Exercise 1.5 – Practice Problems

1. If a ball is thrown into the air with a velocity of 30m/s, its height in metres after t seconds is given by $y = 30t - 4.9t^2$

$\bar{v} = \frac{30t - 4.9t^2 - (30(2) - 4.9(2)^2)}{t-2}$ $\bar{v} = \frac{\Delta s}{\Delta t}$

a) Find the average velocity for the time period beginning when $t = 2$ and lasting

(i) 1s	(ii) 0.5s	(iii) 0.1s	(iv) 0.05s	(v) 0.01s
$\Delta v = \frac{30t - 4.9t^2 - 40.4}{t-2}$ so $t=3$ $2+1$ sec $\Delta v = \frac{30(3) - 4.9(3)^2 - 40.4}{1} = 5.5 m/s$	$\Delta v = 7.95 m/s$	$\Delta v = 9.91 m/s$	$\Delta v = 10.155 m/s$	$\Delta v = 10.351 m/s$

b) Find the instantaneous velocity when $t = 2$

$v(2) \lim_{h\to0} \frac{f(2+h) - f(2)}{h} \to \lim_{h\to0} \frac{30(2+h) - 4.9(2+h)^2 - 40.4}{h} \to \frac{60 + 30h - 4.9(4 + 4h + h^2) - 40.4}{h}$

$v(2) \lim_{h\to0} \frac{60 + 30h - 19.6 - 19.6h - 4.9h^2 - 40.4}{h} \to \frac{10.4h - 4.9h^2}{h} \to \lim_{h\to0} 10.4 - 4.9h = \boxed{10.4 m/s}$

2. The displacement in metres of a particle moving in a straight line is given by $s = t^2 - 4t + 3$, where t is measured in seconds. $\bar{v} = \frac{\Delta s}{\Delta t} = \frac{t^2 - 4t + 3 - (3^2 - 4(3) + 3)}{t-3}$

a) Find the average velocity over the following time periods:

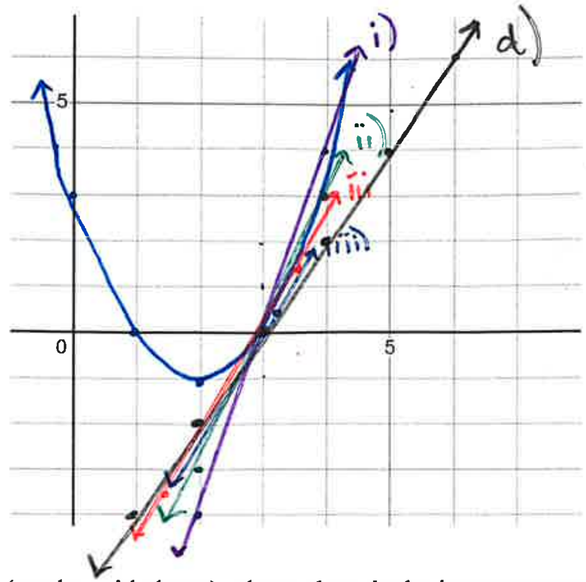
(i) $3 \le t \le 5$	(ii) $3 \le t \le 4$	(iii) $3 \le t \le 3.5$	(iv) $3 \le t \le 3.1$
$\Delta v = \frac{t^2 - 4t + 3 - 0}{t-3}$ From $3 \to 5$ $\frac{5^2 - 4(5) + 3}{5-3} = \frac{8}{2} = \boxed{4 m/s}$	$\frac{4^2 - 4(4) + 3}{4-3}$ $\frac{3}{1}$ $\boxed{3 m/s}$	$\frac{3.5^2 - 4(3.5) + 3}{3.5 - 3}$ $\frac{1.25}{0.5}$ $\boxed{2.5 m/s}$	$\frac{3.1^2 - 4(3.1) + 3}{3.1 - 3}$ $\frac{0.21}{0.1}$ $\boxed{2.1 m/s}$

b) Find the instantaneous velocity when $t = 3$

$v(3) \lim_{h\to0} \frac{f(3+h) - f(3)}{h} \to \frac{(3+h)^2 - 4(3+h) + 3 - (3^2 - 4(3) + 3)}{h} \to \frac{9 + 6h + h^2 - 12 - 4h + 3}{h}$

$\lim_{h\to0} \frac{2h + h^2}{h} \to 2 + h = \boxed{2 m/s}$

c) Draw the graph of s as a function of t and draw the secant lines whose slopes are the average velocities in part (a)



d) Draw the tangent line (on the grid above) whose slope is the instantaneous velocity in part (b)

3. A particle moves in a straight line with position function $s = 2t^2 + 4t - 5$, where t is measured in seconds and s in metres. Find the velocity of the particle at time $t = a$. Use this expression to find the velocity after: 1sec, 2sec, 3sec.

