Review and Preview to Chapter 1

Factoring

Ex. 1
Factor $x^2 - 3x - 18$

Straight up factoring basics

$(x-6)(x+3)$

need $a \cdot b = -18$
$a + b = -3$

those are: $-6, +3$

Some special polynomials can be factored using the following formulas.

$a^2 - b^2 = (a-b)(a+b)$	Difference of Squares
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$	Difference of Cubes
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$	Sum of Cubes

Haven't seen these yet

Ex. 2
Factor.

a) $x^3 + 27$

$a = x$ $b = 3$

$(x+3)(x^2 - 3x + 9)$

3^2

b) $2x^4 - 18x^2$

Factor out first

$2x^2(x^2 - 9)$ Diff of Squares

$2x^2(x+3)(x-3)$

The Factor Theorem
A polynomial $P(x)$ has $x - b$ as a factor if and only if $P(b) = 0$.

Ex. 3
Factor $P(x) = 2x^3 - 5x^2 - 4x + 3$

over ← this

$P(0) = 3$

need this to equal 0

Pick potential factors of ratio of potential factors

Factors: $\frac{3}{2} \rightarrow \frac{\pm 1, \pm 3}{\pm 1, \pm 2}$

try integers first

$\pm 1, \pm 3,$

$P(1) = 2(1)^3 - 5(1)^2 - 4(1) + 3$
$= 2 - 5 - 4 + 3$
$= -4$ NOPE!

$P(-1) = 2(-1)^3 - 5(-1)^2 - 4(-1) + 3$
$= -2 - 5 + 4 + 3$
$= 0$ Yup is a factor

so $(x - (-1)) \rightarrow (x+1)$

Long Division of Polynomials

$$
\begin{array}{r}
2x^2 - 7x + 3 \\
(x+1)\overline{)\,2x^3 - 5x^2 - 4x + 3} \\
-\ \underline{2x^3 + 2x^2} \\
-7x^2 - 4x \\
-\ \underline{-7x^2 - 7x} \\
3x + 3 \\
\underline{3x + 3} \\
0
\end{array}
$$

so:

$P(x) = (x+1)(2x^2 - 7x + 3)$

↓ Factor by Grouping

$P(x) = (x+1)[2x^2 - 6x - x + 3]$

$P(x) = (x+1)[2x(x-3) - 1(x-3)]$

$\boxed{P(x) = (x+1)(2x-1)(x-3)}$

Have $\frac{3}{2}$ and want $\frac{4}{2} = 2$

$-\frac{1}{2} + \frac{4}{2} = \frac{3}{2}$ so factor out $x^{-\frac{1}{2}}$

also

$-\frac{1}{2} + \frac{2}{2} = \frac{1}{2}$

↑

1

Ex. 4

Factor $2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} - 6x^{-\frac{1}{2}}$.

$2x^{-\frac{1}{2}}(x^2 + 2x - 3)$

$\boxed{2x^{-\frac{1}{2}}(x+3)(x-1)}$

Rationalizing

To rationalize a numerator or denominator that contains an expression with one or more radicals such as

$$\sqrt{a} - \sqrt{b}$$

we multiply *both* the numerator and denominator by the ***conjugate radical***

conjugate has the opposite sign in a binomial

$$\sqrt{a} + \sqrt{b}$$

Then using the difference of squares, the product results in:

$$(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$$

Ex. 5

Rationalize the numerator in the expression

$$\frac{(\sqrt{x+4} - 2)}{x} \cdot \frac{(\sqrt{x+4} + 2)}{(\sqrt{x+4} + 2)}$$

Remember this equals 1

conjugate is

$\sqrt{x+4} + 2$

↓

don't water bomb yet! →

$$\frac{(x+4) - 4}{x(\sqrt{x+4} + 2)} \rightarrow \frac{x + 4 - 4}{x(\sqrt{x+4} + 2)} \rightarrow \frac{\cancel{x}}{\cancel{x}(\sqrt{x+4} + 2)}$$

$$\boxed{\frac{1}{\sqrt{x+4} + 2}}$$

Homework Assignment

Exercise 1: #1 – 4 odd; Exercise 2: #1 – 2 odd