Exercise 1.2 – Practice Problems

1. Use the given graph of f to state the value of the limit, if it exists.

a) $\lim_{x \to 3} f(x)$

$\boxed{1}$

b) $\lim_{x \to 2} f(x)$

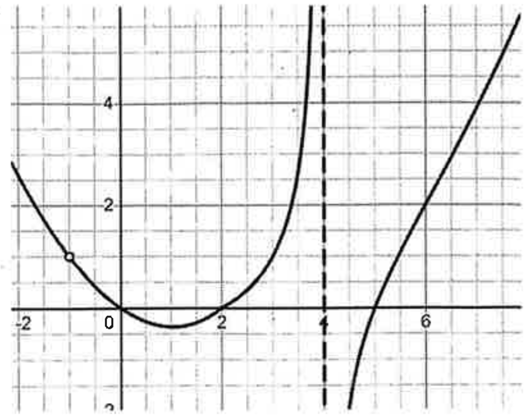
$\boxed{0}$

c) $\lim_{x \to -1} f(x)$

$\boxed{1}$

d) $\lim_{x \to 4} f(x)$

DNE



2. State the value of each limit

a) $\lim_{x \to 2} x^3$ $2^3 = \boxed{8}$

b) $\lim_{x \to \pi} x$ $\boxed{\pi}$

c) $\lim_{x \to 8} 3$ Since no variable. $\boxed{3}$

d) $\lim_{x \to 4} \sqrt{x}$ $\sqrt{4} = \boxed{2}$

e) $\lim_{x \to k} x^6$ $\boxed{k^6}$

f) $\lim_{x \to 0} \pi$ $\boxed{\pi}$

3. Use the properties of limits to evaluate the following.

a) $\lim_{x \to 1} (3x - 7)$

$\lim_{x \to 1} 3x - \lim_{x \to 1} 7$

$3 - 7$ $\boxed{-4}$

b) $\lim_{x \to -1} (2x^2 - 5x + 3)$

$\lim_{x \to -1} 2x^2 - \lim_{x \to -1} 5x + \lim_{x \to -1} 3$

$2(-1)^2 - 5(-1) + 3 \rightarrow 2 + 5 + 3 = \boxed{10}$

c) $\lim_{x \to 2} (x^3 + x^2 - 2x - 8)$

$\lim_{x \to 2} x^3 + \lim_{x \to 2} x^2 - \lim_{x \to 2} 2x - \lim_{x \to 2} 8$

$2^3 + 2^2 - 2(2) - 8$

$8 + 4 - 4 - 8 = \boxed{0}$

d) $\lim_{x \to -2} (x^2 + 5x + 3)^6$

$\left[\lim_{x \to -2} x^2 + \lim_{x \to -2} 5x + \lim_{x \to -2} 3\right]^6$

$\left[(-2)^2 + (5)(-2) + 3\right]^6 \rightarrow \left[4 - 10 + 3\right]^6$

$[-3]^6 = \boxed{729}$

e) $\lim_{x \to 0} \frac{(x-1)}{(x+1)}$

$$\frac{\lim_{x\to 0} x - \lim_{x\to 0} 1}{\lim_{x\to 0} x + \lim_{x\to 0} 1}$$

$$\frac{0-1}{0+1} = \boxed{-1}$$

f) $\lim_{x \to 4} \frac{(x^2+2x-3)}{(x^2+2)}$

$$\frac{\lim_{x\to 4} x^2 + \lim_{x\to 4} 2x - \lim_{x\to 4} 3}{\lim_{x\to 4} x^2 + \lim_{x\to 4} 2}$$

$$\frac{16+8-3}{16+2} \rightarrow \frac{21}{18} = \boxed{\frac{7}{6}}$$

g) $\lim_{t \to 2} \frac{(t^4-3t+1)}{t^2(t-1)^3}$

$$\frac{\lim_{t\to 2} t^4 - \lim_{t\to 2} 3t + \lim_{t\to 2} 1}{\lim_{t\to 2} t^2(\lim_{t\to 2} t - \lim_{t\to 2} 1)^3} \rightarrow \frac{16-6+1}{4(2-1)}$$

