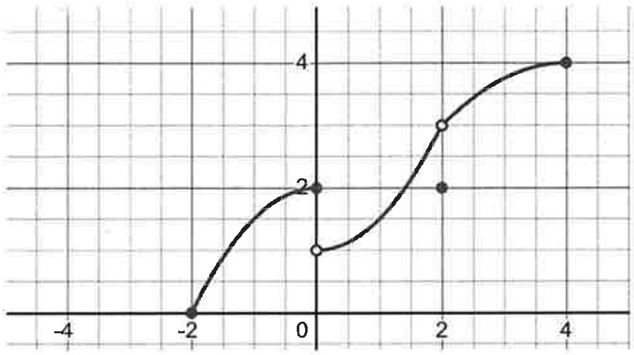
Exercise 1.3 – Practice Problems

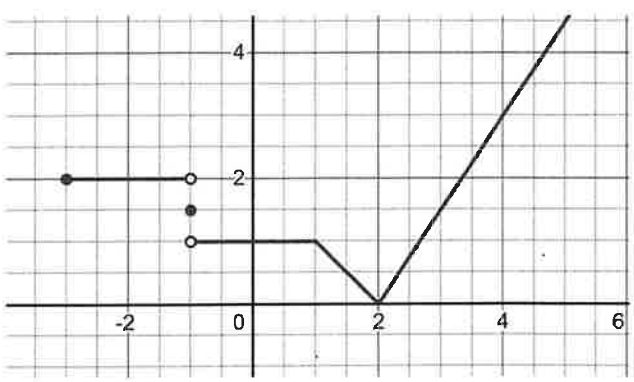
1. Use the given graph of f to state the value of the limit, if it exists.

a) $\lim\limits_{x\to -2^+} f(x)$ 0	b) $\lim\limits_{x\to 0^-} f(x)$ 2	c) $\lim\limits_{x\to 0^+} f(x)$ 1	d) $\lim\limits_{x\to 0} f(x)$ DNE
e) $\lim\limits_{x\to 2^-} f(x)$ 3	f) $\lim\limits_{x\to 2^+} f(x)$ 3	g) $\lim\limits_{x\to 2} f(x)$ 3	h) $\lim\limits_{x\to 4^-} f(x)$ 4



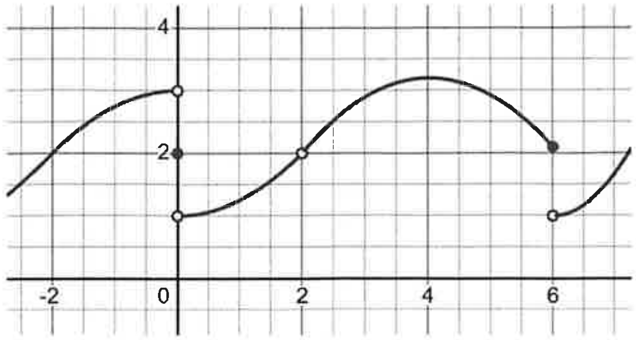
2. Use the given graph of g to state the value of the limit, if it exists.

a) $\lim\limits_{x\to -3^+} g(x)$ 2	b) $\lim\limits_{x\to -1^-} g(x)$ 2	c) $\lim\limits_{x\to -1^+} g(x)$ 1	d) $\lim\limits_{x\to -1} g(x)$ DNE
e) $\lim\limits_{x\to 2^-} g(x)$ 0	f) $\lim\limits_{x\to 2^+} g(x)$ 0	g) $\lim\limits_{x\to 2} g(x)$ 0	h) $\lim\limits_{x\to 1} g(x)$ 1



3. The graph of f is given. State whether f is continuous or discontinuous at each of the following numbers.

a) -2	b) 0	c) 2	d) 4	e) 6
continuous	Discontinuous LHL ≠ RHL	Discontinuous f(2) DNE	Continuous	Discontinuous LHL ≠ RHL



