Exercise 1.1 – Practice Problems

1. State the slopes of the given linear functions.

a) $y = 4x$

$\boxed{m = 4}$

b) $y = 3x - 5$

$\boxed{m = 3}$

c) $f(x) = \frac{1}{3}x - 2$

$\boxed{m = \frac{1}{3}}$

d) $f(x) = 2 - 3x$

$\boxed{m = -3}$

e) $f(x) = \frac{1}{2}(1 - x)$

$f(x) = \frac{1}{2} - \frac{1}{2}x$

$\boxed{m = -\frac{1}{2}}$

f) $x + 2y = 3$

$2y = -x + 3$

$y = -\frac{x}{2} + \frac{3}{2}$

$\boxed{m = -\frac{1}{2}}$

2. Find an equation of the line that passes through the points $(-3, 5)$ and $(4, -5)$

$m = \frac{-5-5}{4-(-3)} = \frac{-10}{7} = -\frac{10}{7}$

$y = -\frac{10}{7}x + b$

$-5 = -\frac{10}{7}(4) + b$

$-5 = -\frac{40}{7} + b$

$-35 = -40 + 7b$

$5 = 7b \qquad b = 5/7$

$\boxed{y = -\frac{10}{7}x + 5/7}$

3. Find a linear function whose graph passes through the points $(-4, -2)$ and $(2, 10)$

$\frac{10-(-2)}{2-(-4)} = \frac{12}{6} = 2$

$y = 2x + b$

$10 = 2(2) + b$

$10 = 4 + b$

$b = 6$

$\boxed{y = 2x + 6}$

4. A linear function is given by the equation $y = 16 + 3x$. How does y change if:

a) x increases by 4.

$m = \frac{\Delta y}{\Delta x} \qquad m = 3$

$3 = \frac{\Delta y}{4}$

$12 = \Delta y$

$\boxed{y \text{ increases by } 12}$

b) x decreases by 2

$m = \frac{\Delta y}{\Delta x} \qquad m = 3$

$3 = \frac{\Delta y}{-2}$

$-6 = \Delta y$

$\boxed{y \text{ decreases by } 6}$

5. A linear function is given by the equation: $y = \frac{1-x}{2}$. How does y change if:

$\frac{1-x}{2} = \frac{1}{2} - \frac{1}{2}x$

$m = -\frac{1}{2}$

a) x increase by 6

$m = \frac{\Delta y}{6}$

$-\frac{1}{2} = \frac{\Delta y}{6}$

$-3 = \Delta y$

$\boxed{y \text{ decreases by } 3}$

b) x decreases by 4

$-\frac{1}{2} = \frac{\Delta y}{-4}$

$2 = \Delta y$

$\boxed{y \text{ increases by } 2}$

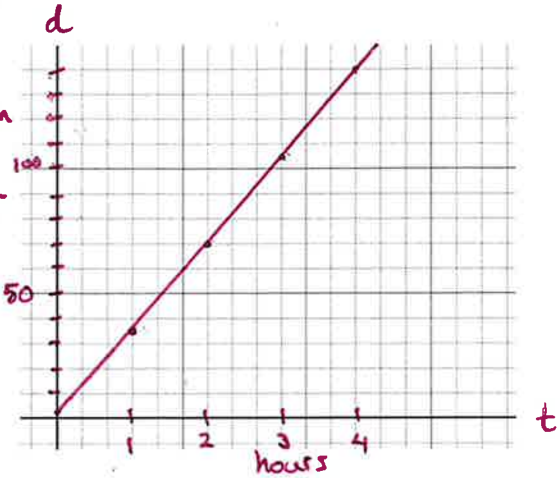
6. A car travels at a constant speed and covers 140km in 4 hours. If s represents the distance travelled (in kilometers) and t represents time elapsed (in hours), express s as a function of $t$and draw its graph. What does the slope of the line represent?

