1.1 Linear Functions and the Tangent Line Problem

A ***linear function*** is a function f of the form

slope

$$f(x) = mx + b$$

y-int

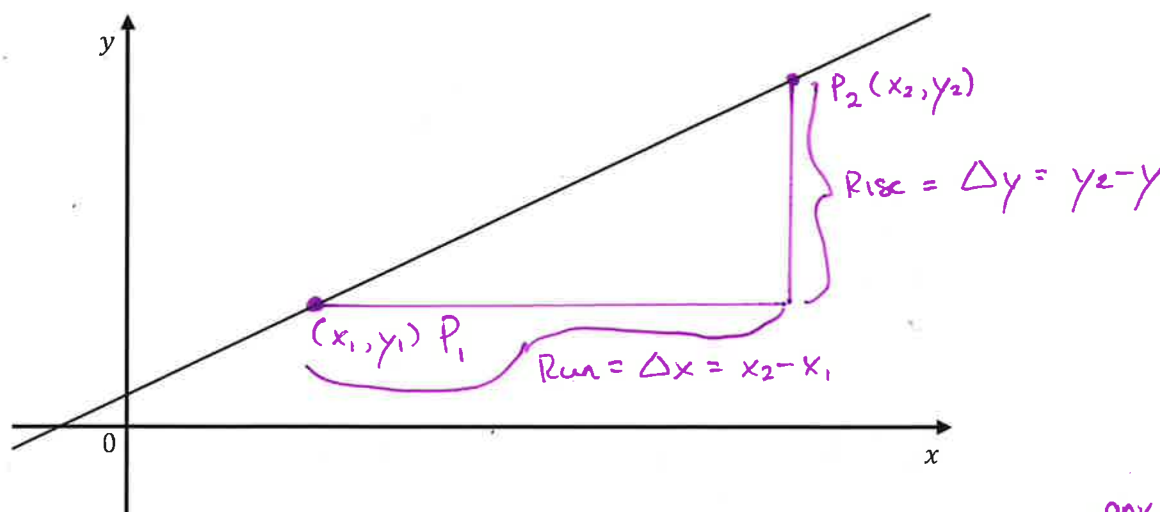
The graph has the form of the equation $y = mx + b$ which is the equation of a line in ***slope***-intercept form with slope m and y-intercept b.

Recall that for any line that is nonvertical, the ***slope*** of the line that passes through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is defined by

Greek Letter Delta means "change in"

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Because slope is a ratio of a change in y to a change in x, it can be interpreted as a **rate of change of y with respect to x.**



Recall that linear functions can also be written in *point-slope* form:

$$y - y_1 = m(x - x_1)$$

algebra → $m = \frac{y - y_1}{x - x_1}$

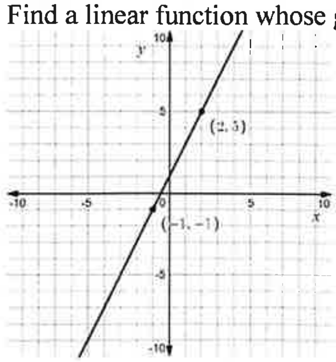
any point on the line

Ex. 1

Find a linear function whose graph passes through the points $(-1, -1)$ and $(2, 5)$.

call this point 1

call this point 2



Find m

$m = \frac{5-(-1)}{2-(-1)} = \frac{6}{3} = 2$

Find b

* use either point (2,5)

$y = mx + b$

$5 = 2(2) + b$

$5 = 4 + b$

$b = 1$

$y = 2x + 1$

* We use calculus to find the equation of the tangent line *

Ex. 2

A linear function is given by $y = 6 - 5x$. If x increases by 2, how does y change?

