Section 11.4 – Practice Problems

1. Evaluate the following indefinite integrals

a)

$$\int x\cos x\ dx$$

let $x = u$ $dv = \cos x\,dx$
$dx = du$ $v = \sin x$

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$$

$$\Rightarrow \boxed{x\sin x + \cos x + C}$$

b)

$$\int x\,e^{2x}\,dx$$

let $u = x$ $dv = e^{2x}dx$
$du = dx$ $v = \frac{1}{2}e^{2x}$

$$\int u\,dv = uv - \int v\,du$$

$$\int xe^{2x}dx = x\cdot\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}dx$$

$$= \boxed{\frac{1}{2}x\cdot e^{2x} - \frac{1}{4}e^{2x} + C}$$

c)

$$\int x\ln x\ dx$$

let $u = \ln x$ $dv = x\,dx$
$du = \frac{1}{x}dx$ $v = \frac{1}{2}x^2$

$$\int x\ln x\,dx = \ln x\cdot\frac{1}{2}x^2 - \int \frac{1}{2}x^2\cdot\frac{1}{x}dx$$

$$= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,dx$$

$$= \frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx$$

$$= \frac{1}{2}x^2\ln x - \frac{1}{2}\cdot\frac{1}{2}x^2 + C$$

$$= \boxed{\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C}$$

d)

$$\int t\sec^2 t\ dt$$

let $u = t$ $dv = \sec^2 t\,dt$
$du = dt$ $v = \tan t$

$$\int t\sec^2 t\,dt = t\tan t - \int \tan t\,dt$$

$$\downarrow$$

$$\int \frac{\sin t}{\cos t}dt$$

let $\cos t = u$
$-\sin t\,dt = du$

$$- \int \frac{du}{u}$$

$$\Rightarrow t\tan t - (-1)\int \frac{du}{u}$$

$$\rightarrow t\tan t + \ln u$$

$$\boxed{t\tan t + \ln(\cos t) + C}$$

$\int u\,dv = uv - \int v\,du$

2. Evaluate the following definite integrals

a)

$$\int_0^{\pi} x\sin x\ dx$$

let $u = x$ $\quad dv = \sin x\,dx$

$du = dx$ $\quad v = -\cos x$

$\int x\sin x\,dx = [x(-\cos x)]_0^{\pi} - \int_0^{\pi} -\cos x\,dx$

$= [-x\cos x]_0^{\pi} + \sin x]_0^{\pi}$

$\rightarrow [-\pi\cos\pi - (0\cos 0)] - [\sin\pi - \sin 0]$

$-\pi(-1) - 0 + [0-0]$

$\boxed{\pi}$

b)

$$\int_0^1 xe^{-x}\,dx$$

let $u = x$

$du = dx$

$dv = e^{-x}dx$

$v = -e^{-x}$

$\int_0^1 xe^{-x}dx = x(-e^{-x}) - \int -e^{-x}dx$

$= [-xe^{-x}]_0^1 + \int e^{-x}dx$

$= -1e^{-1} - 0 + -e^{-x}]_0^1$

$= \frac{-1}{e} - \left[\frac{1}{e} - 1\right]$

$-\frac{1}{e} - \frac{1}{e} + 1 \quad = \boxed{1 - \frac{2}{e}}$

c)

$$\int_1^2 x^4\ln x\,dx$$

let $u = \ln x$ $\quad dv = x^4dx$

$du = \frac{1}{x}dx$ $\quad v = \frac{1}{5}x^5$

$\int x^4\ln x\,dx = \left[\frac{1}{5}x^5\ln x\right]_1^2 - \int_1^2 \frac{1}{5}x^5\cdot\frac{1}{x}dx$

$= \left[\frac{1}{5}x^5\ln x\right]_1^2 - \frac{1}{5}\int_1^2 x^4dx$

$\rightarrow \left(\frac{1}{5}x^5\ln x\right)_1^2 - \frac{1}{5}\cdot\frac{1}{5}x^5\Big]_1^2$