4. Find the following limits, if they exist.

a) $\lim_{x \to 0^+} \sqrt[4]{x}$

$\sqrt[4]{\lim_{x \to 0^+} x} = \sqrt[4]{0} = 0$

b) $\lim_{x \to 3^+} \sqrt{x-3}$

$\sqrt{\lim_{x \to 3^+} x - \lim_{x \to 3^+} 3}$

$\sqrt{3-3} = \sqrt{0} = 0$

c) $\lim_{x \to 1^-} \sqrt{1-x}$

$\sqrt{\lim_{x \to 1^-} 1 - \lim_{x \to 1^-} x}$

$\sqrt{1-1} = \sqrt{0} = 0$

d) $\lim_{x \to \frac{1}{2}^-} \sqrt[4]{1-2x}$

$\sqrt[4]{\lim_{x \to \frac{1}{2}^-} 1 - \lim_{x \to \frac{1}{2}^-} 2x}$

$\sqrt[4]{1-1} = \sqrt[4]{0} = 0$

e) $\lim_{x \to 6^+} |x-6|$

$|\lim_{x \to 6^+} x - \lim_{x \to 6^+} 6|$

$|6-6|$

$|0| = 0$

f) $\lim_{x \to 6^-} |x-6|$

$|\lim_{x \to 6^-} x - \lim_{x \to 6^-} 6|$

$|6-6| = |0| = 0$

g) $\lim_{x \to 6} |x - 6|$

$\boxed{0}$ since both e) and f)

$\lim_{x \to 6^+} = 0$ $\lim_{x \to 6^-} = 0$

h) $\lim_{x \to 0^+} \frac{|x|}{x}$

$\frac{|\lim_{x \to 0^+} x|}{\lim_{x \to 0^+} x} = \frac{1}{1} = \boxed{1}$

i) $\lim_{x \to 0^-} \frac{|x|}{x}$

$\frac{|\lim_{x \to 0^-} x|}{\lim_{x \to 0^-} x} = \frac{|-1|}{-1} = \frac{1}{-1}$

$= \boxed{-1}$

j) $\lim_{x \to 0} \frac{|x|}{x}$

DNE

as h) and i) $\neq$

5. Let

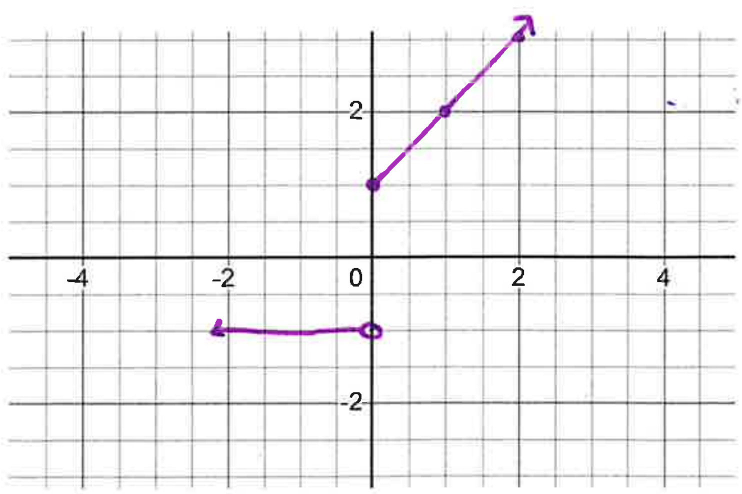
$$f(x) = \begin{cases} -1 & if\ x < 0 \\ x + 1 & if\ x \geq 0 \end{cases}$$

Find the following limits if they exist. Then sketch the graph of f. Use DESMOS to support graphing.

a) $\lim_{x \to 0^-} f(x)$ $\boxed{-1}$

b) $\lim_{x \to 0^+} f(x)$ $\boxed{1}$

c) $\lim_{x \to 0} f(x)$ DNE

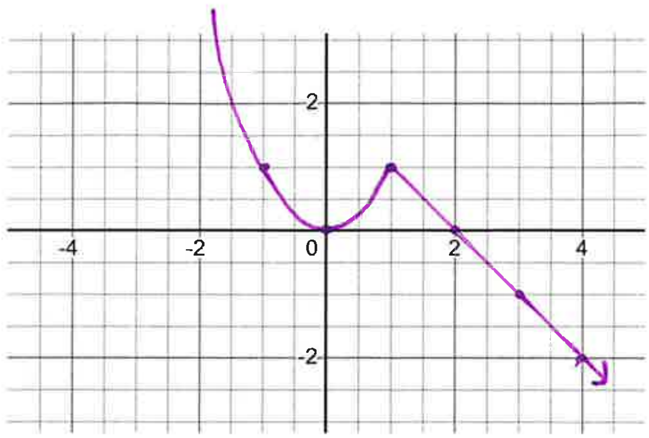


6. Let

$$g(x) = \begin{cases} x^2 & if\ x \le 1 \\ 2 - x & if\ x > 1 \end{cases}$$

Find the following limits if they exist. Then sketch the graph of g. Use DESMOS to support graphing.