$\Delta x = 2$ $\Delta y = ?$

$m = \dfrac{\Delta y}{\Delta x}$ $\rightarrow$ $\Delta x\, m = \Delta y$

$2m = \Delta y$

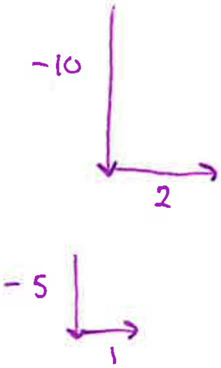
$2(-5) = \Delta y$

$\Delta y = -10$

$y = 6 - 5x$

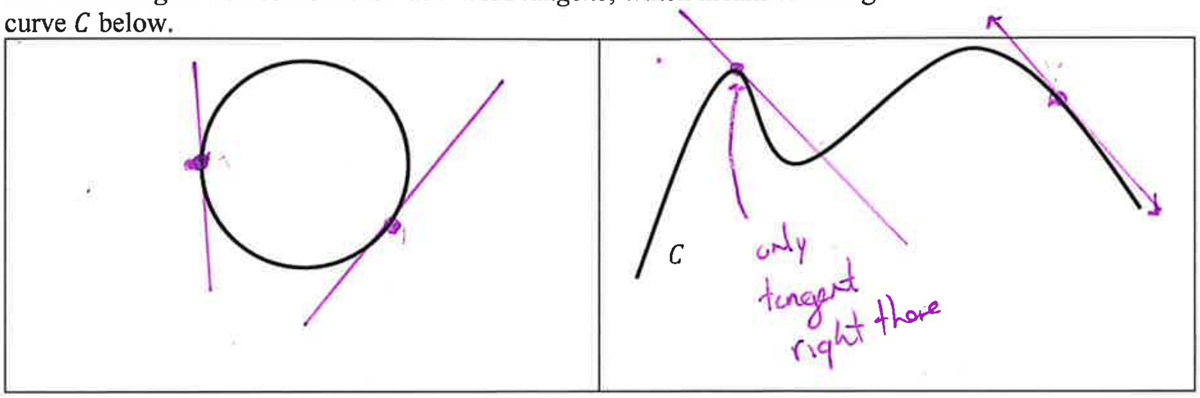
$y = -5x + 6$

m



The Tangent Problem

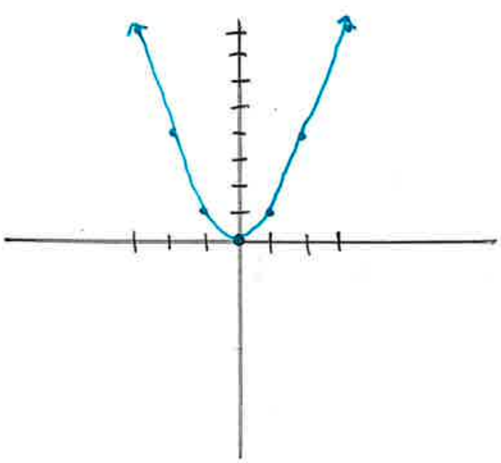
The word *tangent* comes from the Latin word *tangens*, which means touching. Consider the circle and the curve C below.



You can see that for the circle there are many tangent lines that can be drawn, but each tangent line touches the circle in only one place. For curve C however some of the tangent lines that can be drawn cross the curve in more than one place.

Ex. 3

Find the equation of a tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.



We need two points to calculate the slope. Choose $x = 1$ and $x = 1.1$ (close to 1)

when:

$x = 1$
$y = 1$
$(1, 1)$

$x = 1.1$
$y = 1.1^2$
$y = 1.21$
$(1.1, 1.21)$

$m = \dfrac{1.21 - 1}{1.1 - 1}$

$= \dfrac{0.21}{0.1}$

$m = 2.1$ ← this is an estimate!

To improve the estimate, calculate the slope using points that approach $x = 1$ from the *left* and the *right*.

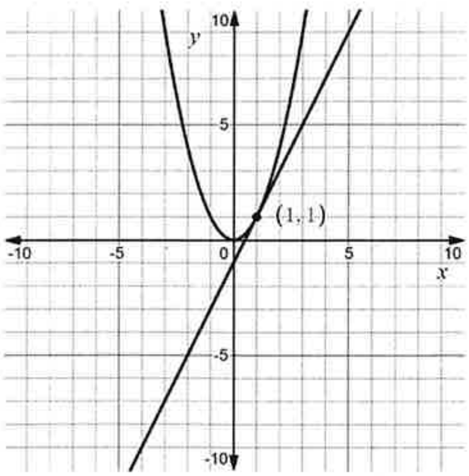
From the Left		**From the Right**	
$x < 1$	m	$x > 1$	m
0	1	2	3
0.5	1.5	1.5	2.5
0.9	1.9	1.1	2.1
0.99	1.99	1.01	2.01
0.999	1.999	1.001	2.001

Approaches 1

Approches 2

As we approach 1 from the left and right the slope approaches 2 from either direction.

- This suggests the tangent line slope at $x=1$ should be $m=2$



Now find b.

$y = 2x + b$ at $(1,1)$

$1 = 2(1) + b$

$1 = 2 + b$

$-1 = b$

The equation of the tangent line to the curve $y=x^2$ at point $(1,1)$ is:

$\boxed{y = 2x - 1}$

Homework Assignment
Exercise 1.1: #1 – 4, 6, 8, 9