Exercise 1.4 – Practice Problems

1.

a) Find the slope of the tangent line to the parabola: $y = 2x - x^2$ at the point (2,0) using

i. Formula 1

$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ where $a = 2$

$\lim_{x \to a} \frac{2x - x^2 - (2(2) - 2^2)}{x - 2}$

$\lim_{x \to 2} \frac{2x - x^2 - 4 + 4}{x - 2}$

$\frac{-x(x-2)}{x-2}$

$-x$

$= \boxed{-2}$

ii. Formula 2 $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ $a = 2$

$\lim_{h \to 0} \frac{2(a+h) - (a+h)^2 - [4 - 4]}{h}$

$\lim_{h \to 0} \frac{2a + 2h - (a^2 + 2ah + h^2)}{h}$ as $a = 2$ $= \frac{\cancel{4} + 2h - \cancel{4} - 4h + h^2}{h}$

$\lim_{h \to 0} \frac{h^2 - 2h}{h} \rightarrow \lim_{h \to 0} \frac{\cancel{h}(h-2)}{\cancel{h}}$

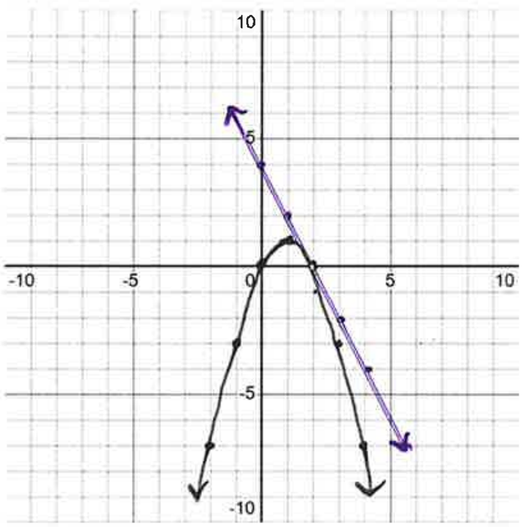
$\lim_{h \to 0} h - 2 = \boxed{-2}$

b) Find the equation of the tangent line

$y = -2x + b$ at $(2,0)$

$0 = -2(2) + b$ $b = 4$ $\boxed{y = -2x + 4}$

c) Graph the parabola and the tangent line.



$y = -x^2 + 2x$

$y = -x(x-2)$

Root: $x = 0$
$x = 2$

vertex: $(1,1)$

opens down
no stretch

$*(1+h)^3 = (1+h)(1+2h+h^2)$
$= h^3 + 3h^2 + 3h + 1$

2.

a) Find the slope of the tangent line to the cubic curve: $y = x^3$ at the point $(1,1)$ using

i. Formula 1

$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ $a = 1$

$\lim_{x \to 1} \frac{x^3 - (1)^3}{x - 1} \rightarrow \lim_{x \to 1} \frac{x^3 - 1}{x - 1} \rightarrow \lim_{x \to 1} \frac{\cancel{(x-1)}(x^2 + x + 1)}{\cancel{(x-1)}}$

$\lim_{x \to 1} = \boxed{3}$

ii. Formula 2 $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ $a = 1$

$\lim_{h \to 0} \frac{(a+h)^3 - 1^3}{h} \rightarrow \lim_{h \to 0} \frac{h^3 + 3h^2 + 3h + 1 - 1}{h}$