$$\boxed{\frac{11}{4}}$$

h) $\lim_{u \to -4} \sqrt{u^4+2u^2}$

$$\sqrt{\lim_{u\to -4} u^4 + 2\lim_{u\to -4} u^2}$$

$$\sqrt{(-4)^4+2(-4)^2} \rightarrow \sqrt{256+32} \rightarrow \sqrt{288}$$

$$\sqrt{2\cdot 144} = \boxed{12\sqrt{2}}$$

i) $\lim_{x \to 5} \sqrt[3]{x^2+2x-8}$

$$\sqrt[3]{\lim_{x\to 5} x^2 + 2\lim_{x\to 5} x - \lim_{x\to 5} 8} \rightarrow \sqrt[3]{25+10-8}$$

$$\boxed{3} \leftarrow \sqrt[3]{27}$$

j) $\lim_{t \to 3} \left(2t^2 + \sqrt{\frac{6+t}{4-t}}\right)$

$$\left(2\lim_{t\to 3} t^2 + \sqrt{\frac{\lim_{t\to 3} 6 + \lim_{t\to 3} t}{\lim_{t\to 3} 4 - \lim_{t\to 3} t}}\right)$$

$$2(9) + \sqrt{\frac{6+3}{4-3}} \rightarrow 18 + \sqrt{\frac{9}{1}} \rightarrow 18+3$$

$$\boxed{21}$$

4. Find the following limits. watch for $\frac{0}{0}$ scenarios

a) $\lim_{x \to -2} \frac{(x+2)}{(x^2-4)}$ DS gives $\frac{0}{0}$

$$\lim_{x\to -2} \frac{\cancel{(x+2)}}{\cancel{(x+2)}(x-2)} \rightarrow \lim_{x\to -2} \frac{1}{x-2}$$

$$\frac{1}{-2-2} = \boxed{\frac{1}{-4}}$$

b) $\lim_{x \to 1} \frac{(x^2-3x+2)}{(x-1)}$ DS gives $\frac{0}{0}$

$$\lim_{x\to 1} \frac{(x-2)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$\lim_{x\to 1} (x-2)$$

$$\boxed{-1}$$

c) $\lim_{x \to 3} \frac{(x^2-2x-3)}{(x^2-4x+3)}$ DS gives $\frac{0}{0}$

$$\lim_{x\to 3} \frac{\cancel{(x-3)}(x+1)}{\cancel{(x-3)}(x-1)}$$

$$\lim_{x\to 3} \frac{(x+1)}{(x-1)} \rightarrow \frac{4}{2} = \boxed{2}$$

d) $\lim_{x \to -2} \frac{(2x^2+5x+2)}{(x^2-2x-8)}$ DS gives $\frac{0}{0}$

$$\lim_{x\to -2} \frac{(2x+1)\cancel{(x+2)}}{\cancel{(x+2)}(x-4)}$$

$$\lim_{x\to -2} \frac{(2x+1)}{(x-4)} \rightarrow \frac{-3}{-6} = \boxed{\frac{1}{2}}$$

e) $\lim_{x \to 1} \frac{(x^3-1)}{(x^2-1)}$ DS gives $\frac{0}{0}$

$\lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+1)} \rightarrow \lim_{x\to 1} \frac{x^2+x+1}{x+1}$

$\boxed{\frac{3}{2}}$

f) $\lim_{x \to -3} \frac{(x+3)}{(x^3+27)}$ DS gives $\frac{0}{0}$

$\lim_{x\to -3} \frac{\cancel{(x+3)}}{\cancel{(x+3)}(x^2-3x+9)}$

$\lim_{x\to -3} \frac{1}{x^2-3x+9} \rightarrow \frac{1}{(-3)^2-3(-3)+9} = \boxed{\frac{1}{27}}$

g) $\lim_{x \to 9} \frac{(x-9)}{(\sqrt{x}-3)} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3}$

$\lim_{x\to 9} \frac{\cancel{(x-9)}(\sqrt{x}+3)}{\cancel{(x-9)}}$

$\lim_{x\to 9} \sqrt{x}+3 \rightarrow \boxed{6}$

h) $\lim_{x \to 2} \frac{\left(\frac{1}{x}-\frac{1}{2}\right)}{(x-2)}$ common denominator

$\frac{\frac{2-x}{2x}}{x-2} \rightarrow \frac{-1(x-2)}{2x} \div (x-2)$

$\lim_{x\to 2} \frac{-1}{2x} \leftarrow \frac{-1\cancel{(x-2)}}{2x} \cdot \frac{1}{\cancel{(x-2)}}$

