Section 11.3 – Practice Problems

1. Suggest and appropriate substitution for each integral

a)

$$\int \sin(x^2)2x\,dx$$

let $u = x^2$

$du = 2x\,dx$

b)

$$\int \frac{(\ln x)^2}{x}\,dx$$

let $u = \ln x$

$du = \frac{1}{x}dx$

c)

$$\int \cos 5x\;dx$$

let $u = 5x$

$du = 5dx$

d)

$$\sqrt{\sin x}\cos x\,dx$$

let $u = \sin x$

$du = \cos x\,dx$

2. Evaluate each integral by making the given substitution

a)

$$\int x(1-x^2)^{10}dx,\;\; u = 1 - x^2$$

then $du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$\int u^{10}\left(-\frac{1}{2}\right)du$

$-\frac{1}{2}\int u^{10}du \rightarrow -\frac{1}{2}\cdot\frac{1}{11}u^{11} \rightarrow -\frac{1}{22}u^{11} + C \rightarrow \boxed{-\frac{1}{22}(1-x^2)^{11} + C}$

b)

$$\int e^{5x}\,dx,\;\; u = 5x$$

then $du = 5dx$

$\frac{1}{5}du = dx$

$\int e^u \frac{1}{5}du \rightarrow \frac{1}{5}\int e^u du \rightarrow \frac{1}{5}e^u + C \rightarrow \boxed{\frac{1}{5}e^{5x} + C}$

c)

$$\int \sqrt{x-1}\,dx, \quad u = x-1$$

$du = 1\,dx$

$\int \sqrt{u}\,du \rightarrow \frac{2}{3}u^{3/2} + C \rightarrow \boxed{\frac{2}{3}(x-1)^{3/2} + C}$

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d)

$$\int \frac{x+1}{x^2+2x-6}\,dx, \quad u = x^2+2x-6$$

$du = 2x+2\,dx$

$\frac{1}{2}du = x+1\,dx$

$\frac{1}{2}\int \frac{1}{u}du \rightarrow \frac{1}{2}\ln u + C \longrightarrow \boxed{\frac{1}{2}\ln|x^2+2x-6| + C}$

3. Evaluate the following indefinite integrals

a)

$$\int x(x^2+4)^8\,dx$$

let $x^2+4 = u$

$2x\,dx = du \rightarrow x\,dx = \frac{1}{2}du$

$\frac{1}{2}\int u^8\,du \rightarrow \frac{1}{2}\cdot\frac{1}{9}u^9 + C$

$\frac{1}{18}u^9 + C \rightarrow \boxed{\frac{1}{18}(x^2+4)^9 + C}$

b)

$$\int x^2\sqrt{x^3+2}\,dx$$

let $x^3+2 = u$

$3x^2\,dx = du$

$x^2\,dx = \frac{1}{3}du$

$\frac{1}{3}\int u^{\frac{1}{2}}du$

$\frac{1}{3}\cdot\frac{2}{3}u^{3/2} \rightarrow \frac{2}{9}u^{3/2} + C \rightarrow \boxed{\frac{2}{9}(x^3+2)^{3/2} + C}$

c)

$$\int (x+6)^{10}\,dx$$

let $x+6 = u$

$dx = du$

$\int u^{10}du \rightarrow \frac{1}{11}u^{11} + C$

$\boxed{\frac{1}{11}(x+6)^{11} + C}$

d)

$$\int \frac{1}{(3x-1)^2}\,dx$$

let $3x-1 = u$

$3dx = du$

$dx = \frac{1}{3}du$

$\int \frac{1}{u^2}\cdot\frac{1}{3}du \rightarrow \frac{1}{3}\int\frac{1}{u^2}du$

$\frac{1}{3}\left(-u^{-1}\right) + C$

$-\frac{1}{3u} + C \rightarrow \boxed{\frac{-1}{3(3x-1)} + C}$

e)

$$\int \sec^2 3x\, dx$$

let $3x = u$

$3\,dx = du$

$dx = \frac{1}{3}du$

$\frac{1}{3}\int \sec^2 u\, du$

$\frac{1}{3}\tan u + C \rightarrow \boxed{\frac{1}{3}\tan 3x + C}$

f)

$$\int (1 + 2x^4)x^3\, dx$$

let $1+2x^4 = u$

$8x^3 dx = du$

$x^3 dx = \frac{1}{8}du$

$\frac{1}{8}\int u\, du \rightarrow \frac{1}{8}\cdot\frac{1}{2}u^2 + C$

$\frac{1}{16}u^2 + C \rightarrow \frac{1}{16}(1+2x^4)^2 + C$

$\boxed{\frac{(1+2x^4)^2}{16} + C}$

g)