$\lim_{h \to 0} \frac{\cancel{h}(h^2 + 3h + 3)}{\cancel{h}}$

$\lim_{h \to 0} \boxed{3}$

b) Find the equation of the tangent line

$y = 3x + b$

$1 = 3(1) + b$

$1 = 3 + b$

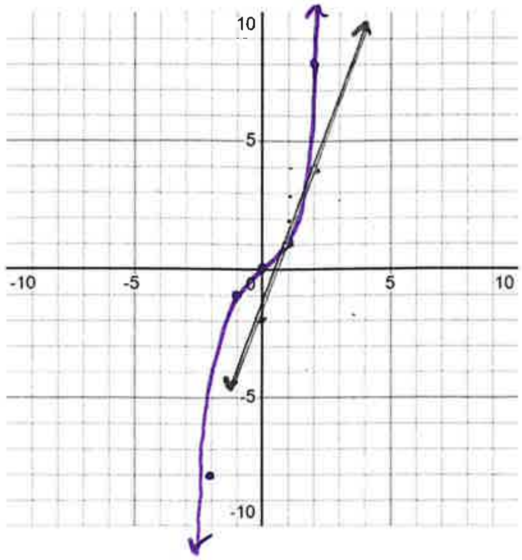
$b = -2$

$\boxed{y = 3x - 2}$

c) Graph the parabola and the tangent line.

$y = x^3$

tangent
$y = 3x - 2$



3. Find the slope in Example 1 (from the notes) using Formula 2.

$y = 2x^2+4x-1$ where $a=2$

$\lim_{h\to0}\frac{f(a+h)-f(a)}{h} \to \lim_{h\to0}\frac{2(a+h)^2+4(a+h)-1-(8+8-1)}{h} \to \lim_{h\to0}\frac{2a^2+4ah+2h^2+4a+4h-1-15}{h}$

$\lim_{h\to0}\frac{8+8h+2h^2+8+4h-16}{h}$

$\lim_{h\to0}\frac{2h^2+12h}{h} \to \frac{\cancel{h}(2h+12)}{\cancel{h}}$

$\lim_{h\to0} = \boxed{12}$

4. Find the slope in Example 2 (from the notes) using Formula 1.

$y=\frac{1}{x}$ $a=-2$

$\lim_{x\to-2}\frac{\frac{1}{x}-\frac{1}{-2}}{x+2} \to \frac{\frac{1}{x}+\frac{1}{2}}{x+2} \to \lim_{x\to-2}\frac{\frac{2+x}{2x}}{x+2} \to \lim_{x\to-2}\frac{1}{2x} = \boxed{-\frac{1}{4}}$

5. Find the slope in Example 3 (from the notes) using Formula 1.

$y=\sqrt{x-2}$ $a=6$

$\lim_{x\to6}\frac{\sqrt{x-2}-\sqrt{6-2}}{x-6} \to \frac{(\sqrt{x-2}-2)(\sqrt{x-2}+2)}{(x-6)(\sqrt{x-2}+2)} \to \frac{x-2-4}{(x-6)(\sqrt{x-2}+2)}$

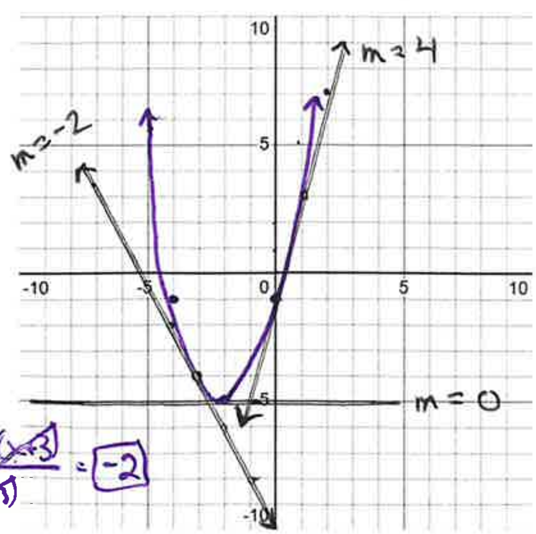
$\lim_{x\to6}\frac{(x-6)}{(x-6)(\sqrt{x-2}+2)} \to \lim_{x\to6}\frac{1}{\sqrt{6-2}+2} \to \lim_{x\to6}\boxed{\frac{1}{4}}$

6. a) Find the slope of the tangent lines to the parabola $y = x^2 + 4x - 1$ at the points whose $x - coordinates$ are:

x^2+4x-1
use Desmos

(i) −3	(ii) −2	(iii) 0
$m=-2$	$m=0$	$m=4$

b) Graph the parabola and the three tangents



Use Formula 1: sub x and a at the end

$\lim_{x\to a}\frac{f(x)-f(a)}{x-a} \to \lim_{x\to a}\frac{x^2+4x-1-(a^2+4a-1)}{x-a}$