$s = f(t)$ time $= a$ $\lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h} \rightarrow \dfrac{2(a+h)^2 + 4(a+h) - 5 - [2a^2 + 4a - 5]}{h}$

$\lim\limits_{h\to 0} \dfrac{\cancel{2a^2} + 4ah + h^2 + \cancel{4a} + 4h \cancel{-5} \cancel{-2a^2} \cancel{-4a} \cancel{+5}}{h} \rightarrow \dfrac{4ah + 4h + h^2}{h} = \boxed{4a+4}$

$\boxed{a=1 \rightarrow 8\,m/s}$ $\boxed{a=2 \rightarrow 12\,m/s}$ $\boxed{a=3 \rightarrow 16\,m/s}$

4.

(a) Use the data of Example 3 to find the average rate of change of temperature with respect to time over the following time intervals: See Table on page 59

(i) $3 \le t \le 5$	(ii) $3 \le t \le 4$	(iii) $1 \le t \le 3$	(iv) $2 \le t \le 3$
$\dfrac{T(5)-T(3)}{5-3} = \dfrac{5.3-6.5}{2}$	$\dfrac{T(4)-T(3)}{4-1}$	$\dfrac{T(3)-T(1)}{3-1}$	$\dfrac{T(3)-T(2)}{3-2}$
$-0.6°/min$	$\dfrac{5.7-6.5}{1}$	$\dfrac{6.5-12}{2}$	$\dfrac{6.5-8.3}{1}$
	$-0.8°/min$	$-2.75°/min$	$-1.8°/min$

(b) Use the graph of T to estimate the instantaneous rate of change of T with respect to t when $t = 3$.

At $t=3$ $\dfrac{\Delta T}{\Delta t} = -1°/min$

* Hard to tell from graph Due to scale factor *

Draw a tangent at $t=3$ and see the behaviour

Growth is $\frac{\text{People}}{\text{Yr}}$

5. The population P of a city from 1982 *to* 1988 is given in the following table:

Year	1982	1983	1984	1985	1986	1987	1988
P (in thousands)	211	219	229	241	255	270	286

(a) Find the average rate of growth $\frac{\Delta P}{\Delta Y}$

(i) From 1984 to 1988

$$\frac{P(1988)-P(1984)}{1988-1984} = \frac{286-229}{4}$$

14.25 thousands/yr

(ii) From 1984 to 1987

$$\frac{P(1987)-P(1984)}{1987-1984} = \frac{270-229}{3}$$

13.7 thousand/yr

(iii) From 1984 to 1986

$$\frac{P(1986)-P(1984)}{1986-1984} = \frac{255-229}{2}$$

13 thousand/yr

(iv) From 1984 to 1985

$$\frac{P(1985)-P(1984)}{1985-1984} = \frac{241-229}{1}$$

12 thousand/yr

(b) Estimate the instantaneous rate of growth in 1984 by measuring the slope of the tangent

Disregard.

Use DESMOS to plot point and try to envision the tangent $\approx \frac{60}{5.5} \approx$ 11 thousand/yr

6.

(a) If $y = \frac{2}{x}$, find the average rate of change of y with respect to x over the interval $3 \le x \le 4$.

Average: $\dfrac{\frac{2}{4}-\frac{2}{3}}{4-3} = \dfrac{\frac{6}{12}-\frac{8}{12}}{1} = -\frac{2}{12} = \boxed{-\frac{1}{6}}$

(b) If $y = \frac{2}{x}$, find the instantaneous rate of change of y with respect to x at $x = 3$.

$\lim_{h\to 0} \dfrac{\frac{2}{x+h} - \frac{2}{x}}{h}$ but $x=3$ $\rightarrow \dfrac{\frac{2}{3+h} - \frac{2}{3}}{h} \rightarrow \dfrac{\frac{6-2(3+h)}{(3+h)3}}{h}$

$\lim_{h\to 0} \dfrac{6-6-2h}{h(3+h)(3)} \rightarrow \dfrac{-2h}{h(3+h)(3)} \rightarrow \dfrac{-2}{3(3+h)} = -\dfrac{2}{9}$

$V = x^3$

7.

(a) A cubic crystal is being grown in a laboratory. Find the average rate of change of the volume of the cube with respect to its edge length x, measured in millimeters, when x changes from:

i. 4 *to* 5

$\frac{V(5)-V(4)}{5-4} = \frac{5^3-4^3}{1}$

$\frac{125-64}{1} = 61\,mm^3/mm$

ii. 4 *to* 4.1

$\frac{V(4.1)-V(4)}{4.1-4} = \frac{4.1^3-4^3}{0.1}$

$\frac{4.921}{0.1} = 49.21\,mm^3/mm$

iii. 4 *to* 4.01

$\frac{V(4.01)-V(4)}{4.01-4} = \frac{4.01^3-4^3}{0.01}$

$\frac{0.4812}{0.01} = 48.12\,mm^3/mm$

(b) Find the instantaneous rate of change when $x = 4$.