$\frac{32}{5}\ln 2 - \frac{1}{5}\cancel{\ln 1}^{0} - \frac{1}{25}(32-1)$

$\boxed{\frac{32}{5}\ln 2 - \frac{31}{25}}$

d)

$$\int_0^{2\pi} x^2\cos x\ dx$$

let $u = x^2$

$du = 2x\,dx$

$dv = \cos x\,dx$

$v = \sin x$

$\int x^2\cos x\,dx = [x^2\sin x]_0^{2\pi} - \int_0^{2\pi}\sin x\ 2x\,dx$

let $u = 2x$ $\leftarrow$ $\uparrow$ have to do this again

$du = 2dx$

$dv = \sin x\,dx$

$v = -\cos x$

$[x^2\sin x]_0^{2\pi} - [[2x(-\cos x)]_0^{2\pi} - \int_0^{2\pi} -\cos x\cdot 2dx$

$[x^2\sin x]_0^{2\pi} - [-2x\cos x]_0^{2\pi} + 2\int_0^{2\pi}\cos x\,dx$

$[x^2\sin x]_0^{2\pi} - [-2x\cos x]_0^{2\pi} + 2\sin x]_0^{2\pi}$

$[4\pi^2\sin 2\pi - 0] - [-4\pi\cos 2\pi - 0] + 2[\sin 2\pi - \sin 0]$

$0 - (-4\pi) + 2[0] \quad = \boxed{4\pi}$

3. Evaluate the following integral by making the substitution $t = \sqrt{x}$ and then integrating by parts.

$$\int e^{\sqrt{x}}\,dx$$

if $t = \sqrt{x}$

$dt = \frac{1}{2\sqrt{x}}\,dx$

$2\,dt = \frac{1}{\sqrt{x}}\,dx$

$2\,dt = \frac{1}{t}\,dx$

$2t\,dt = dx$

$\int e^{\sqrt{x}}\,dx = \int e^{t}\cdot 2t\,dt$

$\rightarrow \int 2te^{t}\,dt$

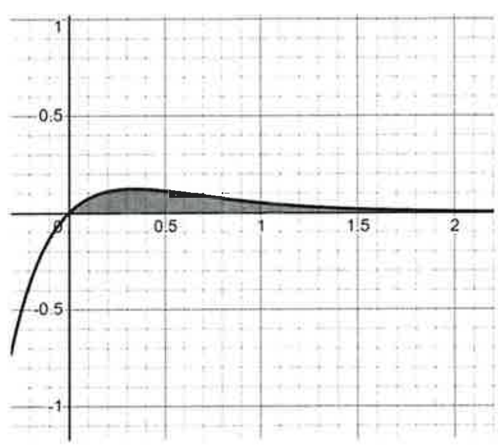
let $u = 2t$ $\quad dv = e^{t}\,dt$
$\quad du = 2\,dt$ $\quad v = e^{t}$

$\int 2te^{t}\,dt = 2te^{t} - \int e^{t}\cdot 2\,dt$

$= 2te^{t} - 2\int e^{t}\,dt \longrightarrow 2te^{t} - 2e^{t}$

$2e^{t}(t-1) \rightarrow \boxed{2e^{\sqrt{x}}(\sqrt{x}-1)+C}$

4. Find the area under curve $y = xe^{-3x}$ from 0 to 2



$$\int_0^2 xe^{-3x}\,dx$$

let $u = x$

$du = dx$

$dv = e^{-3x}\,dx$

$v = -\frac{1}{3}e^{-3x}$

$\int_0^2 xe^{-3x}\,dx = x\left(-\frac{1}{3}e^{-3x}\right) - \int_0^2 -\frac{1}{3}e^{-3x}\,dx$

$= \left[-\frac{1}{3}xe^{-3x}\right]_0^2 + \frac{1}{3}\int_0^2 e^{-3x}\,dx$

$= \left[-\frac{2}{3}e^{-6} - 0\right] + \frac{1}{3}\cdot\left[-\frac{1}{3}e^{-3x}\right]_0^2$

$-\frac{2}{3e^{6}} - \frac{1}{9}\left[e^{-3x}\right]_0^2$

$-\frac{2}{3e^{6}} - \frac{1}{9}(e^{-6} - e^{0})$

$-\frac{2}{3e^{6}} - \frac{1}{9}(e^{-6} - 1)$

$-\frac{2e^{-6}}{3} - \frac{e^{-6}}{9} + \frac{1}{9}$

$-\frac{6e^{-6}}{9} - \frac{e^{-6}}{9} + \frac{1}{9}$

$\frac{1}{9} - \frac{7e^{-6}}{9}$

$\boxed{\frac{1}{9}(1 - 7e^{-6})}$