$\lim_{x\to a}\frac{x^2+4x-1-a^2-4a+1}{x-a}$

i) $\lim_{x\to-3}\frac{x^2+4x-a^2-4a}{x-a} \to \lim_{x\to-3}\frac{x^2+4x-9+12}{x+3}$

$\lim_{x\to-3}\frac{x^2+4x+3}{(x+3)} \to \frac{(x+1)\cancel{(x+3)}}{\cancel{(x+3)}} = \boxed{-2}$

ii) $\lim_{x\to-2}\frac{x^2+4x-4+8}{x+2} \to \lim_{x\to-2}\frac{x^2+4x+4}{(x+2)} \to \frac{(x+2)\cancel{(x+2)}}{\cancel{x+2}} = \boxed{0}$

iii) $\lim_{x\to0}\frac{x^2+4x-0-0}{x} \to \frac{\cancel{x}(x+4)}{\cancel{x}}$

$\lim_{x\to0} = \boxed{4}$

Use either Formula, I like Formula 2

7. For each of the following curves

   a) Find the slope of the tangents at the given point
   b) Find the equation of the tangent at the given point
   c) Graph the curve and the tangent.

i. $y = 4 - x^2$ at $(-2, 0)$

a) $\lim\limits_{h\to 0} \frac{f(a+h)-f(a)}{h}$ $a=-2$ $\rightarrow \lim\limits_{h\to 0} \frac{4-(-2+h)^2-(4-a^2)}{h}$

$\lim\limits_{h\to 0} \frac{4-(4-4h+h^2)-(4-4)}{h} \rightarrow \lim\limits_{h\to 0} \frac{4-4+4h-h^2-0}{h}$

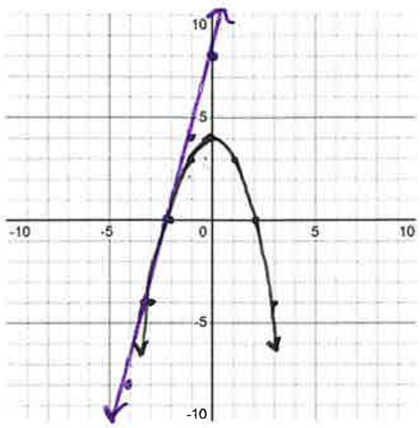
$\lim\limits_{h\to 0} \frac{-\cancel{h}(h-4)}{\cancel{h}} = \boxed{+4}$

b) $y = +4x + b$ at $(-2, 0)$

$0 = +4(-2) + b$

$+8 = b$

$\boxed{y = +4x + 8}$



ii. $y = x^2 - 6x + 5$ at $(2, -3)$

$(x-1)(x-5)$ vertex: $(3, -4)$

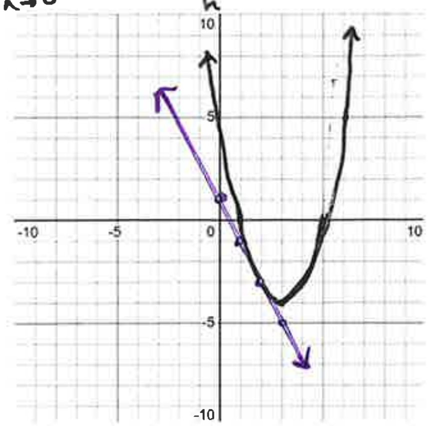
at $a=2$

a) $\lim\limits_{h\to 0} \frac{(a+h)^2 - 6(a+h) + 5 - (a^2 - 6a + 5)}{h}$

$\lim\limits_{h\to 0} \frac{(2+h)^2 - 6(2+h) + 5 - (4 - 12 + 5)}{h}$

$\lim\limits_{h\to 0} \frac{4+4h+h^2-12-6h+5+3}{h} \rightarrow \lim\limits_{h\to 0} \frac{h^2-2h}{h}$