$f(x) = x^3$

$f(4) = 4^3$

$\lim_{h\to 0} \frac{(4+h)^3-4^3}{h} \rightarrow \frac{(\cancel{4}+h-\cancel{4})((4+h)^2+4(4+h)+16)}{h}$

$\lim_{h\to 0} \frac{\cancel{h}(16+8h+h^2+16+4h+16)}{\cancel{h}} = 16+16+16 = 48\,mm^3/mm$

8. If a tank holds 1000L of water, which takes an hour to drain from the bottom of the tank, then the volume V of water remaining in the tank after t minutes is:

$$V = 1000\left(1-\frac{t}{60}\right)^2 \qquad 0 \le t \le 60$$

Find the rate at which the water is flowing out of the tank (the instantaneous rate of change V with respect to t) after 10mins.

Use: $\frac{V(t)-V(a)}{t-a}$ when $a = 10$

$\lim_{t\to 10} \frac{1000\left(1-\frac{t}{60}\right)^2 - \left[1000\left(1-\frac{10}{60}\right)^2\right]}{t-10} \rightarrow \frac{1000\left(1-\frac{1}{30}t+\frac{t^2}{3600}\right)-\left[1000\left(\frac{50}{60}\right)^2\right]}{t-10}$

$\lim_{t\to 10} \frac{1000-\frac{100t}{3}+\frac{5}{18}t^2-\frac{6250}{9}}{t-10} \rightarrow \frac{\frac{36000}{36}-\frac{1200t}{36}+\frac{10t^2}{36}-\frac{25000}{36}}{t-10}$

$\lim_{t\to 10} \frac{10t^2-1200t+11000}{36}\cdot\frac{1}{(t-10)} \rightarrow \frac{10(t^2-120t+1100)}{36(t-10)} = \frac{10\cancel{(t-10)}(t-110)}{36\cancel{(t-10)}}$

$\lim_{t\to 10} \frac{10(t-110)}{36} = \frac{10(-100)}{36} = -\frac{1000}{36} = \boxed{-\frac{250\,L}{9\,min}}$

9. If an arrow is shot upward on the moon with a velocity of 50m/s, its height in metres after t secondsis given by $s = 50t - 0.83t^2$.

$\Delta v = \frac{f(t)-f(1)}{t-1} = \frac{50t-0.83t^2-49.17}{t-1}$

(a) Find the average velocity for the time period beginning when $t = 1$ and lasting:

i. 1 sec	ii. 0.5 sec	iii. 0.1 sec	iv. 0.05 sec	v. 0.01 sec
$t=2$ $\frac{50(2)-0.83(2)^2-49.17}{2-1}$ 47.51m/s	$t=1.5$ 47.93m/s	$t=1.1$ 48.26m/s	$t=1.05$ 48.30m/s	$t=1.01$ 48.33m/s

(b) Find the instantaneous velocity when $t = 1$

$\lim_{h\to0}\frac{50(1+h)-0.83(1+h)^2-[49.17]}{h} \to \frac{50+50h-0.83(1+2h+h^2)-49.17}{h}$

$\lim_{h\to0} \to \frac{\cancel{50}+50h-0.\cancel{83}-1.66h-0.83h^2-\cancel{49.17}}{h} \to \frac{\cancel{h}(50-1.66-0.83h)}{\cancel{h}} = \boxed{48.34m/s}$

(c) Find the velocity after t seconds

$\lim_{h\to0}\frac{50(t+h)-0.83(t+h)^2-(50t-0.83t^2)}{h} = \frac{\cancel{50t}+50h-0.\cancel{83t^2}-1.66th-0.83h^2-\cancel{50t}+\cancel{0.83t^2}}{h}$

$\lim_{h\to0}\frac{50h-1.66th-0.83h^2}{h} \to \boxed{50-1.66t}$

(d) When will the arrow hit the moon?

Need $s(t)=0$ $\quad 50t-0.83t^2=0 \to t(50-0.83t)=0$

$\boxed{t=0}$ or $50-0.83t=0 \to -0.83t=-50 \quad t=\frac{50}{0.83} = \boxed{60.24\ sec}$

(e) With what velocity will the arrow hit the moon?

use velocity from (c) and time 0 is rejected so use

$t=60.24$

$v(t) = 50-1.66t$

$v(60.24) = 50-1.66(60.24)$

$= -50m/s$

$\boxed{\text{Downward velocity of } 50 m/s}$