$s = \frac{d}{t}$ $\quad \frac{km}{h}$

so: $(0,0)$ $(4,140)$

$m = \frac{140-0}{4-0} = \frac{140}{4} = 35$

$\boxed{s = 35t}$



7. The point $P(1,3)$ lies on the curve $y = 2x + x^2$

a) If Q is the point $(x, 2x + x^2)$, find the slope of the secant line PQ for the following value of x

i) 2 ii) 1.5 iii) 1.1 iv) 1.01 v) 1.001
vi) 0 vii) 0.5 viii) 0.9 ix) 0.99 x) 0.999

i) $\frac{8-3}{2-1} = 5$

ii) $\frac{5.25-3}{1.5-1} = 4.5$

iii) $\frac{3.41-3}{1.1-1} = 4.1$

iv) $\frac{3.0401-3}{1.01-1} = 4.01$

v) $\frac{3.004001-3}{1.001-1} = 4.001$

vi) $\frac{0-3}{0-1} = 3$

vii) $\frac{1.25-3}{0.5-1} = 3.5$

viii) $\frac{2.61-3}{0.9-1} = 3.9$

ix) $\frac{2.9601-3}{0.99-1} = 3.99$

x) $\frac{2.996001-3}{0.999-1} = 3.999$

b) Using the results from part *a*), estimate the value of the slope of the tangent line to the curve at $P(1,3)$.

slope converges on 4 from either direction so:

at $(1,3)$ $m=4$

c) Using the slope from part *b*), find the equation of the tangent line to the curve at $P(1,3)$

$y=4x+b \rightarrow 3=4(1)+b$

$3=4+b$

$-1=b$

$\boxed{y=4x-1}$

d) Sketch the curve, two secant lines, and the tangent line

$y=2x+x^2$

$y=x(x+2)$

Roots: $(0,0)$ $(-2,0)$

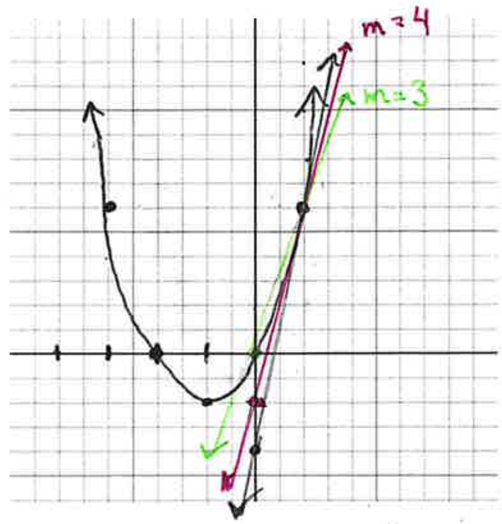
vertex: $(-1,-1)$

$m=3$ when $x=0$ $y=0$

$m=5$ when $x=2$ $y=8$

$8=5(2)+b$

$-2=b$



8. The point $P(2,0)$ lies on the curve $y=-x^2+6x-8$

a) If Q is the point $(\underset{x_2}{x}, \underbrace{-x^2+6x-8}_{y_2})$, find the slope of the secant line PQ for the following value of x

i) 3	ii) 2.5	iii) 2.1	iv) 2.01	v) 1
vi) 1.5	vii) 1.9	viii) 1.99		

i) $\frac{1-0}{3-2}=\frac{1}{1}=1$

ii) $\frac{0.75-0}{2.5-2}=1.5$

iii) $\frac{0.19-0}{2.1-2}=1.9$

iv) $\frac{0.0199-0}{2.01-2}=1.99$

v) $\frac{-3-0}{1-2}=3$

vi) $\frac{-1.25-0}{1.5-2}=2.5$

vii) $\frac{-0.21-0}{1.9-2}=2.1$

viii) $\frac{-0.0201}{1.99-2}=2.01$

b) Using the results from part a), estimate the value of the slope of the tangent line to the curve at $P(2,0)$.

From the left and right slopes converge at 2

so at $P(2,0)$ $m = 2$

c) Using the slope from part b), find the equation of the tangent line to the curve at $P(2,0)$