$\lim\limits_{h\to 0} \frac{\cancel{h}(h-2)}{\cancel{h}} = \boxed{-2}$



b) $y = -2x + b$

$-3 = -2(2) + b$

$-3 = -4 + b$

$b = 1$

$\boxed{y = -2x + 1}$

iii. $y = 1 - x^3$ at $(0, 1)$

a) $\lim\limits_{h\to 0} \frac{1-(a+h)^3-(1-a^3)}{h} \rightarrow \lim\limits_{h\to 0} \frac{1-(h)^3-(1-0)}{h}$

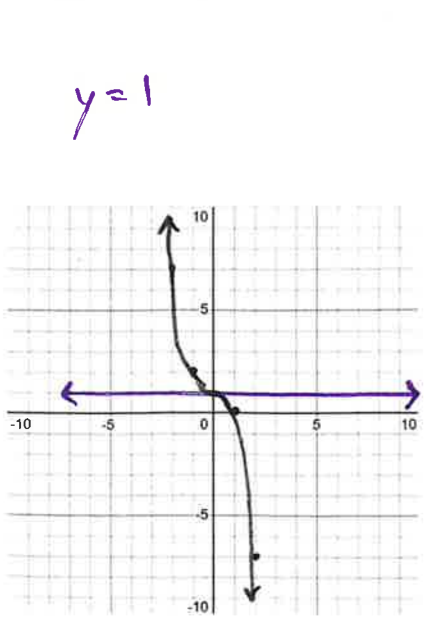
$\lim\limits_{h\to 0} \frac{1-h^3-1}{h} \rightarrow \lim\limits_{h\to 0} \frac{-h^3}{h} = -h^2 = \boxed{0}$

b) $y = 0x + b$

$1 = 0 + b$

$b = 1$

$y = 1$



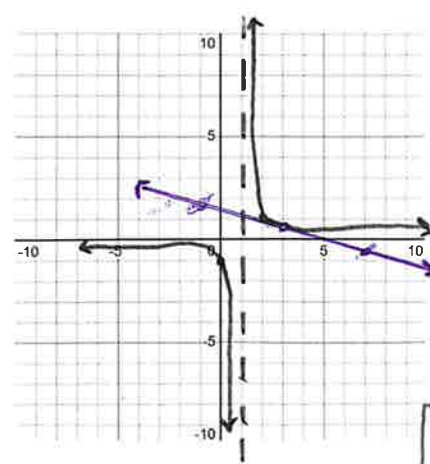
$y = -x^3 + 1$

iv. $y = \frac{1}{x-1}$ at $(3, ½)$ $a=3$ $x \neq 1$

a) $\lim\limits_{h\to 0} \frac{\frac{1}{(a+h)-1} - \frac{1}{a-1}}{h} \rightarrow \frac{\frac{1}{(3+h)-1} - \frac{1}{2}}{h}$

$\lim\limits_{h\to 0} \frac{\frac{1}{(h+2)} - \frac{1}{2}}{h} \rightarrow \lim\limits_{h\to 0} \frac{2-(h+2)}{2(h+2)} \cdot \frac{1}{h}$

$\lim\limits_{h\to 0} \frac{2-h-2}{2h(h+2)} \rightarrow \lim\limits_{h\to 0} \frac{-h}{2h(h+2)} \rightarrow \frac{-1}{2(h+2)} = \frac{-1}{4}$



b) $y = -\frac{1}{4}x + b$

$\frac{1}{2} = -\frac{1}{4}(3) + b$

$\frac{1}{2} = -\frac{3}{4} + b$

$\frac{3}{4} + \frac{1}{2} = b$ $b = 5/4$

$\boxed{y = -\frac{1}{4}x + \frac{5}{4}}$

D: $x \geq -3$

v. $y = \sqrt{x+3}$ at $(6, 3)$ ← a

a) $\lim_{h\to0} \frac{\sqrt{a+h+3} - \sqrt{a+3}}{h} \rightarrow \lim_{h\to0} \frac{\sqrt{6+h+3} - \sqrt{9}}{h}$

$\lim_{h\to0} \frac{\sqrt{h+9} - 3}{h} \cdot \frac{(\sqrt{h+9}+3)}{\sqrt{h+9}+3} \rightarrow \lim_{h\to0} \frac{h+9-9}{h(\sqrt{h+9}+3)}$