a) $\lim_{x \to 1^-} g(x)$ $\boxed{1}$ | b) $\lim_{x \to 1^+} g(x)$ $\boxed{1}$ | c) $\lim_{x \to 1} g(x)$ $\boxed{1}$

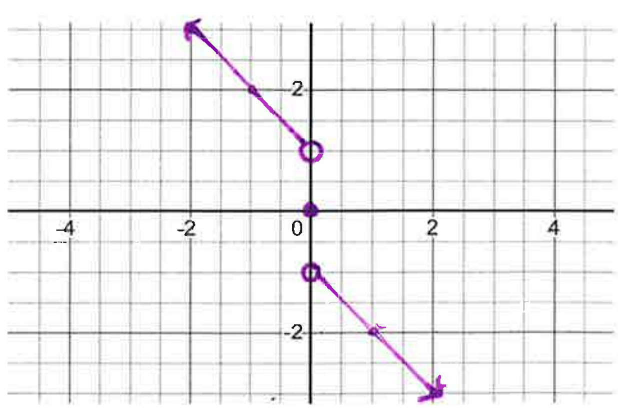


7. Let

$$h(x) = \begin{cases} 1 - x & if\ x < 0 \\ 0 & if\ x = 0 \\ -x - 1 & if\ x > 0 \end{cases}$$

Find the following limits if they exist. Then sketch the graph of h. Use DESMOS to support graphing.

i. $\lim_{x \to 0^-} h(x)$ $\boxed{1}$ | ii. $\lim_{x \to 0^+} h(x)$ $\boxed{-1}$ | iii. $\lim_{x \to 0} h(x)$ DNE



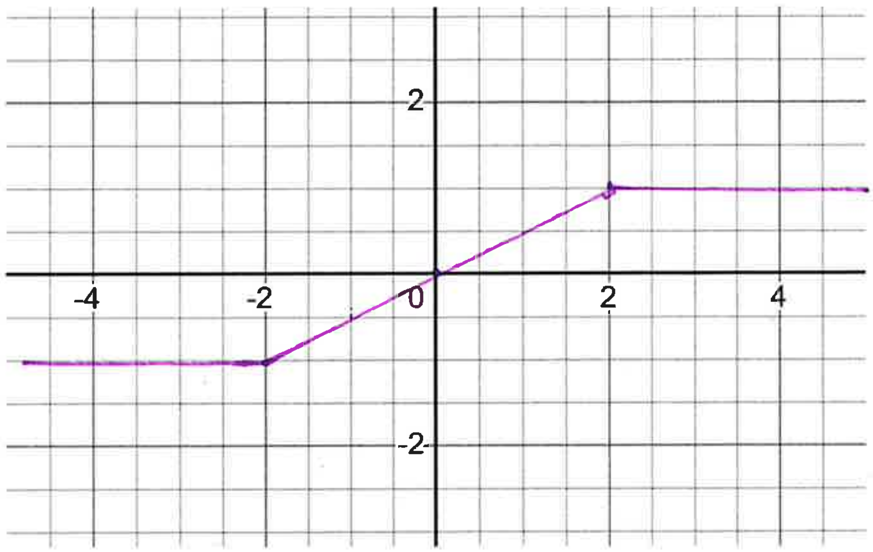
8. Let

$$f(x) = \begin{cases} -1 & if\ x \le -2 \\ \frac{1}{2}x & if -2 < x < 2 \\ 1 & if\ x \ge 2 \end{cases}$$

a) Find the following limits.

i. $\lim_{x \to -2^-} f(x)$ $\boxed{-1}$ | ii. $\lim_{x \to -2^+} f(x)$ $\boxed{-1}$ | iii. $\lim_{x \to 2^-} f(x)$ $\boxed{1}$ | iv. $\lim_{x \to 2^+} f(x)$ $\boxed{1}$

b) Sketch the Graph. Use DESMOS to support graphing.



c) Where is the graph discontinuous?

