+4x+16)}{\cancel{(x-4)}(x\sqrt{x}+8)}$

$$\boxed{\frac{x^2+4x+16}{x\sqrt{x}+8}}$$

d) $\frac{\sqrt{2+h}+\sqrt{2-h}}{h} \cdot \frac{\sqrt{2+h}-\sqrt{2-h}}{\sqrt{2+h}-\sqrt{2-h}}$

$\frac{2+h-(2-h)}{h(\sqrt{2+h}-\sqrt{2-h})} \rightarrow \frac{2+h-2+h}{h(\sqrt{2+h}-\sqrt{2-h})}$

$\frac{2h}{2(\sqrt{2+h}-\sqrt{2-h})} \rightarrow \boxed{\frac{h}{\sqrt{2+h}-\sqrt{2-h}}}$

e) $\frac{\sqrt{x^2+3x+4}-x}{\sqrt{x^2+3x+4}+x} \cdot \frac{\sqrt{x^2+3x+4}+x}{\sqrt{x^2+3x+4}+x}$

$\frac{\cancel{\sqrt{x^2+3x+4}}-x}{\sqrt{x^2+3x+4}+x}$

$$\boxed{\frac{3x+4}{\sqrt{x^2+3x+4}+x}}$$

f) $\frac{\sqrt{x^2+x}-\sqrt{x^2-x}}{\sqrt{x^2+x}+\sqrt{x^2-x}} \cdot \frac{\sqrt{x^2+x}+\sqrt{x^2-x}}{\sqrt{x^2+x}+\sqrt{x^2-x}}$

$\frac{x^2+x-(x^2-x)}{\sqrt{x^2+x}+\sqrt{x^2-x}} \rightarrow \frac{\cancel{x^2}+x-\cancel{x^2}+x}{\sqrt{x^2+x}+\sqrt{x^2-x}}$

$$\boxed{\frac{2x}{\sqrt{x^2+x}+\sqrt{x^2-x}}}$$

2. Rationalize the Denominator

$$a) \frac{1}{\sqrt{x+1}-1} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$$

$$\frac{\sqrt{x+1}+1}{x+1-1} \rightarrow \boxed{\frac{\sqrt{x+1}+1}{x}}$$

$$b) \frac{4}{\sqrt{x+2}+\sqrt{x}} \cdot \frac{\sqrt{x+2}-\sqrt{x}}{\sqrt{x+2}-\sqrt{x}}$$

$$\frac{4(\sqrt{x+2}-\sqrt{x})}{x+2-x}$$

$$\frac{4(\sqrt{x+2}-\sqrt{x})}{2}$$

$$\boxed{2[\sqrt{x+2}-\sqrt{x}]}$$

$$c) \frac{x}{\sqrt{x^2+1}+x} \cdot \frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}-x}$$

$$\frac{x(\sqrt{x^2+1}-x)}{x^2+1-x^2}$$

$$\frac{x(\sqrt{x^2+1}-x)}{x^2-x^2+1}$$

$$\boxed{x(\sqrt{x^2+1}-x)}$$

$$d) \frac{x^2}{\sqrt{x+1}-\sqrt{x-1}} \cdot \frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}}$$

$$\frac{x^2(\sqrt{x+1}+\sqrt{x-1})}{x+1-(x-1)}$$

$$\frac{x^2(\sqrt{x+1}+\sqrt{x-1})}{2}$$

$$\boxed{\frac{1}{2}x^2(\sqrt{x+1}+\sqrt{x-1})}$$