$\lim_{h\to0} \frac{h}{h(\sqrt{h+9}+3)} \rightarrow \lim_{h\to0} \frac{1}{\sqrt{h+9}+3} = \frac{1}{3+3} = \boxed{\frac{1}{6}}$

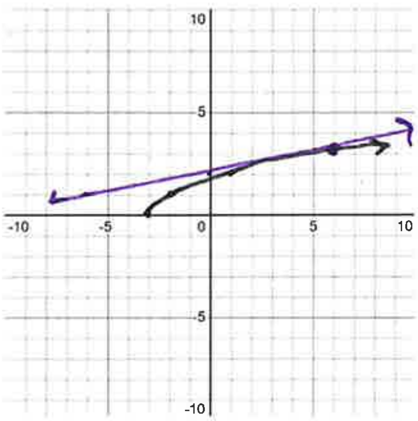
b) $y = \frac{1}{6}x + b$

$3 = \frac{1}{6}(6) + b$

$3 = 1 + b$

$b = 2$

$y = \frac{1}{6}x + 2$



vi. $y = 2x^4$ at $(-1, 2)$ ← a

a) $\lim_{h\to0} \frac{2(a+h)^4 - 2a^4}{h} \rightarrow \lim_{h\to0} \frac{2(-1+h)^4 - 2(-1)^4}{h}$

$\lim_{h\to0} \frac{2(h^4 - 4h^3 + 6h^2 - 4h + 1) - 2}{h}$ *

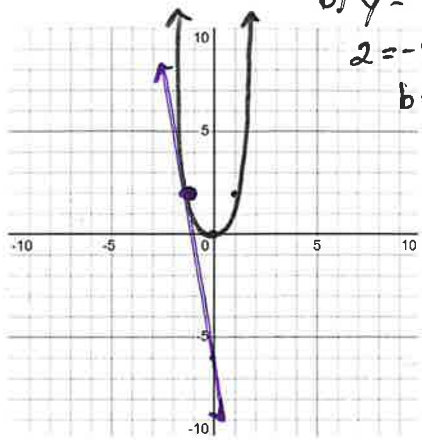
$\lim_{h\to0} \frac{2h^4 - 8h^3 + 12h^2 - 8h + 2 - 2}{h}$

$\lim_{h\to0} \frac{\cancel{h}(2h^3 - 8h^2 + 12h - 8}{\cancel{h}} = -8$

b) $y = -8x + b$

$2 = -8(-1) + b$ $\boxed{y = -8x - 6}$

$b =$



↓ see examples online about Pascal's Triangle and higher power binomials

8. Find the equation of the tangent line to the graph of the given function at the given point. NO GRAPHS NECESSARY

a) $f(x) = 4 - x + 3x^2$ at $(-1, 8)$

$\lim_{h\to0} \frac{4 - (a+h) + 3(a+h)^2 - [4 - a + 3a^2]}{h}$

$\lim_{h\to0} \frac{4 + 1 - h + 3(1 - 2h + h^2) - [4 + 1 + 3]}{h}$

$\lim_{h\to0} \frac{\cancel{5} - h + \cancel{3} - 6h + 3h^2 - \cancel{8}}{h}$

$\lim_{h\to0} \frac{3h^2 - 7h}{h}$

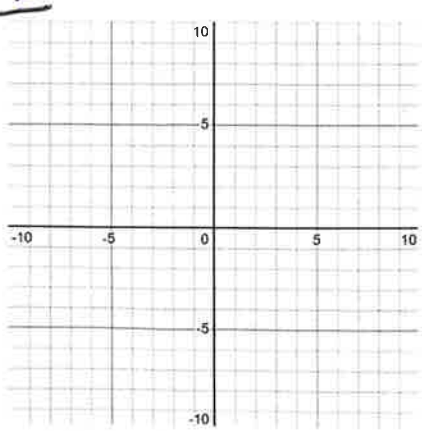
$\lim_{h\to0} \frac{\cancel{h}(3h - 7)}{\cancel{h}}$