f is continuous everywhere

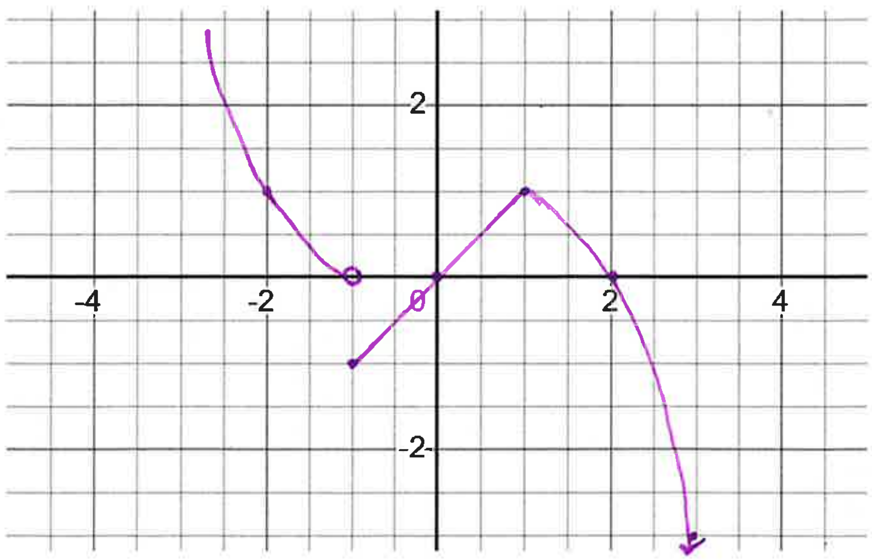
9. Let

$$f(x) = \begin{cases} (x+1)^2 & if\ x < -1 \\ x & if\ -1 \le x \le 1 \\ 2x - x^2 & if\ x > 1 \end{cases}$$

a) Find the following limits, if they exist.

i. $\lim_{x \to -1^-} f(x)$ $\boxed{0}$	ii. $\lim_{x \to -1^+} f(x)$ $\boxed{-1}$	iii. $\lim_{x \to -1} f(x)$ DNE
iv. $\lim_{x \to 1^-} f(x)$ $\boxed{1}$	v. $\lim_{x \to 1^+} f(x)$ $\boxed{1}$	vi. $\lim_{x \to 1} f(x)$ $\boxed{1}$

b) Sketch the Graph



c) Where is the graph discontinuous?

Discontinuous at $x = -1$

10. Where are the following functions discontinuous?

a)

$$f(x) = \begin{cases} 2x+3 & \text{if } x \neq 4 \\ 12 & \text{if } x = 4 \end{cases}$$

$\lim\limits_{x\to4^+} 2x + \lim\limits_{x\to4^-} 3 = 11$

just check $\lim\limits_{x\to4^-}$ and $\lim\limits_{x\to4^+}$

$\lim\limits_{x\to4^-} 2x + \lim\limits_{x\to4^-} 3 \to 8+3=11$

but $f(4)=12$ so $\boxed{\text{Discontinuous at } x=4}$

b)

$$f(x) = \begin{cases} 1-x^2 & \text{if } x \leq 0 \\ x+1 & \text{if } 0 < x \leq 1 \\ (x-1)^2 & \text{if } x > 1 \end{cases}$$

only potential discontinuities $x=0$, $x=1$

$\lim\limits_{x\to0^+} f(x) = 1$

$\lim\limits_{x\to0^-} f(x) = 1$ ✓

$\lim\limits_{x\to1^-} f(x) = 2$

$\lim\limits_{x\to1^+} f(x) = 0$ ×

$\boxed{\text{Discontinuity at } x=1}$

c)

$$f(x) = \begin{cases} -x & \text{if } x < -1 \\ x^3 & \text{if } -1 \leq x \leq 1 \\ x & \text{if } x > 1 \end{cases}$$

possible at $x=-1$, $x=1$

$\boxed{\text{Discontinuity at } x=-1}$

$\lim\limits_{x\to-1^-} f(x) = 1$

$\lim\limits_{x\to-1^+} f(x) = -1$ ×

$\lim\limits_{x\to1^-} f(x) = 1$

$\lim\limits_{x\to1^+} f(x) = 1$ ✓

d)

$$f(x) = \begin{cases} x & \text{if } 0 \leq x < 1 \\ x-2 & \text{if } 1 < x < 3 \\ x-4 & \text{if } 3 \leq x \leq 4 \end{cases}$$

