Section 11.2 – Practice Problems

1. Evaluate the following definite integrals.

a) $\int_{-6}^{7} 2dx$ → $2x\Big]_{-6}^{7}$

$2(7)-2(-6)$

$14+12$

26

b) $\int_{-1}^{5} (6x-7)dx$

$3x^2-7x\Big]_{-1}^{5}$

$3(5)^2-7(5)-[3(-1)^2-7(-1)]$

$75-35-[3+7]$

$40-10=$ 30

c) $\int_{1}^{2} (5+4x-6x^2)dx$

$5x+2x^2-2x^3\Big]_{1}^{2}$

$5(2)+2(4)-2(8)-[5+2-2]$

$10+8-16-5$

-3

d) $\int_{0}^{1} (t^2+6t-1)dt$ → $\frac{1}{3}t^3+3t^2-t\Big]_{0}^{1}$

$\frac{1}{3}+3-1-0$

$\frac{1}{3}+\frac{9}{3}-\frac{3}{3}$

$\frac{7}{3}$

e) $\int_{-1}^{2} (x^3-x^2+4x)dx$

$\frac{1}{4}x^4-\frac{1}{3}x^3+2x^2\Big]_{-1}^{2}$

$\frac{16}{4}-\frac{8}{3}+8-[\frac{1}{4}+\frac{1}{3}+2]$

$4-\frac{8}{3}+8-[\frac{31}{12}]$

$\frac{27}{4}$

f) $\int_{0}^{1} (x^{99}+1)dx$

$\frac{1}{100}x^{100}+x\Big]_{0}^{1}$

$\frac{1}{100}+1-0$

1.01

g) $\int_2^3 \frac{1}{t^2}dt \rightarrow \int_2^3 t^{-2} \rightarrow -t^{-1}\Big]_2^3$

$-\frac{1}{3}-\left(-\frac{1}{2}\right)$

$-\frac{1}{3}+\frac{1}{2} = \boxed{\frac{1}{6}}$

h) $\int_1^4 (x-\sqrt{x})dx \rightarrow \frac{1}{2}x^2-\frac{2}{3}x^{3/2}\Big]_1^4$

$\frac{16}{2}-\frac{2}{3}(4)^{3/2}-\left[\frac{1}{2}-\frac{2}{3}\right]$

$\frac{16}{2}-\frac{16}{3}+\frac{1}{6} = \boxed{\frac{17}{6}}$

i) $\int_0^1 \sqrt[4]{x^5}dx \rightarrow \int_0^1 x^{5/4} \rightarrow \frac{4}{9}x^{9/4}\Big]_0^1$

$\frac{4}{9}(1)^{9/4}-0 = \boxed{\frac{4}{9}}$

j) $\int_1^8 \frac{2}{\sqrt[3]{x}}dx \rightarrow \int_1^8 2x^{-1/3} \rightarrow 3x^{2/3}\Big]_1^8$

$3(8)^{2/3}-3(1)^{2/3}$

$3(4)-3$

$12-3$

$\boxed{9}$

k) $\int_1^2 \frac{x^3+x^2+1}{x^3}dx \rightarrow \int_1^2 1+\frac{1}{x}+\frac{1}{x^3}dx$

$x+\ln|x|-\frac{1}{2}x^{-2}\Big]_1^2$

$2+\ln 2-\frac{1}{8}-\left[1+\cancel{\ln 1}^{0}-\frac{1}{2}\right]$

$\frac{15}{8}+\ln 2-\frac{1}{2}$

$\boxed{\frac{11}{8}+\ln 2}$

l) $\int_1^4 \left(\frac{\sqrt{x}+1}{x}\right)dx \rightarrow \int_1^4 \left(\frac{1}{\sqrt{x}}+\frac{1}{x}\right)dx$