$= -7$

$y = -7x + b$

$8 = -7(-1) + b$

$b = 1$



$\boxed{y = -7x + 1}$

b) $f(x) = x^3 - x$ at $(0, 0)$

$\lim_{h\to0} \frac{(a+h)^3 - (a+h) - (a^3 - a)}{h}$

$\lim_{h\to0} \frac{(0+h)^3 - (0+h) - (0-0)}{h} \rightarrow \lim_{h\to0} \frac{h^3 - h - 0}{h}$

$\lim_{h\to0} \frac{\cancel{h}(h^2-1)}{\cancel{h}} \rightarrow h^2 - 1 = -1$

$y = -1x + b$ $\boxed{y = -x}$

$0 = 0 + b$

$b = 0$

c) $g(x)=\frac{2x+1}{x-1}$ at $(2,5)$ a

$$\lim_{h\to 0}\frac{\frac{2(a+h)+1}{(a+h)-1}-\left[\frac{2(a)+1}{a-1}\right]}{h}$$

$$\lim_{h\to 0}\frac{\frac{2(2+h)+1}{2+h-1}-\left[\frac{4+1}{2-1}\right]}{h}$$

$$\lim_{h\to 0}\frac{\frac{4+2h+1}{h+1}-\frac{5}{1}}{h}\rightarrow\lim_{h\to 0}\frac{\frac{5+2h-5(h+1)}{h+1}}{h}$$

$$\lim_{h\to 0}\frac{5+2h-5h-5}{h(h+1)}\rightarrow\lim_{h\to 0}\frac{-3h}{h(h+1)}$$

$$\lim_{h\to 0}\frac{-3}{h+1}=-3$$

$$\boxed{y=-3x+11}$$

$y=-3x+b$

$5=-3(2)+b$

$b=11$

d) $g(x)=\frac{1}{\sqrt{x}}$ at $(1,1)$

$$\lim_{h\to 0}\frac{\frac{1}{\sqrt{a+h}}-\frac{1}{\sqrt{a}}}{h}\rightarrow\lim_{h\to 0}\frac{\frac{1}{\sqrt{1+h}}-\frac{1}{1}}{h}$$

$$\lim_{h\to 0}\frac{1-\sqrt{1+h}}{\sqrt{1+h}}\cdot\frac{1}{h}\rightarrow\lim_{h\to 0}\frac{1-\sqrt{1+h}}{h(\sqrt{1+h})}$$

$$\lim_{h\to 0}\frac{1-\sqrt{1+h}}{h(\sqrt{1+h})}\cdot\frac{1+\sqrt{1+h}}{1+\sqrt{1+h}}$$

$$\lim_{h\to 0}\frac{1-(1+h)}{h(\sqrt{1+h})(1+\sqrt{1+h})}\rightarrow\lim_{h\to 0}\frac{1-1-h}{h\sqrt{1+h}(1+\sqrt{1+h})}$$

$$\lim_{h\to 0}\frac{-\cancel{h}}{\cancel{h}\sqrt{1+h}(1+\sqrt{1+h})}=\frac{-1}{\sqrt{1}(1+\sqrt{1})}=\boxed{\frac{-1}{2}}$$

$y=-\frac{1}{2}x+b$ → $b=3/2$

$1=-\frac{1}{2}(1)+b$

$$\boxed{y=-\frac{1}{2}x+\frac{3}{2}}$$

9.

a) Find the slope of the tangent line to the parabola $y=x^2+x+1$ at the general point whose $x-coordinate$ is a.