$y = 2x + b \rightarrow 0 = 2(2) + b$

$-4 = b$

$\boxed{y = 2x - 4}$

d) Sketch the curve, two secant lines, and the tangent line

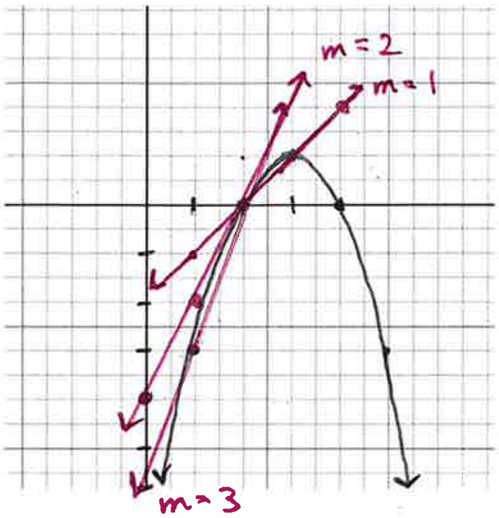
$-x^2 + 6x - 8 = -(x^2 - 6x + 8)$

$-(x-4)(x-2)$

vertex: $(3,1)$

at $(3,1)$ $m = 1$

at $(1,-3)$ $m = 3$



9. The point $P(1, \frac{1}{4})$ lies on the curve $y = \frac{1}{4}x^3$

a) If Q is the point $\left(x, \frac{1}{4}x^3\right)$, find the slope of the secant line PQ for the following value of x

i) 2	ii) 1.5	iii) 1.1	iv) 1.01	v) 1.001
vi) 0	vii) 0.5	viii) 0.9	ix) 0.99	x) 0.999

i) $\frac{2 - \frac{1}{4}}{2 - 1} = 1.75$

ii) $\frac{0.84375 - \frac{1}{4}}{1.5 - 1} = 1.1875$

iii) $\frac{0.33275 - \frac{1}{4}}{1.1 - 1} = 0.8275$

iv) $\frac{0.257... - \frac{1}{4}}{1.01 - 1} = 0.757525$

You got it now!

v) 0.75075

vi) 0.25

vii) 0.4375

viii) 0.6775

ix) 0.742525

x) 0.74925

b) Using the results from part *a*), estimate the value of the slope of the tangent line to the curve at $P\left(1, \frac{1}{4}\right)$.

We can see that as x approaches 1 from the left and right m approaches 0.75 or 3/4

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c) Using the slope from part *b*), find the equation of the tangent line to the curve at $P(1, \frac{1}{4})$

$y = \frac{3}{4}x + b$ $\quad \frac{1}{4} = \frac{3}{4}(1) + b$ $\quad b = -\frac{1}{2}$ $\quad \boxed{y = \frac{3}{4}x - \frac{1}{2}}$

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d) Sketch the curve, two secant lines, and the tangent line

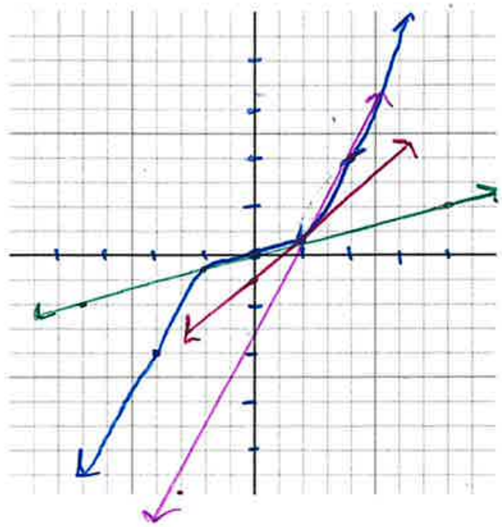
$y = \frac{1}{4}x^3$

$x = 0 \quad y = 0$

$m = \frac{1}{4}$

$x = 2 \quad y = 2$

$m = \frac{7}{4}$



10. The point $P(0.5, 2)$ lies on the curve $y = \frac{1}{x}$

a) If Q is the point $\left(x, \frac{1}{x}\right)$, find the slope of the secant line PQ for the following value of x

i) 2	ii) 1	iii) 0.9	iv) 0.8	v) 0.7
vi) 0.6	vii) 0.55	viii) 0.51	ix) 0.45	x) 0.49

i) $\dfrac{\frac{1}{2} - 2}{2 - 0.5} = -1$

ii) $m = -2$

iii) $m = -2.222$

iv) $m = -2.5$

v) $m = -2.8571$

vi) $m = -3.333$

vii) $m = -3.6364$

viii) -3.9121

ix) -4.444

x) -4.0816

b) Using the results from part a), estimate the value of the slope of the tangent line to the curve at $P(0.5, 2)$.