possible at $x=1$, $x=3$

$\boxed{\text{Discontinuity at } x=1 \text{ and } x=3}$

$\lim\limits_{x\to1^-} f(x) = 1$

$\lim\limits_{x\to1^+} f(x) = -1$ ×

$\lim\limits_{x\to3^-} f(x) = 1$

$\lim\limits_{x\to3^+} f(x) = -1$ ×

11. Postal rates for a first-class letter up to 200g are given in the following chart.

Up to and Including	30g	50g	100g	200g
Mailing Cost	\$0.38	\$0.59	\$0.76	\$1.14

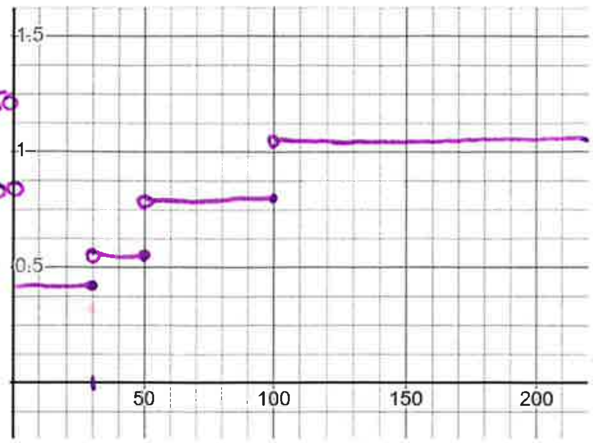
Draw the graph of the cost C, in dollars, of mailing a first-class letter as a function of its mass x, in grams. What are the discontinuities of the function?

$C(x) = 0.38$ if $x \leq 30$

$C(x) = 0.59$ if $30 < x \leq 50$

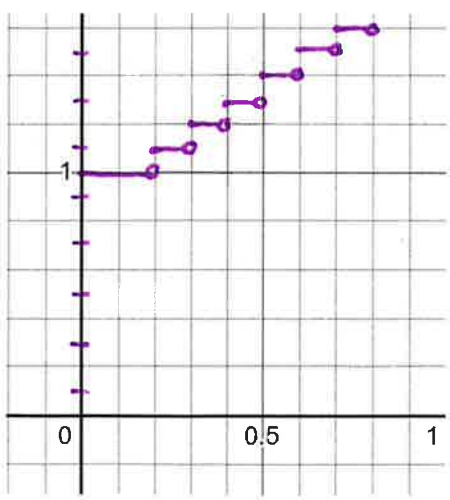
$C(x) = 0.76$ if $50 < x \leq 100$

$C(x) = 1.14$ if $100 < x \leq 200$



Discontinuities at:
0, 30, 50, and 100

12. A taxi company charges \$1.00 for the first 0.2km (or part of) and \$0.10 for each additional 0.1km (or part of). Draw the graph of the cost C of a taxi ride, in dollars, as a function of the distance travelled, x (in kilometers). Where are the discontinuities of this function?

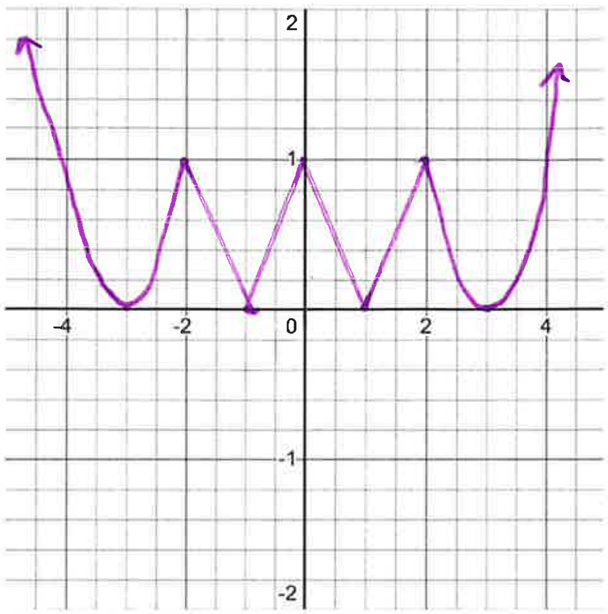


Discontinuity at
0.2, 0.3, 0.4, ...

13. Let

$$f(x) = \begin{cases} 1-|x| & \text{if } |x| \le 1 \\ |x|-1 & \text{if } 1<|x|\le 2 \\ (x-3)^2 & \text{if } x>2 \\ (x+3)^2 & \text{if } x<-2 \end{cases}$$

Sketch the graph of f and determine any values of x at which f is discontinuous.



f is continuous everywhere

14. For what values of the constant c is the function below continuous at every number?

$$f(x) = \begin{cases} x+c & \text{if } x<2 \\ cx^2+1 & \text{if } x\ge 2 \end{cases}$$

only possible discontinuity is a $x=2$

$$\left.\begin{array}{l} \lim_{x\to 2^-}(x+c) \\ \\ \lim_{x\to 2^+}(cx^2+1) \end{array}\right\} \text{have to be equal when } x=2$$

$x+c = cx^2+1$ when $x=2$

$2+c = c(2)^2+1$

$2+c = 4c+1 \longrightarrow 1 = 3c$ $\boxed{c=\frac{1}{3}}$