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}\rightarrow\frac{(a+h)^2+(a+h)+1-(a^2+a+1)}{h}\rightarrow\frac{\cancel{a^2}+2ah+h^2+\cancel{a}+h+\cancel{1}-\cancel{a^2}-\cancel{a}-\cancel{1}}{h}$$

$$\lim_{h\to 0}\frac{h^2+2ah+h}{h}\rightarrow\lim_{h\to 0}\frac{\cancel{h}(h+2a+1)}{\cancel{h}}=\lim_{h\to 0}h+2a+1=2a+1$$

b) Find the slopes of the tangents to this parabola at the points whose $x-coordinates$ are:

$a=$ -1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
$m=2(-1)+1$	$2(-\frac{1}{2})+1$	$2(0)+1$	$2(\frac{1}{2})+1$	$2(1)+1$
$=-1$	$=0$	$=1$	$=2$	$=3$

10.

a) Find the slope of the tangent line to the parabola $y = 3x^2 + 2x$ at the point whose $x - coordinate$ is a

$$\lim_{h\to 0} \frac{3(a+h)^2+2(a+h)-(3a^2+2a)}{h} \to \lim_{h\to 0}\frac{3(a^2+2ah+h^2)+2a+2h-3a^2-2a}{h}$$

$$\lim_{h\to 0}\frac{\cancel{3a^2}+6ah+3h^2+\cancel{2a}+2h-\cancel{3a^2}-\cancel{2a}}{h} \to \lim_{h\to 0}\frac{3h^2+6ah+2h}{h} \to \frac{\cancel{h}(3h+6a+2)}{\cancel{h}} \to \lim_{h\to 0}\boxed{6a+2}$$

b) At what point on the parabola is the tangent line parallel to the line $y = 10x - 2$

Same slope for parrallel

$m = 10$

$10 = 6a + 2$

$8 = 6a$

$a = 8/6$ or $4/3$ ← plug in for x to solve y

$\boxed{(4/3, 8)}$

$$y = \left(3\left(\tfrac{4}{3}\right)^2\right) + 2\left(\tfrac{4}{3}\right) \to 3\left(\tfrac{16}{9}\right) + \tfrac{8}{3} \to \tfrac{16}{3} + \tfrac{8}{3} = \tfrac{24}{3} = 8$$

11. Find the points of intersection of the parabolas $y = \frac{1}{2}x^2$ and $y = 1 - \frac{1}{2}x^2$. Show that at each of these points the tangent lines to the two parabolas are perpendicular.

Intersection is where they are equal $\quad \frac{1}{2}x^2 = 1 - \frac{1}{2}x^2 \to 1x^2 = 1$

$\boxed{x = \pm 1}$

when $x = \pm 1$ $y = \frac{1}{2}$

For $y = \frac{1}{2}x^2$

$$\lim_{h\to 0} \frac{\frac{1}{2}(a+h)^2 - \frac{1}{2}a^2}{h}$$

$$\lim_{h\to 0} \frac{\frac{1}{2}(a^2+2ah+h^2) - \frac{1}{2}a^2}{h}$$

$$\lim_{h\to 0} \left(\frac{\cancel{a^2}+2ah+h^2}{2} - \frac{\cancel{a^2}}{2}\right)\frac{1}{h}$$

$$\lim_{h\to 0} \frac{h^2+2ah}{2h} \to \lim_{h\to 0}\frac{\cancel{h}(h+2a)}{2\cancel{h}}$$

$= a$

For $x = 1$ $\quad y = \frac{1}{2}x^2 \quad m = 1$] perp
$\quad y = 1 - \frac{1}{2}x^2 \quad m = -1$

For $x = -1$ $\quad y = \frac{1}{2}x^2 \quad m = -1$] perp
$\quad y = 1 - \frac{1}{2}x^2 \quad m = 1$

For $y = -\frac{1}{2}x^2 + 1$

$$\lim_{h\to 0} \frac{\frac{-1(a+h)^2}{2} + \frac{1}{1} - \left[-\frac{1}{2}a^2 + 1\right]}{h}$$

$$\lim_{h\to 0} \frac{\frac{-(a^2+2ah+h^2)+2}{2} + \frac{a^2-2}{2}}{h}$$

$$\lim_{h\to 0} \frac{\cancel{-a^2}-2ah-h^2\cancel{+2}\cancel{+a^2}\cancel{-2}}{2h}$$

$$\lim_{h\to 0} \frac{-h^2-2ah}{2h} \to \frac{-\cancel{h}(h+2a)}{2\cancel{h}}$$

$$\lim_{h\to 0} -\frac{2a}{2} = -a$$