As x approaches 0.5 from either side m approaches −4

c) Using the slope from part b), find the equation of the tangent line to the curve at $P(0.5, 2)$

$y = -4x + b$ $\quad 2 = -4(0.5) + b \quad 2 = -2 + b \quad b = 4$ $\quad \boxed{y = -4x + 4}$

d) Sketch the curve, two secant lines, and the tangent line

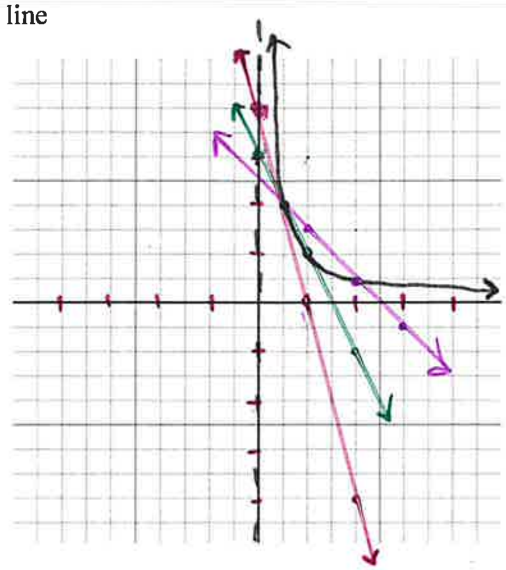
$y = \frac{1}{x} \quad x \neq 0$

$x = 1 \quad y = 1$

$m = -2$

$x = 2 \quad y = \frac{1}{2}$

$m = -1$



11. As dry air moves upward, it expands and in so doing cools at a rate of around 1°C for each 100m rise, up to about 12km.

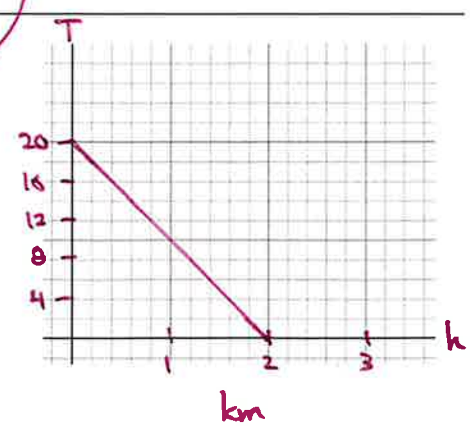
a) If the ground temperature is 20°C, find an expression for the temperature T as a function of the height h.

$m = \frac{\Delta T}{\Delta h} = \frac{-1^\circ}{100m}$ $\quad T = -\frac{1}{100}h + 20$ (h in meters)

b) Sketch the graph of T. What does the slope represent?

$T = 20 - 10h$ (when h is km)

The slope represents the rate change in temperature as altitude changes



12. The monthly cost of owning a car depends on the number of kilometers driven. Jeff Davidson found that in May it cost him $500 to drive 800km and in June it cost him $650 to drive 1400km.

a) Express the monthly cost C as a function of distance driven d, assuming that a linear function is a suitable model.

$\boxed{C = \frac{1}{4}d + 300}$

Looking for $/km Have (800, 500) and (1400, 650)

$m = \frac{500-650}{800-1400} = \frac{-150}{-600} = \frac{1}{4}$

$C = \frac{1}{4}d + b$

$650 = \frac{1}{4}(1400) + b$

$650 = 350 + b$ $b = 300$

b) Use this function to predict the cost of driving 2000km per month.

$C = \frac{1}{4}d + 300$ when $d = 2000$

$C = \frac{1}{4}(2000) + 300 \rightarrow 500 + 300$

$\boxed{C = \$800}$

c) What does the slope of the function represent?

$\frac{\$1}{4\text{km}} = \$0.25/\text{km}$

The slope represents the cost per kilometer of driving the car.

d) What is the monthly cost if she does not drive her car at all? Is this reasonable?

If $d = 0$ then $\boxed{C = \$300}$

Reasonable yup.

Insurance, maintenance, parking, etc.

e) Why is a linear function a suitable model for this situation?

Because we have fixed expenses to operate vehicle and then a cost per kilometer.