$\boxed{\frac{-1}{4}}$

5. Evaluate the following.

a) $\lim_{h \to 0} \frac{(4+h)^3-64}{h}$

$\lim_{h\to 0} \frac{(4+h)(4+h)(4+h)-64}{h}$

$\lim_{h\to 0} \frac{h^3+12h^2+48h+64-64}{h}$

$\lim_{h\to 0} \frac{\cancel{h}(h^2+12h+48)}{\cancel{h}} \rightarrow 0^2+12(0)+48$

$\boxed{48}$

b) $\lim_{h \to 0} \frac{(h-2)^2-4}{h}$

$\lim_{h\to 0} \frac{h^2-4h+4-4}{h}$

$\lim_{h\to 0} \frac{h^2-4h}{h} \rightarrow \lim_{h\to 0} \frac{\cancel{h}(h-4)}{\cancel{h}}$

$\lim_{h\to 0} h-4 = \boxed{-4}$

c) $\lim_{h \to 0} \frac{\left(\frac{1}{1+h}-1\right)}{h}$

$\lim_{h\to 0} \frac{\frac{1-(1+h)}{1+h}}{h} \rightarrow \lim_{h\to 0} \frac{1-1-h}{(1+h)h}$

$\lim_{h\to 0} \frac{-\cancel{h}}{(1+h)\cancel{h}} \rightarrow \lim_{h\to 0} \frac{-1}{1+0}$

$\boxed{-1}$

d) $\lim_{h \to 0} \frac{\cancel{(x-9)}\ (2+h)^4-16}{h}$

$\lim_{h\to 0} \frac{(4+4h+h^2)(4+4h+h^2)-16}{h}$

$\lim_{h\to 0} \frac{h^4+8h^3+24h^2+32h\cancel{+16}\cancel{-16}}{h}$

$\lim_{h\to 0} \frac{\cancel{h}(h^3+8h^2+24h+32)}{\cancel{h}}$

$\boxed{32}$

e) $\lim_{h \to 0} \frac{(\sqrt{9+h}-3)}{h} \frac{(\sqrt{9+h}+3)}{(\sqrt{9+h}+3)}$

$\lim_{h \to 0} \frac{9+h-9}{h(\sqrt{9+h}+3)} \rightarrow \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h}+3)}$

$\lim_{h \to 0} \frac{1}{\sqrt{9+h}+3} = \boxed{\frac{1}{6}}$

f) $\lim_{h \to 0} \frac{\left(\frac{1}{(2+h)^2} - \frac{1}{4}\right)}{h} \rightarrow \frac{\frac{4-(2+h)^2}{4(2+h)^2}}{h}$

$\lim_{h \to 0} \frac{4-(4+4h+h^2)}{4h(2+h)^2} \rightarrow \lim_{h \to 0} \frac{\cancel{4}-\cancel{4}-4h-h^2}{4h(2+h)^2}$

$\lim_{h \to 0} \frac{-4h-h^2}{4h(2+h)^2} \rightarrow \frac{\cancel{h}(-4-h)}{4\cancel{h}(2+h)^2}$

$\lim_{h \to 0} \frac{-4-0}{4(2+0)^2} = \frac{-4}{16} = \boxed{-\frac{1}{4}}$

6. Find the following limits, if they exist.

a) $\lim_{x \to 3} \frac{1}{(x-3)^2}$

Nothing can be simplified

Limit Does Not Exist

b) $\lim_{x \to -8} \frac{x^2+16x+64}{(x+8)}$

$\lim_{x \to -8} \frac{\cancel{(x+8)}(x+8)}{\cancel{(x+8)}}$

$\lim_{x \to -8} (x+8) = \boxed{0}$

c) $\lim_{x \to 1} \frac{x^4-1}{x-1}$

$\lim_{x \to 1} \frac{(x^2-1)(x^2+1)}{(x-1)}$

$\lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}(x^2+1)}{\cancel{(x-1)}}$