$$\int \sin^2 x \cos x\; dx$$

let $u = \sin x$

$du = \cos x\, dx$

$\int u^2 du$

$\frac{1}{3}u^3 + C \rightarrow \boxed{\frac{1}{3}\sin^3 x + C}$

h)

$$\int \frac{\sqrt{\ln x}}{x}\, dx$$

let $u = \ln x$

$du = \frac{1}{x}dx$

$\int \sqrt{u}\, du \rightarrow \frac{2}{3}u^{3/2} + C$

$\boxed{\frac{2}{3}(\ln|x|)^{3/2} + C}$

i)

$$\int t^2 e^{t^3} dt$$

let $u = t^3$

$du = 3t^2 dt$

$\frac{1}{3}du = t^2 dt$

$\frac{1}{3}\int e^u du$

$\frac{1}{3}e^u \rightarrow \boxed{\frac{1}{3}e^{t^3} + C}$

j)

$$\int \frac{1}{1 - x}\, dx$$

let $1 - x = u$

$-1dx = du$

$dx = -du$

$-\int \frac{1}{u}du$

$-\ln u$

$\boxed{-\ln|1-x| + C}$

k) $\int \frac{(3x^2-2)}{(x^3-2x+1)^3}dx$

Let $x^3-2x+1=u$

$3x^2-2\,dx=du$

$\int \frac{1}{u^3}du$

$-\frac{1}{2}u^{-2}+C \rightarrow -\frac{1}{2u^2}+C$

$\boxed{-\frac{1}{2(x^3-2x+1)^2}+C}$

l) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

Let $u=\sqrt{x}$

$du=\frac{1}{2\sqrt{x}}dx$

$2du=\frac{1}{\sqrt{x}}dx$

$2\int \sin u\,du$

$2(-\cos u)+C$

$\boxed{-2\cos\sqrt{x}+C}$

m) $\int e^{3-x}dx$

Let $u=3-x$

$du=-1\,dx$

$-du=dx$

$-\int e^u$

$-e^u+C$

$\boxed{-e^{3-x}+C}$

n) $\int e^{\cos x}\sin x\,dx$

Let $u=\cos x$

$du=-\sin x\,dx$

$-du=\sin x\,dx$

$-\int e^u du$

$-e^u+C$

$\boxed{-e^{\cos x}+C}$

o) $\int \sqrt{1+\tan x}\sec^2 x\;dx$

Let $1+\tan x=u$

$\sec^2 x\,dx=du$

$\int \sqrt{u}\,du$

$\frac{2}{3}u^{3/2}+C$

$\boxed{\frac{2}{3}(1+\tan x)^{3/2}+C}$

p) $\int x\sin(x^2)dx$

Let $u=x^2$

$du=2x\,dx$

$\frac{1}{2}du=x\,dx$

$\frac{1}{2}\int \sin u\,du$

$\frac{1}{2}(-\cos u)+C$

$\boxed{-\frac{1}{2}\cos x^2+C}$

4. Evaluate the following definite integrals

a)

$$\int_0^1 e^{2x+1}dx$$

let $u = 2x+1$

$du = 2dx$

$\frac{1}{2}du = dx$

boundaries

$u = 2(1)+1 = 3$

and

$u = 2(0)+1 = 1$

$\frac{1}{2}\int_1^3 e^u du$

$\frac{1}{2}e^u\Big]_1^3 \rightarrow \frac{1}{2}(e^3 - e^1)$

$\boxed{\frac{e(e^2-1)}{2}}$

b)

$$\int_0^2 \frac{1}{(1+5x)^4}dx$$

let $1+5x = u$

$5dx = du$

$dx = \frac{1}{5}du$

Bound

$11 \downarrow 1$

$\frac{1}{5}\int_1^{11} \frac{1}{u^4}du$

$\frac{1}{5}\cdot\frac{-1}{3}u^{-3}\Big]_1^{11}$

$-\frac{1}{15}\left(11^{-3} - 1^{-3}\right)$

$-\frac{1}{15}\left(\frac{1}{11^3} - 1\right) = \boxed{\frac{266}{3993}}$

c)

$$\int_0^2 x\sqrt{(4-x^2)}dx$$

let $u = 4-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

Bound

$0 \downarrow 4$

$-\frac{1}{2}\int_4^0 u^{1/2}du \rightarrow -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\Big]_4^0$