$\int_1^4 \left(x^{-\frac{1}{2}}+\frac{1}{x}\right)dx$

$2x^{\frac{1}{2}}+\ln x\Big]_1^4$

$2(4)^{\frac{1}{2}}+\ln 4-\left[2+\cancel{\ln 1}^{0}\right]$

$4-2+\ln 4$

$\boxed{2+\ln 4}$

m) $\int_0^{64} \sqrt{y}(1+\sqrt[3]{y})dy \rightarrow \int_0^{64} \sqrt{y} + \sqrt[6]{y^5}\, dy$

$\frac{2}{3}y^{3/2} + \frac{6}{11}y^{11/6}\Big]_0^{64}$

$\frac{2}{3}(64)^{3/2} + \frac{6}{11}(64)^{11/6} - 0$

$\frac{1024}{3} + \frac{12288}{11} = \boxed{\frac{48128}{33}}$

n) $\int_0^{\frac{\pi}{2}} (8x + \cos x)dx$

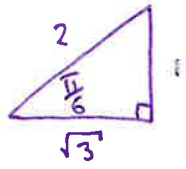
$4x^2 + \sin x\Big]_0^{\frac{\pi}{2}}$

$4\left(\frac{\pi}{2}\right)^2 + \sin\frac{\pi}{2} - [4(0)^2 + \sin 0]$

$\pi^2 + 1 - 0 + 0$

$\boxed{\pi^2 + 1}$

o) $\int_0^{\frac{\pi}{6}} (\sec x \tan x)\, dx$

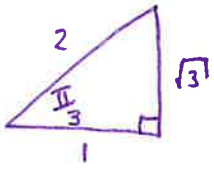


$\sec x\Big]_0^{\pi/6}$

$\sec\frac{\pi}{6} - \sec 0$

$\frac{2}{\sqrt{3}} - 1 \rightarrow \boxed{\frac{2}{\sqrt{3}} - 1}$

p) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (3\sin\theta - \sec^2\theta)\, d\theta$



$3(-\cos\theta) - \tan\theta\Big]_{\pi/4}^{\pi/3}$

$-3\cos\frac{\pi}{3} - \tan\frac{\pi}{3} - \left[-3\cos\frac{\pi}{4} - \tan\frac{\pi}{4}\right]$

$-3\left(\frac{1}{2}\right) - \sqrt{3} + 3\left(\frac{1}{\sqrt{2}}\right) - 1$

$\boxed{-\frac{3}{2} - \sqrt{3} + \frac{3}{\sqrt{2}} - 1}$

2. Find the following general indefinite integrals.

a) $\int (x^5 - 2x^3 + 4)dx$

$\boxed{\frac{1}{6}x^6 - \frac{1}{2}x^4 + 4x + C}$

b) $\int x^2\sqrt{x}\, dx$

$\int x^2 \cdot x^{\frac{1}{2}}\, dx$

$\int x^{5/2} dx \rightarrow \boxed{\frac{2}{7}x^{\frac{7}{2}} + C}$

c)
$\int \left(t + \frac{2}{t}\right) dt$

$\frac{1}{2}t^2 + 2\ln|t| + C$

d)
$\int (1 + \sqrt{x})^2 dx$

$\int (1 + 2\sqrt{x} + x)\, dx$

$x + \frac{4}{3}x^{3/2} + \frac{1}{2}x^2 + C$

e)
$\int \frac{x - 5}{\sqrt[4]{x}} dx \rightarrow \int \frac{x^{4/4}}{x^{1/4}} - 5x^{-1/4}\, dx$

$\int x^{3/4} - 5x^{-1/4}\, dx$

$\frac{4}{7}x^{7/4} - \frac{20x^{3/4}}{3} + C$

f)
$\int (\cos\theta + \sin\theta) d\theta$

$\sin\theta - \cos\theta + C$

g)
$\int (5x^4 - 2\csc x \cot x) dx$

$x^5 - 2\csc x + C$

h)
$\int (2\csc^2 x + 1) dx$

$2(-\cot x) + x + C$

$x - 2\cot x + C$