$\lim_{x \to 1} (x+1)(x^2+1)$

$\lim_{x \to 1} (1+1)(1^2+1) = \boxed{4}$

d) $\lim_{x \to -1} \frac{x-1}{x^2-1}$

$\lim_{x \to -1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$

$\lim_{x \to -1} \frac{1}{(x+1)}$ * Didn't help

Limit Does Not Exist

e) $\lim\limits_{x\to 1}\frac{x^2+x-2}{x^2-2x+1}$

$\lim\limits_{x\to 1}\frac{(x+2)\cancel{(x-1)}}{(x-1)\cancel{(x-1)}}$

$\lim\limits_{x\to 1}\frac{(x+2)}{(x-1)}$

Limit DNE

f) $\lim\limits_{x\to -2}\frac{x^2-x-2}{x^2+3x+2}$

$\lim\limits_{x\to -2}\frac{(x-2)\cancel{(x+1)}}{(x+2)\cancel{(x+1)}}$

$\lim\limits_{x\to -2}\frac{(x-2)}{(x+2)}$

Limit DNE

g) $\lim\limits_{x\to 3}\frac{x^{-2}-3^{-2}}{x-3}$

$\lim\limits_{x\to 3}\frac{\frac{1}{x^2}-\frac{1}{3^2}}{x-3} \rightarrow \frac{\frac{9-x^2}{9x^2}}{x-3}$

$\lim\limits_{x\to 3}\frac{-1(x^2-9)}{9x^2(x-3)} \rightarrow \lim\limits_{x\to 3}\frac{-1(x+3)\cancel{(x-3)}}{9x^2\cancel{(x-3)}}$

$\lim\limits_{x\to 3}\frac{-1(x+3)}{9x^2} = -\frac{6}{81} = \boxed{\frac{-2}{27}}$

h) $\lim\limits_{x\to 4}\frac{\left(\frac{1}{\sqrt{x}}-\frac{1}{2}\right)}{(x-4)} \rightarrow \lim\limits_{x\to 4}\frac{\frac{2-\sqrt{x}}{2\sqrt{x}}}{(x-4)}$

$\lim\limits_{x\to 4}\frac{-1(\sqrt{x}-2)}{2\sqrt{x}(x-4)}$

still a diff of squares

$\lim\limits_{x\to 4}\frac{-1\cancel{(\sqrt{x}-2)}}{2\sqrt{x}(\sqrt{x}+2)\cancel{(\sqrt{x}-2)}}$

$\lim\limits_{x\to 4}\frac{-1}{2\sqrt{x}(\sqrt{x}+2)} = \boxed{-\frac{1}{16}}$

i) $\lim\limits_{x\to 1}\frac{x^3-1}{x^3-x^2-4x+4}$

$\lim\limits_{x\to 1}\frac{(x-1)(x^2+x+1)}{(x^3-x^2)(-4x+4)}$ factor by grouping

$\lim\limits_{x\to 1}\frac{(x-1)(x^2+x+1)}{x^2(x-1)-4(x-1)}$

$\lim\limits_{x\to 1}\frac{\cancel{(x-1)}(x^2+x+1)}{(x^2-4)\cancel{(x-1)}}$

$\lim\limits_{x\to 1}\frac{(x^2+x+1)}{(x^2-4)} = \frac{3}{-3} = \boxed{-1}$

j) $\lim\limits_{x\to 1}\frac{x-1}{\sqrt{x}-x}\cdot\frac{(\sqrt{x}+x)}{\sqrt{x}+x}$

$\lim\limits_{x\to 1}\frac{(x-1)(\sqrt{x}+x)}{x-x^2}$

$\lim\limits_{x\to 1}\frac{\cancel{(x-1)}(\sqrt{x}+x)}{-x\cancel{(x-1)}}$

$\lim\limits_{x\to 1}\frac{\sqrt{x}+x}{-x} = \frac{2}{-1} = \boxed{-2}$

7. a) Use your calculator to evaluate $f(x) = (1+x)^{\frac{1}{x}}$ correct to six decimals places for:

$x = 1, 0.1, 0.001, 0.000\,1, 0.000\,01, 0.000\,001, \textit{and } 0.000\,000\,1$

$f(1) = 2.000\,000$ $f(0.0001) = 2.718146$ $f(0.0000001) = 2.718282$

$f(0.1) = 2.593742$ $f(0.00001) = 2.718268$

$f(0.001) = 2.716924$ $f(0.000\,001) = 2.718280$

b) Estimate the value of the limit below to five decimal places:

$$\lim_{x\to 0}(1+x)^{\frac{1}{x}}$$

so $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}} \doteq 2.71828$

8. a) Use your calculator to evaluate $g(x) = \dfrac{2^x - 1}{x}$ correct to four decimals places for:

$x = 1, 0.1, 0.01, 0.001, 0.000\,1$

$g(1) = 1.0000$ $g(0.01) = 0.6956$ $g(0.0001) = 0.6932$

$g(0.1) = 0.7177$ $g(0.001) = 0.6934$

b) Estimate the value of the limit below to three decimal places:

$$\lim_{h\to 0}\frac{2^h - 1}{h}$$

so $\lim\limits_{h\to h}\dfrac{2^h - 1}{h}$ is $\doteq 0.693$

9. Evaluate the following limits

a) $\lim\limits_{x\to 8}\dfrac{x-8}{\sqrt[3]{x}-2}$ ← can be diff of cubes

$\lim\limits_{x\to 8}\dfrac{\cancel{(\sqrt[3]{x}-2)}(\sqrt[3]{x}^2 + 2\sqrt[3]{x} + 4)}{\cancel{(\sqrt[3]{x}-2)}} = 8^{2/3} + 2\sqrt[3]{8} + 4$

$= 4+4+4$

$= \boxed{12}$

Radicalize numerator

b) $\lim\limits_{x\to 2}\dfrac{\sqrt{6-x}-2}{\sqrt{3-x}-1} \cdot \dfrac{(\sqrt{6-x}+2)}{\sqrt{6-x}+2} \to \dfrac{6-x-4}{(\sqrt{3-x}-1)(\sqrt{6-x}+2)} \to \dfrac{2-x}{}$ → denom

← still gives 0 so radicalize denominator

$\dfrac{(2-x)}{(\sqrt{6-x}+2)(\sqrt{3-x}-1)} \cdot \dfrac{(\sqrt{3-x}+1)}{(\sqrt{3-x}+1)} \to \lim\limits_{x\to 2}\dfrac{(2-x)(\sqrt{3-x}+1)}{(\sqrt{6-x}+2)(3-x-1)} = \dfrac{\cancel{(2-x)}(\sqrt{3-x}+1)}{(\sqrt{6-x}+2)\cancel{(2-x)}} = \dfrac{2}{4}$

FOIL

$\boxed{\frac{1}{2}}$

10. If $f(x) = 2x + 3$, show that:

$$|f(x) - 7| < 0.01 \quad \text{if} \quad |x - 2| < 0.005$$

$$|2x+3-7| < 0.01$$
$$|2x-4| < 0.01$$
$$|2(x-2)| < 0.01$$
$$2|(x-2)| < 0.01 \rightarrow |(x-2)| < 0.05 \checkmark$$

11. How close to 1 do we have to take x so that $\frac{16x^2 - 1}{4x - 1}$ is within a distance of 0.001 from 5

$$\frac{16x^2-1}{4x-1} = \frac{(\cancel{4x-1})(4x+1)}{(\cancel{4x-1})} = (4x+1)$$

so we need

$$|4x+1-5| < 0.001$$
$$|4x-4| < 0.001$$
$$4|x-1| < 0.001$$
$$|x-1| < 0.00025$$

x must be within $\boxed{0.00025 \text{ of } 1}$

12. Show that the limit below Does Not Exist.

$$\lim_{x \to 0} \frac{|x|}{x}$$

If $x > 0$

then $\frac{|x|}{x} = 1$

since $|x| = x$

so as $x \to 0^+$ $\frac{|x|}{x} \to 1$

If $x < 0$

then $\frac{|x|}{x} = -1$

since $|x| = x$

$\frac{+}{-} = -1$

so as $x \to 0^-$ $\frac{|x|}{x} \to -1$

Since LHL $\neq$ RHL the Limit DNE

13. Find functions f and g such that $\lim_{x \to 0}[f(x) + g(x)]$ exists but $\lim_{x \to 0} f(x)$ and $\lim_{x \to 0} g(x)$ do not exist.

Many possible solutions

Take $f(x) = \frac{1}{x}$ and $g(x) = -\frac{1}{x}$ but $\lim_{x \to 0} \frac{1}{x}$ DNE

$$\lim_{x \to 0} \left[\frac{1}{x} + \frac{-1}{x}\right]$$

$$\lim_{x \to 0} -\frac{1}{x} \text{ DNE}$$

$$\lim_{x \to 0} [0] = 0$$