$-\frac{1}{3}\left[0 - 4^{3/2}\right]$

$-\frac{1}{3}[0-8]$

$\boxed{8/3}$

d)

$$\int_0^1 \sin \pi t\, dt$$

Bound

$\pi \downarrow 0$

let $\pi t = u$

$\pi dt = du$

$dt = \frac{1}{\pi}du$

$\frac{1}{\pi}\int_0^{\pi} \sin u\, du$

$\frac{1}{\pi}(-\cos u)_0^{\pi}$

$-\frac{1}{\pi}(\cos\pi - \cos 0)$

$-\frac{1}{\pi}(-1-1)$

$\boxed{\frac{2}{\pi}}$

e)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos\theta}{\sin^3\theta}\,d\theta$$

Let $\sin\theta = u$

$\cos\theta\, d\theta = du$

Bound

1
$\downarrow$
$\frac{1}{2}$

$$\int_{\frac{1}{2}}^{1} \frac{1}{u^3}\,du \rightarrow \int_{\frac{1}{2}}^{1} u^{-3}\,du$$

$$-\frac{1}{2}u^{-2}\Big]_{\frac{1}{2}}^{1}$$

$$-\frac{1}{2}\left(\frac{1}{1^2} - \frac{1}{(\frac{1}{2})^2}\right) \rightarrow -\frac{1}{2}(1-4)$$

$$\boxed{\frac{3}{2}}$$

f)

$$\int_0^1 x^4(x^5+1)^5\,dx$$

Bound

2
$\downarrow$
1

Let $x^5+1 = u$

$5x^4\,dx = du$

$x^4\,dx = \frac{1}{5}du$

$$\frac{1}{5}\int_1^2 u^5\,du$$

$$\downarrow$$

$$\frac{1}{5}\cdot\frac{1}{6}u^6\Big]_1^2$$

$$\frac{1}{5}\cdot\frac{1}{6}\left(2^6 - 1^6\right)$$

$$\frac{1}{30}(64-1) \Rightarrow \frac{63}{30} = \frac{21}{10} = \boxed{2.1}$$

g)

$$\int_{\frac{1}{2}}^{1} \frac{\left(1+\frac{1}{x}\right)^5}{x^2}\,dx$$

Let $u = 1 + x^{-1}$

$du = -x^{-2}\,dx$

$-du = x^{-2}\,dx$

$-du = \frac{1}{x^2}dx$

Bound

2
$|$
3

$$-\int_3^2 u^5\,du$$

$$-\frac{1}{6}u^6\Big]_3^2$$

$$-\frac{1}{6}\left[2^6 - 3^6\right]$$

$$-\frac{1}{6}[64-729] = \boxed{\frac{665}{6}}$$

h)

$$\int_1^2 (x+1)e^{(3x^2+6x-4)}\,dx$$

Let $3x^2+6x-4 = u$

$6x+6\,dx = du$

$x+1\,dx = \frac{1}{6}du$

Bound

20
$\downarrow$
5

$$\frac{1}{6}\int_5^{20} e^u\,du$$

$$\frac{1}{6}\left[e^u\right]_5^{20}$$

$$\boxed{\frac{1}{6}\left(e^{20} - e^5\right)}$$

5. Find

a) $\int \tan x \, dx$

$\int \frac{\sin x}{\cos x} dx$

let $u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$-\int \frac{1}{u} du$

$-\ln|u| + C$

$\boxed{-\ln|\cos x| + C}$

b) $\int \cot x \, dx$

$\int \frac{\cos x}{\sin x} dx$

let $\sin x = u$

$\cos x \, dx = du$

$\int \frac{1}{u} du \rightarrow \ln|u| + C$

$\boxed{\ln|\sin x| + C}$

6. Find the area under the curve $y = \sqrt{4x+1}$ from 0 to 10.

$y = \sqrt{4x+1}$ is ≥ 0 on $[0,10]$ so area is under the curve and above the x-axis

let $u = 4x+1$

$du = 4dx$

$\frac{1}{4} du = dx$

$u = 4(10)+1$

$u = 41$

$u = 4(0)+1$

$= 1$

$A = \int f(x)$

$= \int_0^{10} \frac{1}{4}\sqrt{u} \, du$

$\frac{1}{4}\int_0^{10} \sqrt{u} \, du$

$\frac{1}{4}\left[\frac{2}{3}u^{3/2}\right]_1^{41}$

$\frac{1}{4}\left[\frac{2}{3}(41)^{3/2} - \frac{2}{3}(1)^{3/2}\right]$

$\frac{2}{12}\left[41^{3/2} - 1\right]$

$\boxed{\frac{1}{6}(41^{3/2} - 1)}$